AE 429 - Aircraft Performance and Flight Mechanics

Power Required and Available

Power

- Power is energy per unit time

$$POWER = P \equiv \frac{FORCE * DISTANCE}{TIME} = FV_\infty \text{ if } V_\infty \text{ is constant}$$

- For an airplane in level, unaccelerated flight, power $P_R = T_R V_\infty$ required is

$$P_R = T_R V_\infty = \frac{W}{C_L / C_D} V_\infty \text{ but } V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

  - So

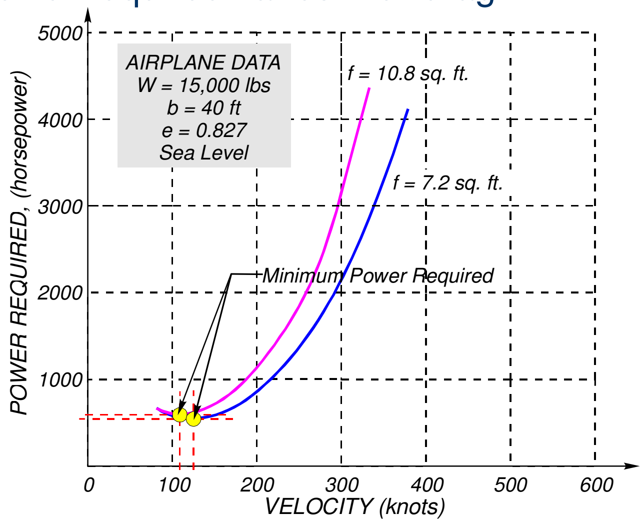
$$P_R = \frac{W}{C_L / C_D} \sqrt{\frac{2W}{\rho_\infty S C_L}} = \left(\sqrt{\frac{2W^3}{\rho_\infty S}} \right) \left(\sqrt{\frac{C_D^2}{C_L^3}} \right)$$

  - That is,

$$P_R \propto \frac{1}{C_L^{3/2} / C_D}$$

Power

- How power required varies with drag



Power

- Let's analyze minimum power required
  - As we observed before for a parabolic drag polar

$$P_R = T_R V_\infty = DV_\infty = q_\infty S\left(C_{D0} + \frac{C_L^2}{\pi eAR}\right)V_\infty$$

$$P_R = \underbrace{q_\infty SC_{D0}V_\infty}_{\text{Parasite power required}} + \underbrace{q_\infty SV_\infty \frac{C_L^2}{\pi eAR}}_{\text{Induced power required}}$$

  - In level, unaccelerated flight

$$C_L = \frac{2L}{\rho_\infty V_\infty^2 S} = \frac{2W}{\rho_\infty V_\infty^2 S};$$

$$P_R = \frac{1}{2}\rho_\infty V_\infty^3 SC_{D_0} + \frac{1}{2}\rho_\infty V_\infty^3 S\frac{C_L^2}{\pi eAR} = \frac{1}{2}\rho_\infty V_\infty^3 SC_{D_0} + \frac{1}{2}\rho_\infty V_\infty^3 S\frac{4W^2}{\rho_\infty^2 V_\infty^4 S^2 \pi eAR}$$

$$P_R = \frac{1}{2}\rho_\infty V_\infty^3 SC_{D_0} + \frac{2W^2}{\rho_\infty V_\infty S\pi eAR}$$

Power

- Still looking for the minimum P_R point
  - Differentiating this last expression for P_R and setting this result to zero

$$\frac{dP_R}{dV_\infty} = \frac{3}{2}\rho_\infty V_\infty^2 S C_{D_0} - \frac{2W^2}{\rho_\infty V_\infty^2 S \pi e AR}$$
$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D_0} - \frac{4W^2}{3\rho_\infty^2 V_\infty^4 S^2 \pi e AR}\right)$$
$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D_0} - \frac{C_L^2}{3\pi e AR}\right)$$
$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D_0} - \frac{C_{D_i}}{3}\right)$$

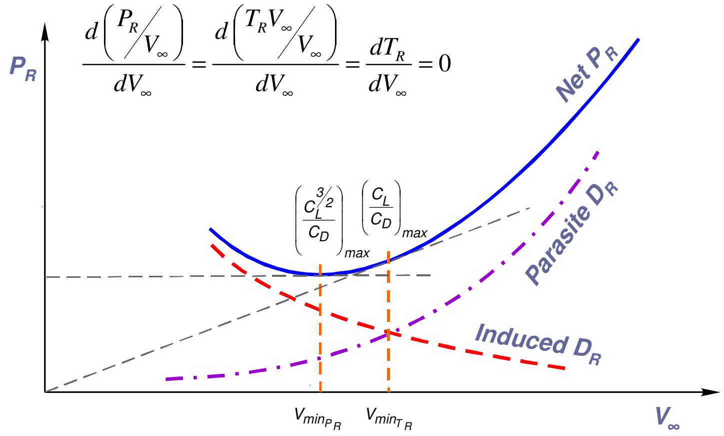
  - Thus, the relationship between C_{D0} and C_{Di} at minimum power required is

$$C_{D_0} = \frac{C_{D_i}}{3}$$

Power

- In level flight, $D = T_R$

$$\frac{d\left(P_R/V_\infty\right)}{dV_\infty} = \frac{d\left(T_R V_\infty/V_\infty\right)}{dV_\infty} = \frac{dT_R}{dV_\infty} = 0$$



Altitude effects on P_R

- Power required and velocity for level unaccelerated flight
  - At sea level

$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}} \Rightarrow P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

