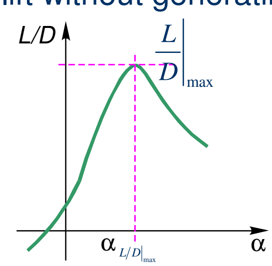
AE 429 - Aircraft Performance and Flight Mechanics

Steady Flight

Aerodynamic efficiency

- Lift/drag ratio is a measure of aerodynamic efficiency
  - It indicates the ability to produce lift without generating excessive drag

MAX L/D	VEHICLE
0.3-0.4	GEMINI
12.8	T-38
26	SAILPLANE



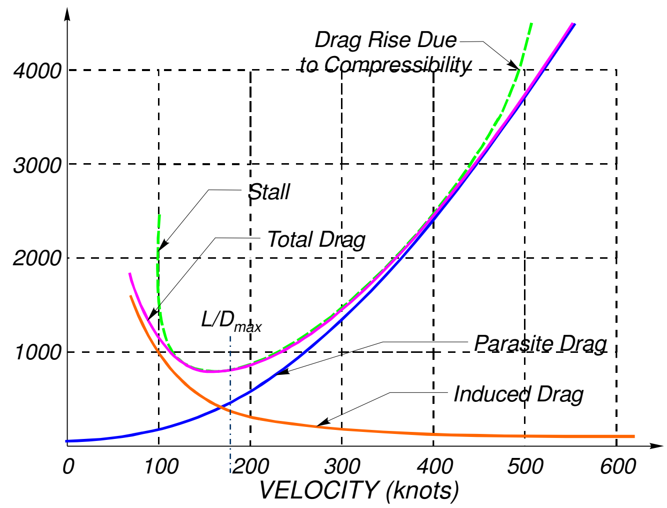
- Character of drag

$$C_L = \frac{2L}{\rho V^2 S} = \frac{2W}{\rho V^2 S};\ C_D = C_{D_0} + \frac{C_L^2}{\pi eAR} = C_{D_0} + KC_L^2 = C_{D_0} + \frac{4W^2}{\rho^2 V^4 S^2 \pi eAR}$$

$$D = \frac{1}{2}\rho V^2 SC_{D_0} + \frac{2W^2}{\rho V^2 S\pi eAR} = \underbrace{\frac{1}{2}\rho V^2 SC_{D_0}}_{\text{Zero lift drag}} + \underbrace{\frac{1}{2}\rho V^2 SKC_L^2}_{\text{Drag due to lift}}$$

Aerodynamic efficiency

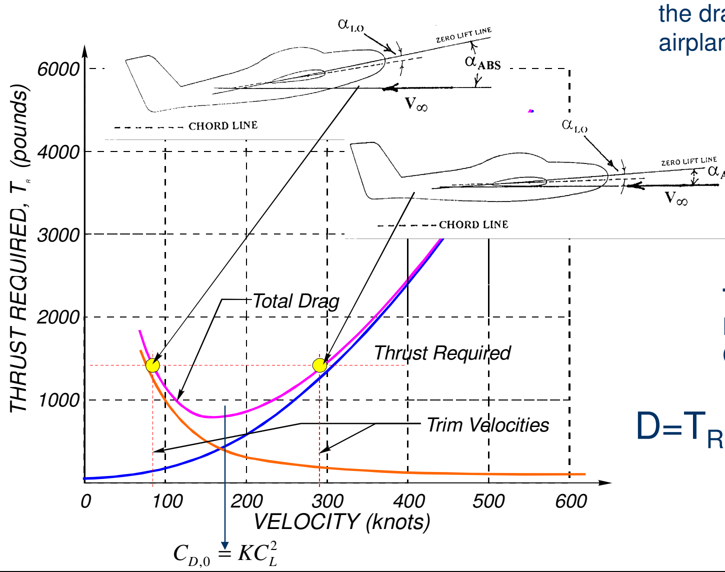
Parasite and induced drag



Aerodynamic efficiency

In level flight, D = thrust required T_R

To maintain certain speed and altitude, enough thrust must be generated to overcome the drag and to keep the airplane going.



