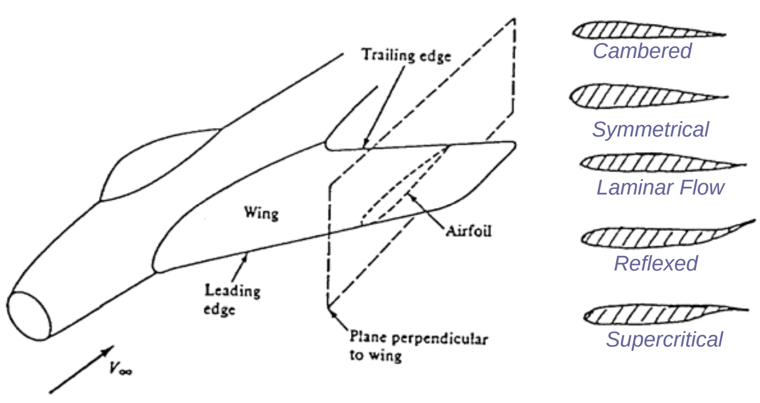
AE 429 - Aircraft Performance and Flight Mechanics

Aerodynamics of the Airplane

Airfoils

- An airfoil is a section of a wing **(or a fin, or a stabilizer, or a propeller, etc.)**



Airfoil Nomenclature

- chord line
  straight line connecting the LE and TE
- mean camber line
  locus of points halfway between upper and lower surfaces
- camber
  maximum distance between mean camber line and chord line

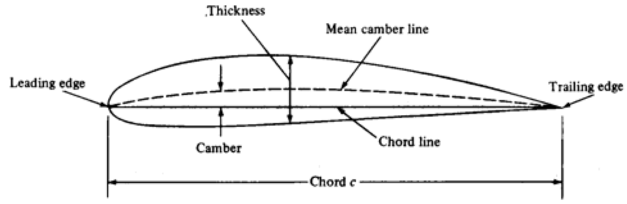
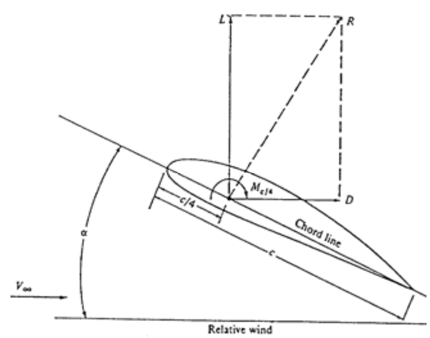


Figure 5.2 Airfoil nomenclature. The shape shown here is a NACA 4415 airfoil.

More definitions

- **relative wind**
  direction of v_∞
- **angle-of-attack**
  angle between relative wind and chord line
- **drag**
  component of resultant aerodynamic force parallel to the relative wind
- **lift**
  component of aerodynamic force perpendicular to relative wind
- **moment**
  pressure distribution also produces rotational torque
  - $M_{c/4} \equiv$ Moment about the wing quarter-chord point



Review of Aircraft Metrics

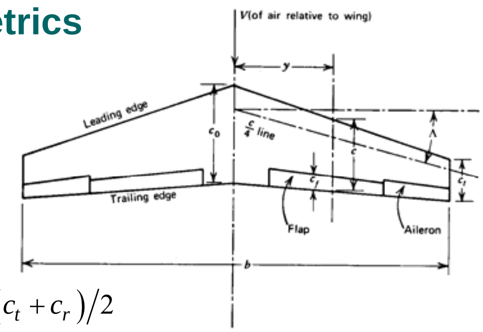


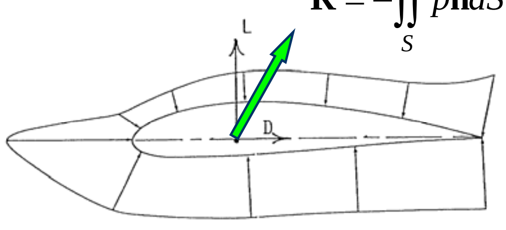
Figure 3.1 Top view of a wing planform.