  - At altitude

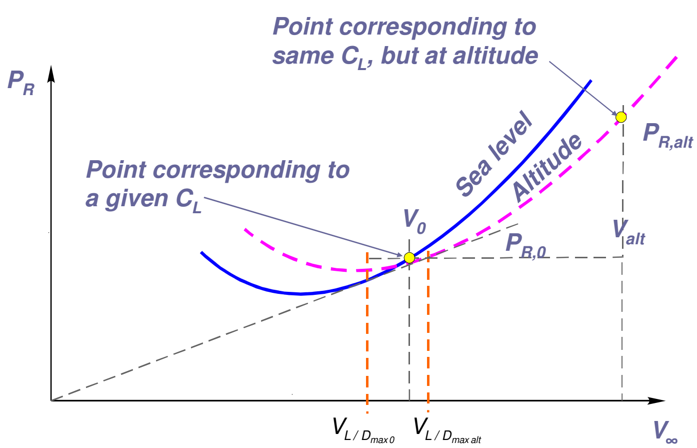
$$V_{alt} = \sqrt{\frac{2W}{\rho S C_L}} \Rightarrow P_{R,alt} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

  - Dividing each altitude equation by the s.l. one

$$V_{alt} = V_0\sqrt{\frac{\rho_0}{\rho}} = V_0\sqrt{\frac{1}{\sigma}} \quad \Rightarrow \quad P_{R,alt} = P_{R,0}\sqrt{\frac{\rho_0}{\rho}} = P_{R,0}\sqrt{\frac{1}{\sigma}}$$

  - On a power required curve, any point associated with a given C_L at sea level, moves to the right and up as altitude is increased

Altitude effects on P_R



Weight effects on P_R

$$P_R = \frac{W}{C_L/C_D}\sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$= W^{3/2}\sqrt{\frac{2}{\rho_\infty S}}\left(\sqrt{\frac{1}{C_L^{3/2}/C_D}}\right)$$

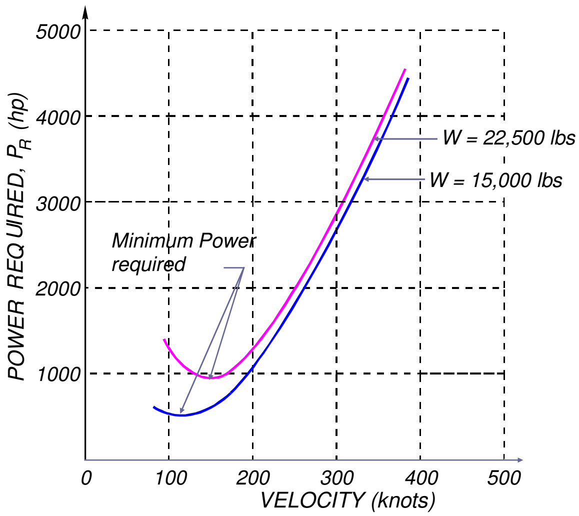
$$V = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

– Define

$$\omega \equiv \frac{W}{W_{std}}$$

$$V = V_{std}\sqrt{\omega}$$

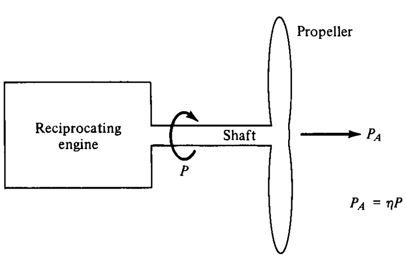
$$P_R = P_{R_{std}}\sqrt{\omega^3}$$

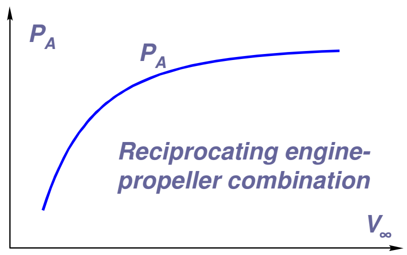


Power available P_A

- Propeller efficiency is important for any powerplant driving a propeller $P_A = \eta P$
  - η is propeller efficiency
  - P is brake horsepower
  - P_A is available horsepower < P

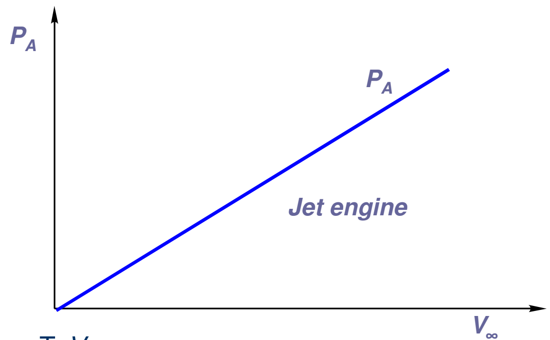
1 hp = 550 ft – lb / sec = 746 watts





Power available P_A

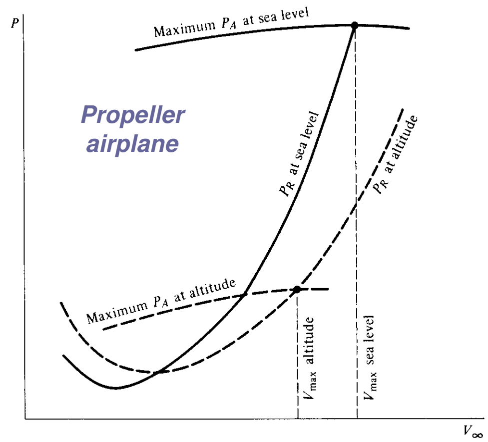
- Power available from a jet engine has quite a different shape



- $P_A = T_A V_\infty$

Power available P_A

- Altitude effects on P_A are kin to P_R
  - Propeller aircraft



Power available P_A

- Similarly for a jet-powered airplane