Thrust Required Curve

$D=T_R = f(h,V,W)$

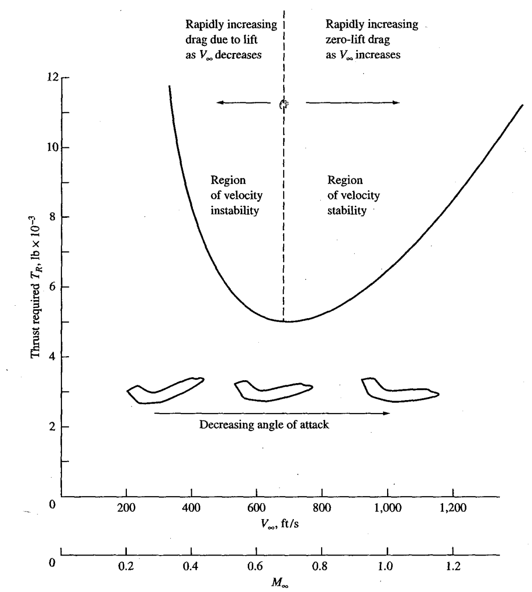


Figure 5.4 Thrust required curve for the Gulfstream IV at the conditions for Example 5.1, illustrating the regions of velocity instability and stability, and the direction of decreasing angle of attack with increasing velocity. Altitude = 30,000 ft; W = 73,000 lb.

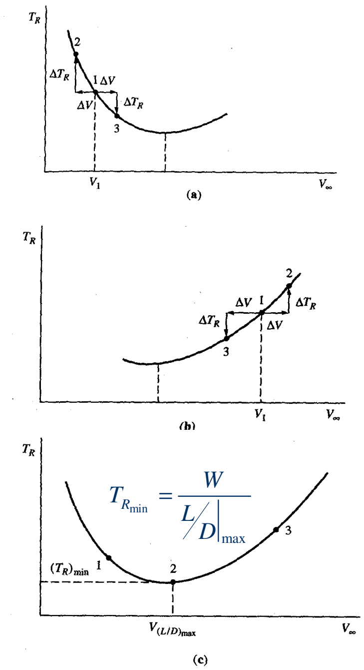


Figure 5.6 (a) The mechanism of velocity instability. (b) The mechanism of velocity stability. (c) Maximum T_R occurs at maximum lift-to-drag ratio, point 2. Points 1, 2, and 3 correspond to points 1, 2, and 3, respectively, in Fig. 5.7.

Drag = T_R

$$D = qSC_D = \frac{1}{2}\rho V^2 S\left[C_{D_0} + KC_L^2\right] = \frac{1}{2}\rho V^2 S\left[C_{D_0} + \frac{4KW^2}{\left(\rho V^2 S\right)^2}\right]$$

Since for L=W $\quad C_L = \dfrac{2W}{\left(\rho V^2 S\right)}$

$$D = \frac{1}{2}\rho V^2 SC_{D_0} + \frac{2SK}{\rho V^2}\left(\frac{W}{S}\right)^2 \qquad \text{=f(h,V,W)}$$

$$K = \frac{1}{e\pi AR}$$

Important parameters:

Thrust to Weight ratio T_R/W
Wing loading W/S,
Polar drag (C_{D0} and K)

Aerodynamic efficiency

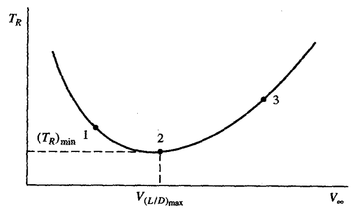
- Relationship of C_{D0} and C_{Di} for L/D_{max}

$$T_R = C_{D_0} q_\infty S + \frac{W^2}{q_\infty S \pi e AR}$$

  - Differentiating with respect to q_∞: $$\frac{dT_R}{dq_\infty} = C_{D_0} S - \frac{W^2}{q_\infty^2 S \pi e AR} = 0$$
  - Solving for C_{D0} $$C_{D_0} = \frac{W^2}{q_\infty^2 S^2 \pi e AR} = \frac{KW^2}{q_\infty^2 S^2}$$
  - And observing that $$\frac{W^2}{q_\infty^2 S^2} = \left(\frac{W}{q_\infty S}\right)^2 = C_L^2$$