- Wing Area = S
- Wing Span = b
- Mean Chord = $\bar{c} = S/b = (c_t + c_r)/2$
- Root Chord = $c_0 = c_r$
- Tip Chord = c_t
- Taper Ratio = $\lambda = c_t/c_r$
- Aspect Ratio = $AR = b/\bar{c} = b^2/S$

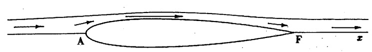
The source of aerodynamic forces

- Force due to pressure and force due to friction

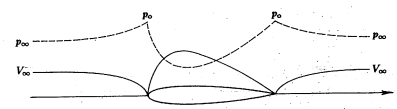
$$\mathbf{R} = -\iint_S p\mathbf{n}dS + \iint_S \tau\mathbf{k}dS$$



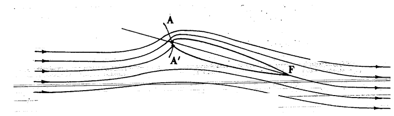
Velocity and pressure around an airfoil

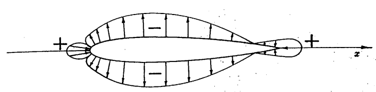


Streamline upper surface

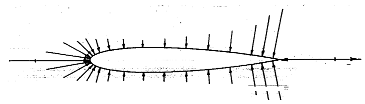


Pressure and velocity distributions α = 0





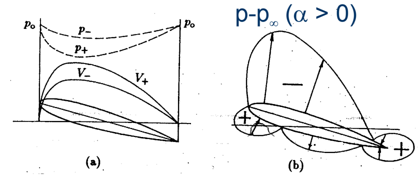
p-p$_\infty$ (α = 0)



Pressure distributions α = 0

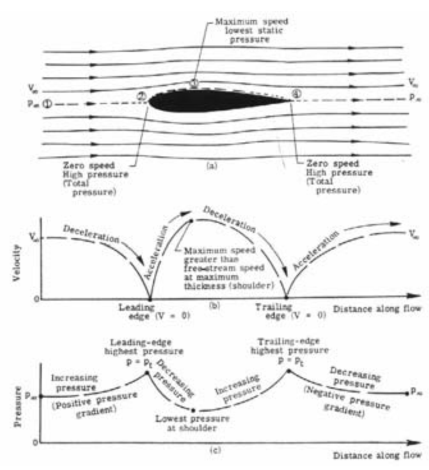
Symmetric airfoil

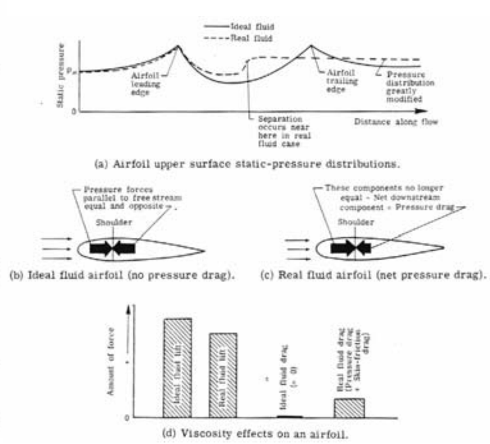
p-p$_\infty$ (α > 0)



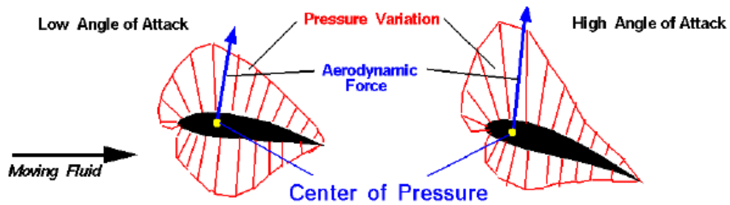
Pressure and velocity distributions α > 0

