AE 429 - Aircraft Performance and Flight Mechanics

Equations of Motion

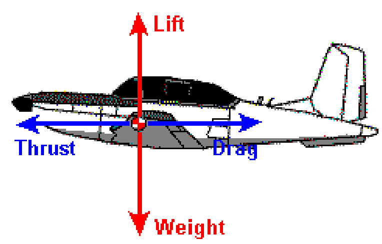
Performance equations

- Forces considered
  - Lift, drag, thrust, and weight
    - Drag polar known
    - Weight known
    - Thrust available known
  - Lift is perpendicular to flight path
  - Drag is parallel to flight path
  - Thrust is inclined wrt the flight path
  - Weight acts vertically (toward center of earth)
- Assumptions
  - The aircraft is a rigid body
  - Rotational motions are not important
  - The entire mass of the vehicle can be treated as though it were concentrated at a point

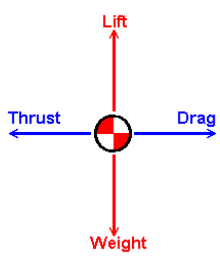
The four forces of flight are lift, drag, thrust, and weight.

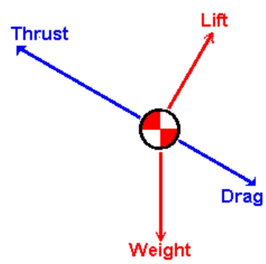
Lift is the upward force created by the wings moving through the air that sustains the airplane in flight. Lift operates to overcome weight. It must be equal to or greater than the weight of the object in flight and acting in the opposite direction. Lift can be increased by increasing the forward speed of the aircraft or by increasing the angle of attack

Drag is the resistance of the airplane to forward motion. It is directly opposed to thrust and is caused by the resistance of air.



The four forces of flight are lift, drag, thrust, and weight.





Thrust is the force exerted by the engine and its propeller(s). It pushes the air backward with the object of causing movement of the airplane in the forward direction.

Weight is the downward force due to the weight of the airplane and its load. It is directly opposed to lift

An aircraft is in a state of equilibrium when the thrust and drag are equal and opposite. It will continue to move forward at the same uniform speed. If thrust or drag becomes greater than the opposite force, the aircraft loses its state of equilibrium. If thrust is greater than drag, the aircraft will accelerate. If drag is greater than thrust, the aircraft will lose speed and eventually descend.

When lift and weight are equal and opposite, the airplane is in a state of equilibrium. If lift is greater than weight, the aircraft will climb. If weight is greater than lift, the airplane will descend.

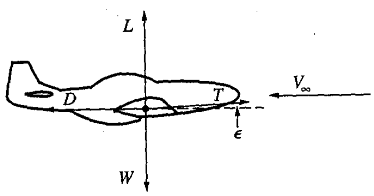


Figure 4.1 Four forces of flight—lift, drag, thrust, and weight. Illustration shows the case of a horizontal flight path. *Note:* For ordinary flight, lift and weight are much larger than thrust and drag; that is, for typical airplanes, $L/D \approx 10$ to 15.

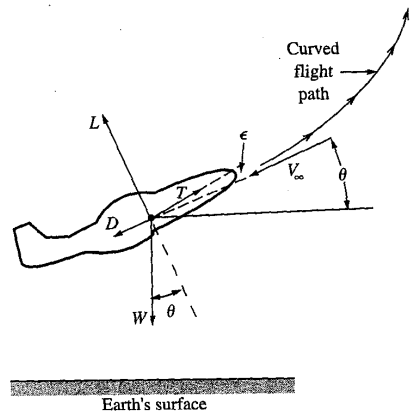


Figure 4.2 Climbing flight.