– Thus, at max L/D

$$C_{D_0} = \frac{C_L^2}{\pi e AR} = C_{D_i}$$



(c)

Figure 5.6 (a) The mechanism of velocity instability. (b) The mechanism of velocity stability. (c) Maximum T_R occurs at maximum lift-to-drag ratio, point 2. Points 1, 2, and 3 correspond to points 1, 2, and 3, respectively, in Fig. 5.7.

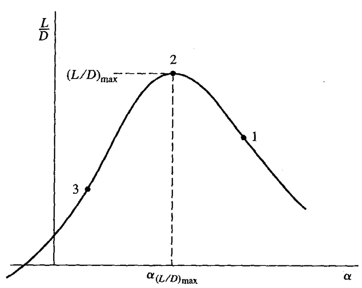


Figure 5.7 Schematic of the variation of lift-to-drag ratio for a given airplane as a function of angle of attack. Points 1, 3, and 3 correspond to points 1, 2, and 3, respectively, in Fig. 5.6c.

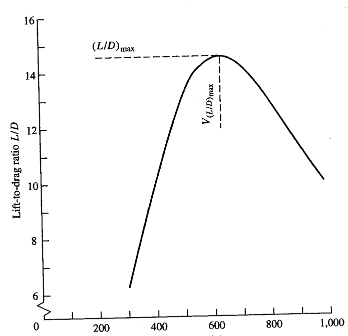


Figure 5.8 Variation of L/D with velocity for the Gulfstream IV at the conditions for Example 5.1. Altitude = 30,000 ft; W = 73,000 lb.

Equation (5.12) can be used to find the flight velocities for a given value of T_R. Writing Eq. (5.12) in terms of the dynamic pressure $q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$ and noting that $D = T_R$, we obtain

$$T_R = q_\infty S C_{D,0} + \frac{KS}{q_\infty}\left(\frac{W}{S}\right)^2 \tag{5.13}$$

Multiplying Eq. (5.13) by q_∞, and rearranging, we have

$$q_\infty^2 S C_{D,0} - q_\infty T_R + KS\left(\frac{W}{S}\right)^2 = 0 \tag{5.14}$$

Note that, being a quadratic equation in q_∞, Eq. (5.14) yields two roots, that is, two solutions for q_∞. Solving Eq. (5.14) for q_∞ by using the quadratic formula results in

$$\begin{aligned} q_\infty &= \frac{T_R \pm \sqrt{T_R^2 - 4SC_{D,0}K(W/S)^2}}{2SC_{D,0}} \\ &= \frac{T_R/S \pm \sqrt{(T_R/S)^2 - 4C_{D,0}K(W/S)^2}}{2C_{D,0}} \end{aligned} \tag{5.15}$$

By replacing q_∞ with $\frac{1}{2}\rho_\infty V_\infty^2$, Eq. (5.15) becomes

$$V_\infty^2 = \frac{T_R/S \pm \sqrt{(T_R/S)^2 - 4C_{D,0}K(W/S)^2}}{\rho_\infty C_{D,0}} \tag{5.16}$$

The parameter T_R/S appears in Eq. (5.16); analogous to the wing loading W/S, the quantity T_R/S is sometimes called the *thrust loading*. However, in the hierarchy of parameters important to airplane performance, T_R/S is not quite as fundamental as the wing loading W/S or the thrust-to-weight ratio T_R/W (as will be discussed in the next section). Indeed, T_R/S is simply a combination of T_R/W and W/S via

$$\frac{T_R}{S} = \frac{T_R}{W}\frac{W}{S} \tag{5.17}$$

Substituting Eq. (5.17) into (5.16) and taking the square root, we have our final expression for velocity:

$$\boxed{V_\infty = \left[\frac{(T_R/W)(W/S) \pm (W/S)\sqrt{(T_R/W)^2 - 4C_{D,0}K}}{\rho_\infty C_{D,0}}\right]^{1/2}} \tag{5.18}$$