Ideal and real fluid flow about an airfoil





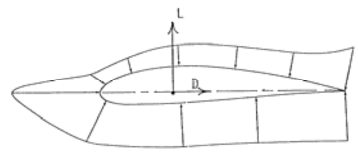
Center of Pressure



Center of Pressure is the average location of the pressure.
Pressure varies around the surface of an object. P = P(x)

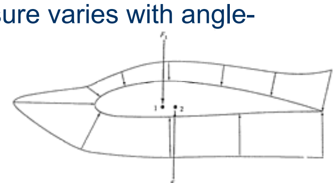
$$cp = \frac{\int x\ p(x)\ dx}{\int p(x)\ dx}$$

Aerodynamic force acts through the center of pressure.
Center of pressure moves with angle of attack.



Center of Pressure c.p.

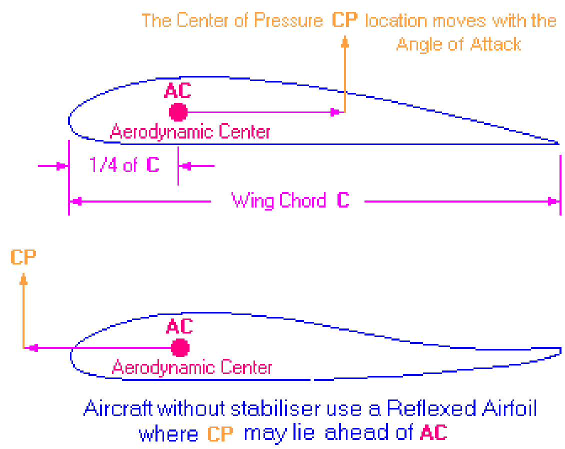
- the resultant forces (lift and drag) acting at the center of pressure produce no moment (use centroid rule to find R location)
- since the pressure distribution over the airfoil changes with angle-of-attack, the location of the center of pressure varies with angle-of-attack



Aerodynamic Center a.c.

- the a.c. is the point on the airfoil about which moments do not vary with angle of attack, assuming v_∞ is constant
- if L = 0, the moment is a pure couple equal to the moment about the aerodynamic center, $m_{a.c.}$
- simple airfoil theory places the a.c.
  - at the quarter chord point for low speed airfoils.
  - at the half chord point for supersonic airfoils

Aerodynamic Center and Center of Pressure



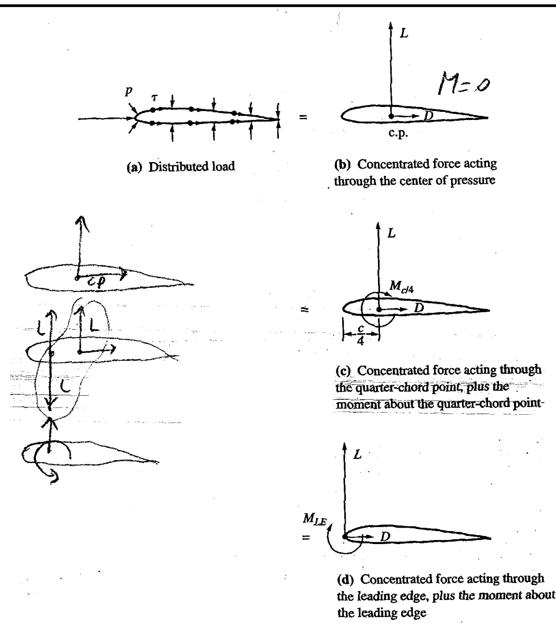


Figure 2.5 Three ways of representing the actual distributed load exerted by pressure and shear stress on the *surface of the airfoil* by *a concentrated* force at a point and the moment at that point.