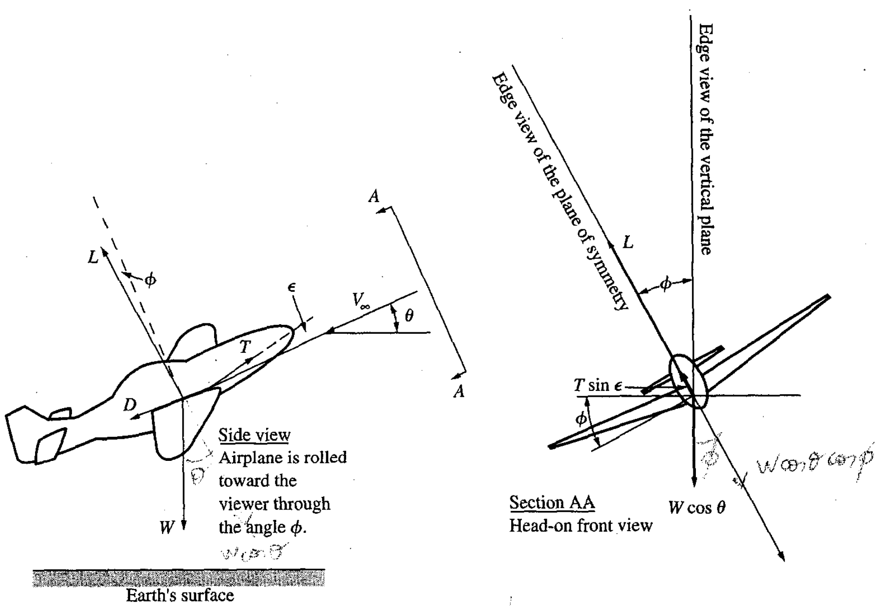


Figure 4.3 Airplane in climbing flight and rolled through angle ϕ.

Equations Of Motion

In the direction of the flight path

$$m\frac{dV_\infty}{dt} = T\cos\varepsilon - D - W\sin\theta$$

Perpendicular to the flight path

$$m\frac{V_\infty^2}{r_1} = L\cos\phi + T\sin\varepsilon\cos\phi - W\cos\theta$$

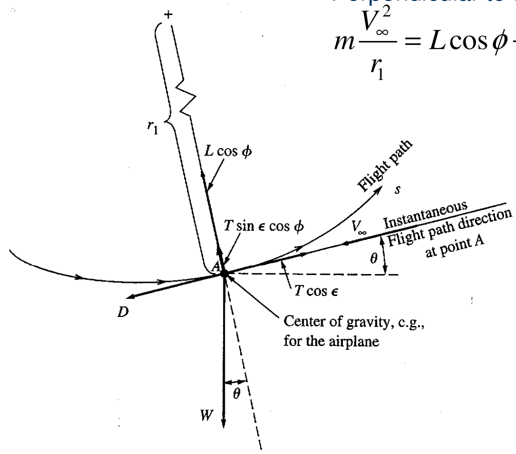


Figure 4.4 Forces projected into the plane formed by the local free-stream velocity V_∞ and the vertical (perpendicular to the surface of the earth).

Perpendicular to flight path in the horizontal plane

$$m\frac{(V_\infty\cos\theta)^2}{r_2} = L\sin\phi + T\sin\varepsilon\sin\phi$$

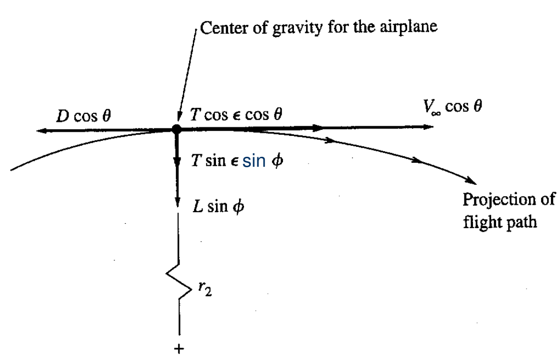


Figure 4.5 Forces projected into the horizontal plane parallel to the flat earth.

Force balance

- For an airplane following a curved flight path (ϕ=0)

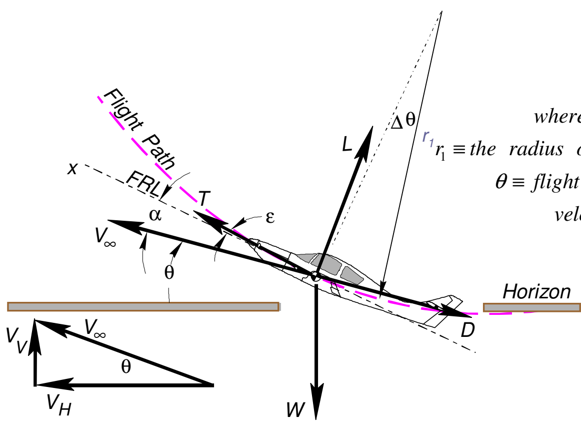
$$T\cos\varepsilon - D - W\sin\theta = m\frac{dV_\infty}{dt}$$

$$L + T\sin\varepsilon - W\cos\theta = m\frac{V_\infty^2}{r_1}$$

where: $m \equiv$ *the mass of the airplane*

$r_1 \equiv$ *the radius of curvature of the flight path*

$\theta \equiv$ *flight path angle (angle between the velocity direction and the horizon)*



Force balance

- Now consider unaccelerated level flight
  - That is: $\dfrac{dV_\infty}{dt} = 0$ and $r_1 \Rightarrow \infty$
  - Then $T\cos\varepsilon - D = 0$ and $L + T\sin\varepsilon - W = 0$
  - And, if ε is small, L = W and T = D
  - We could also write lift and drag in terms of nondimensional coefficients