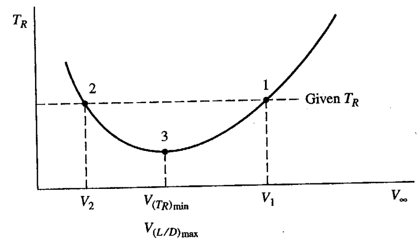


Figure 5.9 At a given T_R larger than the minimum value, there are two corresponding velocities, the low velocity V_2 and the high velocity V_1.

When the discriminant in Eq. (5.18) equals zero, then only one solution for V_∞ is obtained. This corresponds to point 3 in Fig. 5.9, namely, the point of minimum T_R. That is, in Eq. (5.18) when

$$\left(\frac{T_R}{W}\right)^2 - 4C_{D,0}K = 0 \qquad \textbf{[5.19]}$$

then the velocity obtained from Eq. (5.18) is

$$V_{(T_R)_{\min}} = \left[\frac{1}{\rho_\infty C_{D,0}}\left(\frac{T_R}{W}\right)_{\min}\frac{W}{S}\right]^{1/2} \qquad \textbf{[5.20]}$$

The value of $(T_R/W)_{\min}$ is given by Eq. (5.19) as

$$\left(\frac{T_R}{W}\right)^2_{\min} = 4C_{D,0}K$$

or

$$\boxed{\left(\frac{T_R}{W}\right)_{\min} = \sqrt{4C_{D,0}K}} \qquad \textbf{[5.21]}$$

Substituting Eq. (5.21) into Eq. (5.20), we have

$$V_{(T_R)_{\min}} = \left(\frac{\sqrt{4C_{D,0}K}}{\rho_\infty C_{D,0}}\frac{W}{S}\right)^{1/2}$$

or

$$\boxed{V_{(T_R)_{\min}} = V_{(L/D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}}\frac{W}{S}\right)^{1/2}} \qquad \textbf{[5.22]}$$

In Eq. (5.22), by stating that $V_{(T_R)_{\min}} = V_{(L/D)_{\max}}$, we are recalling that the velocity for minimum T_R is also the velocity for maximum L/D, as shown in Fig. 5.6. Indeed, since $T_R = D$ and $L = W$ for steady, level flight, Eq. (5.21) can be written as

$$\left(\frac{D}{L}\right)_{\min} = \sqrt{4C_{D,0}K} \qquad \textbf{[5.23]}$$

Since the minimum value of D/L is the reciprocal of the maximum value of L/D, then Eq. (5.23) becomes

$$\boxed{\left(\frac{L}{D}\right)_{\max} = \frac{1}{\sqrt{4C_{D,0}K}}} \qquad \textbf{[5.24]}$$

Effect of weight on T_R

- Drag

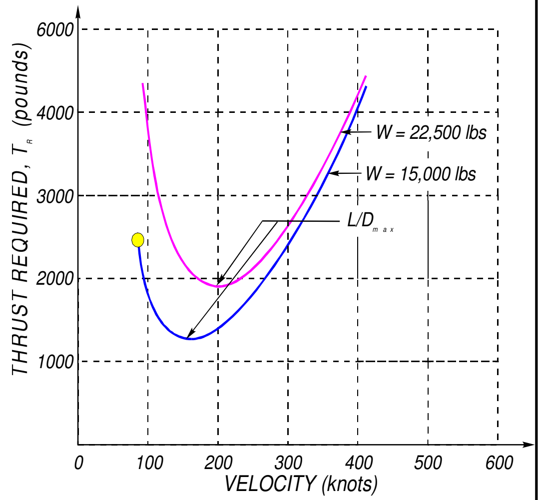
$$D = \frac{C_{D_0} \rho V^2 S}{2} + \frac{2W^2}{\rho V^2 S \pi e AR}$$