Dimensional analysis

- Lift, drag, and pitching moment
  - Are functions of several variables:

$$L = f_1(V_\infty, \rho_\infty, \mu_\infty, S, a_\infty)$$
$$D = f_2(V_\infty, \rho_\infty, \mu_\infty, S, a_\infty)$$
$$M = f_3(V_\infty, \rho_\infty, \mu_\infty, S, a_\infty)$$

  - Because it is not feasible to experimentally vary each of these independent variables, we seek to group parameters and thus reduce our effort
- Dimensional analysis
  - Allows us to intelligently group the variables
  - Is an application of the Buckingham Pi Theorem
  - Assume:

$$L = Z\left(V_\infty^a, \rho_\infty^b, S^d, a_\infty^e, \mu_\infty^f\right)$$

Z, a, b, d, e, f Dimensionless constants

Dimensional analysis

- Principle: dimensions on both sides of the equations must be identical
  - Fundamental units: m, l, t
    - Are related to physical quantities
    - For example: $L \propto \dfrac{ml}{t^2}$
  - Equating the dimensions on left and right of the lift force equation

$$\frac{ml}{t^2} = \left(\frac{l}{t}\right)^a \left(\frac{m}{l^3}\right)^b \left(l^2\right)^d \left(\frac{l}{t}\right)^e \left(\frac{m}{lt}\right)^f$$

    - Equating mass exponents $1 = b + f$
    - Equating length exponents $1 = a - 3b + 2d + e - f$
    - Equating time exponents $-2 = -a - e - f$

Dimensional analysis

- Continuing our analysis
  - solving the preceding three equations for a, b, and d (in terms of e and f)

$$L = Z(V_\infty)^{2-e-f}\,\rho_\infty^{1-f}, S^{1-f/2}, a_\infty^e, \mu_\infty^f$$

$$L = ZV_\infty^2 S\rho_\infty \left(\frac{a_\infty}{V_\infty}\right)^e \left(\frac{\mu_\infty}{\rho_\infty V_\infty \sqrt{S}}\right)^f$$

  - Noting that $\frac{a_\infty}{V_\infty} = \frac{1}{M_\infty}$ and $\sqrt{S}$ has units of length, we choose c as our characteristic length
  - Then we can replace $\frac{\mu_\infty}{\rho_\infty V_\infty \sqrt{S}}$ with $\frac{\mu_\infty}{\rho_\infty V_\infty c}$
  - Now, the lift equation becomes

$$L = ZV_\infty^2 S\rho_\infty \left(\frac{1}{M_\infty}\right)^e \left(\frac{1}{\mathrm{Re}_c}\right)^f$$

Force/moment coefficients

- Now we define the airfoil's section lift coefficient

$$\frac{c_l}{2} \equiv Z\left(\frac{1}{M_\infty}\right)^e \left(\frac{1}{Re_c}\right)^f \quad \Rightarrow \quad L = \frac{1}{2}\rho_\infty V_\infty^2 S c_l$$

  - Or we could have simply defined lift coefficient as

$$c_l \equiv \frac{L}{q_\infty S}$$

Dynamic pressure

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

    - Notice that c_l is dimensionless
    - It is represented as a function of Mach number and Reynolds number, but since the dimensional analysis was carried out for a given angle of attack, an airfoil's section lift coefficient depends on all three of these variables

$$c_l = f(\alpha, M_\infty, Re)$$

Force/moment coefficients

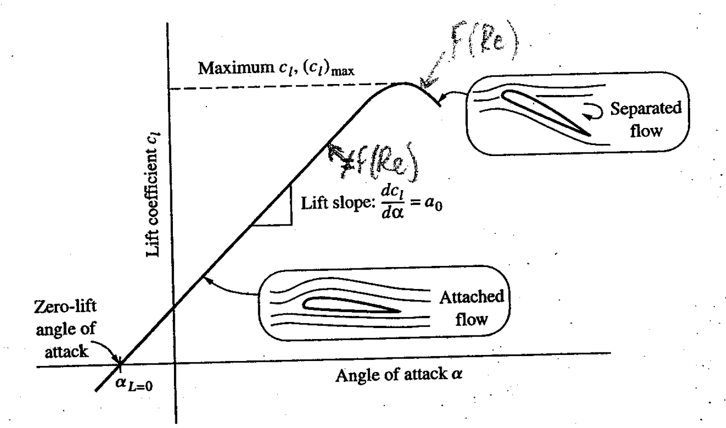
- A similar analysis gives
  - Drag coefficient $D = q_\infty S c_d$
  - Moment coefficient $M = q_\infty S c c_m$
    - Notice that this coefficient has c explicitly included
    - This term accounts for the force x length units
- Summarizing