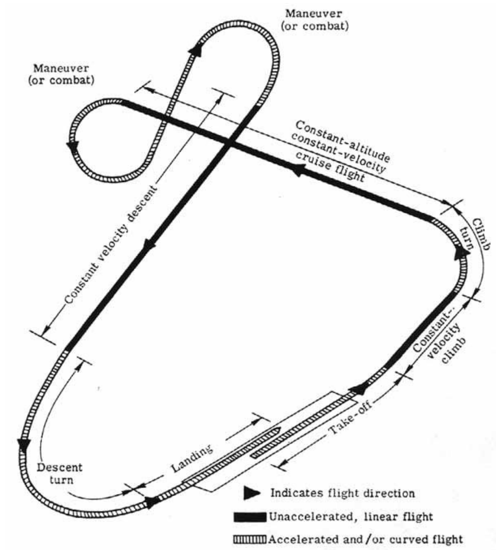
$$T = D = qSC_D \quad \text{and} \quad L = W = qSC_L$$

  - Dividing thrust by lift and solving for thrust for level unaccelerated flight

$$\frac{T}{W} = \frac{D}{L} = \frac{C_D}{C_L} \quad \text{or} \quad T_R = \frac{W}{C_L/C_D} = \frac{W}{L/D}$$

  - The components of drag are: $C_D = C_{D0} + C_{D_i}$

Various flight conditions encountered by an airplane

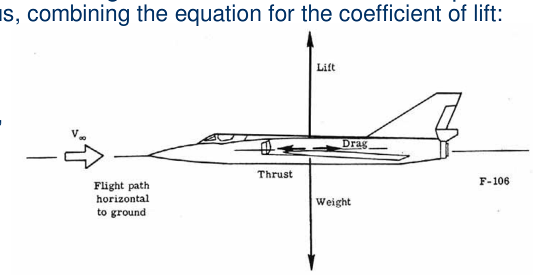


Unaccelerated linear flight — Class 1 motion

- Although straight and level flight may occur only over a small section of the total flight, it is very important since it is usually considered the standard condition in the design of an airplane.
- For straight and level flight, the flight path is horizontal to the Earth's surface and, for simplicity, it is assumed that the thrust always acts along this horizontal plane. For the flight to be horizontal, or at a constant altitude, lift must equal weight. To fly at a constant velocity (unaccelerated), the thrust must equal the drag.
- The velocity of the airplane must be sufficient to produce a lift equivalent to the weight, and there is a range of velocities over which the plane may fly straight and level. Thus, combining the equation for the coefficient of lift:

$C_L = L/qS$

L = Total lift on the wing,
S = wing area, and
q = dynamic pressure



Unaccelerated linear flight — Class 1 motion

- with the condition that Lift = Weight, one obtains

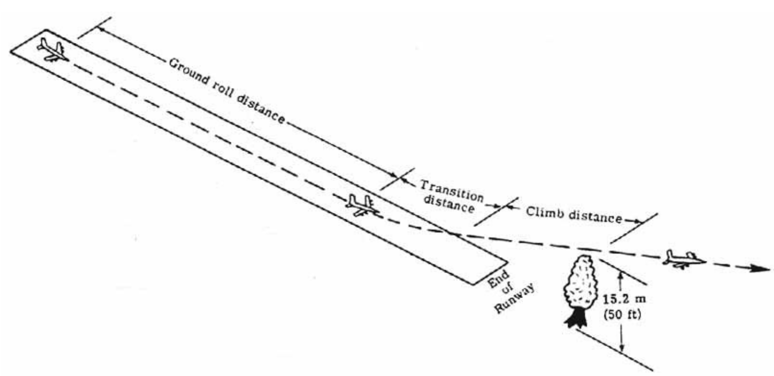
$$W = 1/2\rho V^2 C_L S$$

- If it is assumed that the weight, air density ρ, and wing area S are constant, one observes that as the velocity V increases, the wing lift coefficient C_L decreases, which may be accomplished by a decrease in the wing angle of attack. Minimum flying speed for straight and level flight occurs when the wing is operating at $C_{L,max}$, that is, near the stall angle. The maximum flying speed for straight and level flight is limited by the thrust available from the engine. This condition also requires a small value of C_L and hence a small angle of attack.

- Thus, at low speeds, to fly straight and level, the airplane angle of attack is large whereas for high speeds the airplane angle of attack is small.

Class 2 Motion

- Accelerated motion and curved flight or Class 2 motion, is considered specifically for the cases of takeoff, landing, and the constant-altitude banked turn.



- takeoff distance needed consists of three parts: (1) the ground-roll distance, (2) the transition distance, and (3) the climb out distance over an obstacle