  – Increasing aircraft weight by Δw

$$D = \frac{C_{D_0} \rho V^2 S}{2} + \frac{2(W + \Delta W)^2}{\rho V^2 S \pi e AR}$$

  – Then

$$D = \frac{C_{D_0} \rho V^2 S}{2} + \frac{2W^2}{\rho V^2 S \pi e AR} + \frac{2\left(2W\Delta W + \Delta W^2\right)}{\rho V^2 S \pi e AR}$$



Effect of altitude on T_R

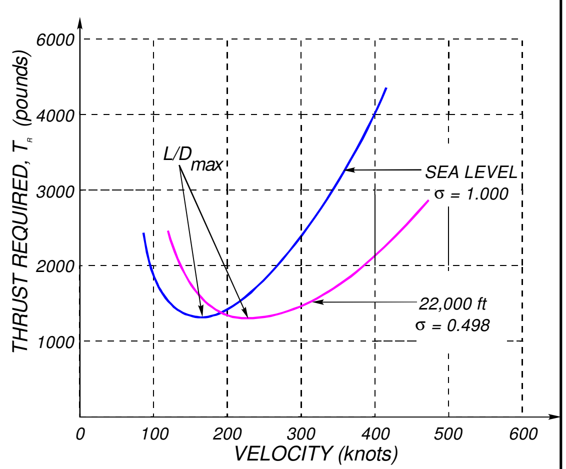
- Multiplying the drag equation by ρ/ρ

$$D = \frac{C_{D_0} \rho_0 V^2 S}{2} + \frac{2W^2}{\rho_0 V^2 S \pi e AR}$$

  – Does not change $T_{R\,min}$

  – Minimum drag occurs at a higher V_∞

  – T_R curve opens up and shifts to the right

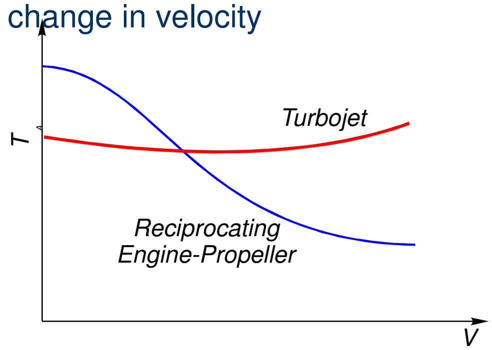


Thrust Available

- Thrust required is dictated by the airframe
  - Shape (airfoil, planform, fuselage, empennage)
  - Size (surface area, frontal area, airfoil)
  - Configuration (clean, gear down, flaps down)
- Thrust available is dictated by the powerplant (engine type, prop)
  - Reciprocating engine-propeller combination
  - Turbojet
  - Turboprop (turgine engine and propeller)
  - Turbofan
  - Ducted propeller
  - Rocket

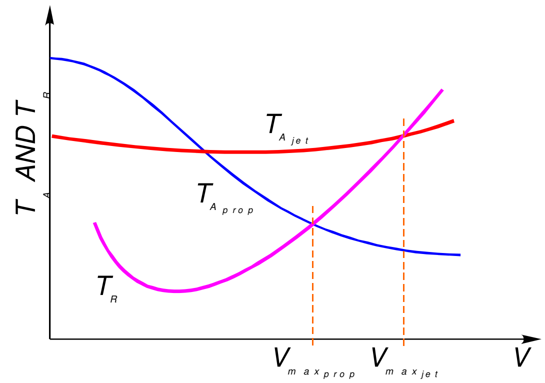
Thrust Available

- Accelerating a mass of air gives T_A
  - Propeller operates on a large volume of air and imparts a small change in velocity
  - A turbojet operates on a smaller mass of air and imparts a larger change in velocity



Thrust Available

- T_A and T_R curve intersections give V_{max}
  - This statement is true for only a single altitude, since T_A and T_R are both altitude dependent
  - T_A goes down as altitude increases