$$c_l \equiv \frac{L}{q_\infty S} \qquad c_d \equiv \frac{D}{q_\infty S} \qquad c_m \equiv \frac{M}{q_\infty S c}$$

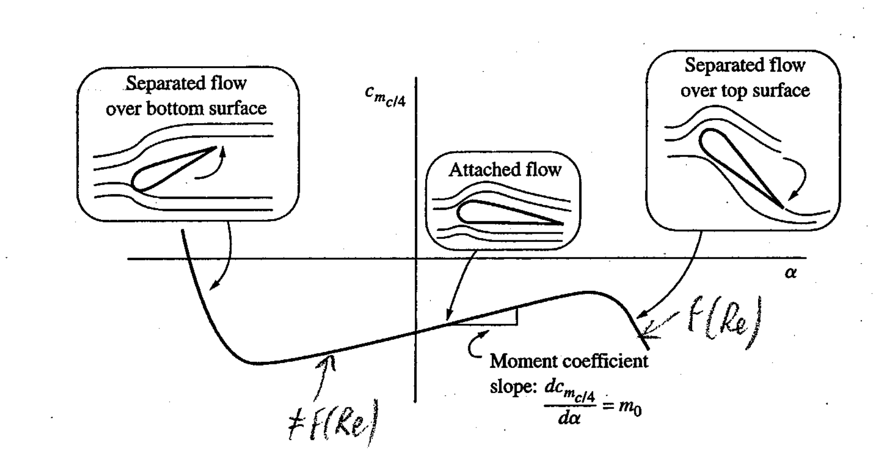
- Where

$$c_l = f_1(\alpha, M_\infty, Re) \qquad c_d = f_2(\alpha, M_\infty, Re) \qquad c_m = f_3(\alpha, M_\infty, Re)$$

Generic lift coefficient variation with α



Generic moment coefficient variation with α



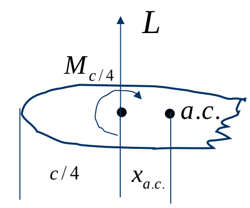
How to find the Aerodynamic Center $x_{a.c.}$?

$$M_{a.c.} = Lx_{a.c.} + M_{c/4}$$

$$\frac{M_{a.c.}}{q_\infty Sc} = \frac{L}{q_\infty S}\frac{x_{a.c.}}{c} + \frac{M_{c/4}}{q_\infty Sc}$$

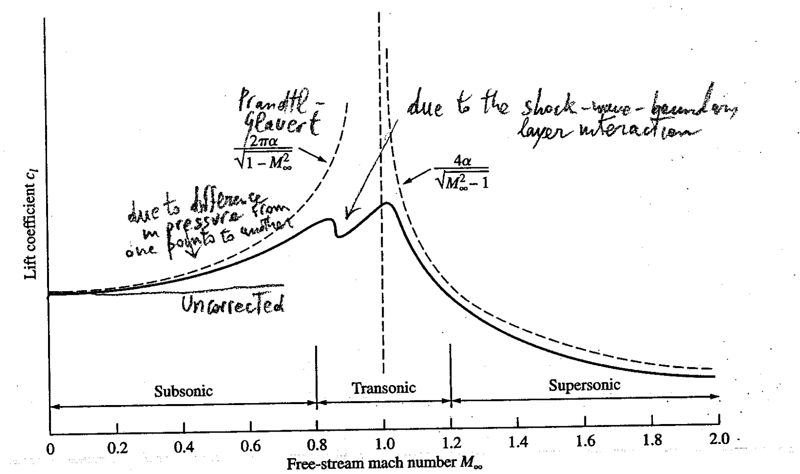
$$c_{ma.c.} = c_l \frac{x_{a.c.}}{c} + c_{mc/4}$$

$$\frac{dc_{ma.c.}}{d\alpha} = \frac{dc_l}{d\alpha}\frac{x_{a.c.}}{c} + \frac{dc_{mc/4}}{d\alpha}$$