Importance of the Ratios C_L/C_D, $C_L^{3/2}/C_D$, $C_L^{1/2}/C_D$

$\left.\dfrac{C_L}{C_D}\right|_{max}$ ⇒ Max Range for reciprocating engine/propeller airplanes
Max Endurance for jet-propelled airplanes

$\left.\dfrac{C_L^{3/2}}{C_D}\right|_{max}$ ⇒ Max Endurance for reciprocating engine/propeller

$\left.\dfrac{C_L^{1/2}}{C_D}\right|_{max}$ ⇒ Max Range for jet-propelled airplanes

Aerodynamic Relations Associated with Maximum C_L/C_D, $C_L^{3/2}/C_D$, and $C_L^{1/2}/C_D$

$$\frac{L}{D} = \frac{C_L}{C_D} = \frac{C_L}{C_{D,0} + KC_L^2}$$

$$\frac{d(C_L/C_D)}{dC_L} = 0 \Rightarrow \boxed{C_{D,0} = KC_L^2}$$

$$\boxed{\left(\frac{L}{D}\right)_{\max} = \left(\frac{C_L}{C_D}\right)_{\max} = \sqrt{\frac{1}{4C_{D,0}K}}}$$

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$\boxed{V_{(L/D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}\frac{W}{S}}\right)^{1/2}}$$

$$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D,0} + KC_L^2}$$

$$\frac{d\left(C_L^{3/2}/C_D\right)}{dC_L} = 0 \Rightarrow \boxed{C_{D,0} = \tfrac{1}{3}KC_L^2}$$

$$\boxed{\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{1}{4}\left(\frac{3}{KC_{D,0}^{1/3}}\right)^{3/4}}$$

$$\boxed{V_{(C_L^{3/2}/C_D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}\frac{W}{S}}\right)^{1/2}}$$

$$V_{(C_L^{3/2}/C_D)_{\max}} = \left(\frac{1}{3}\right)^{1/4} V_{(L/D)_{\max}}$$

Aerodynamic Relations Associated with Maximum C_L/C_D, $C_L^{3/2}/C_D$, and $C_L^{1/2}/C_D$

$$(C_L^{1/2}/C_D)_{\max} \Rightarrow \boxed{C_{D,0} = 3KC_L^2}$$

$$\boxed{\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{3}{4}\left(\frac{1}{3KC_{D,0}^3}\right)^{1/4}}$$

$$\boxed{V_{(C_L^{1/2}/C_D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{3K}{C_{D,0}}\frac{W}{S}}\right)^{1/2}}$$

$$V_{(C_L^{1/2}/C_D)_{\max}} = 3^{1/4} V_{(L/D)_{\max}}$$

$$V_{(C_L^{3/2}/C_D)_{\max}} < V_{(C_L/C_D)_{\max}} < V_{(C_L^{1/2}/C_D)_{\max}}$$

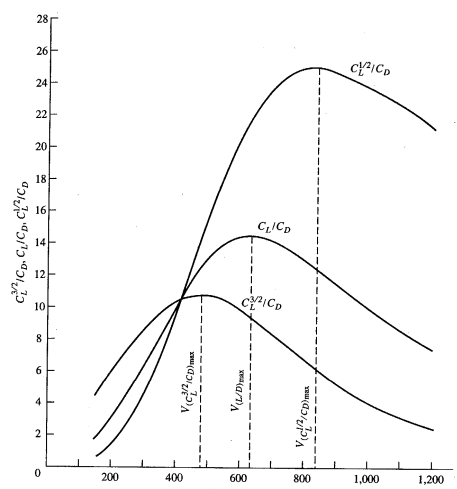


Figure 5.11 Variation of $C_L^{3/2}/C_D$, C_L/C_D, and $C_L^{1/2}/C_D$ versus velocity for the Gulfstream IV at the conditions set in Example 5.1. Altitude = 30,000 ft, W = 73,000 lb.