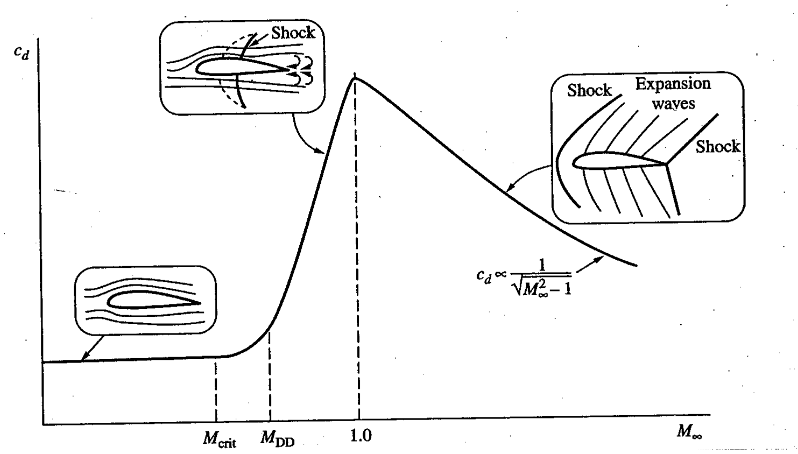


$$0 = a_0 \frac{x_{a.c.}}{c} + m_0 \qquad \frac{x_{a.c.}}{c} = -\frac{m_0}{a_0} = const$$

Generic lift coefficient variation with Mach number

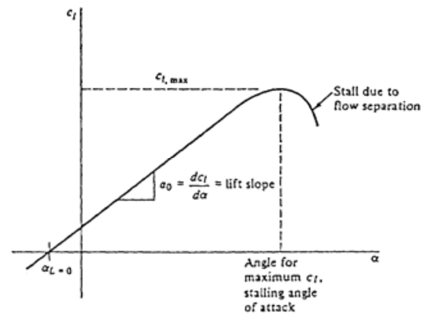


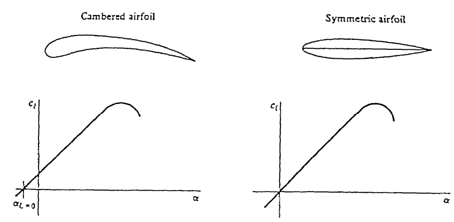
Generic drag coefficient variation with Mach number



C_l - α

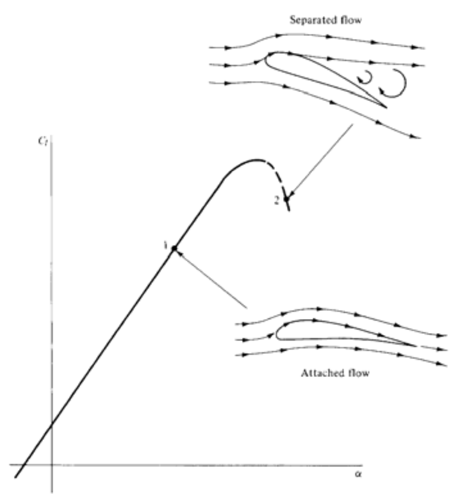
- Experimental data are essential to aircraft design
  - NACA/NASA data
- C_l varies linearly with α
  - camber changes $\alpha_{L=0}$
- This linear relationship breaks down when stall occurs





C_l - α

- At high α, the boundary layer tends to separate
  - Lift decreases
  - Drag increases
  - Moment becomes nose down

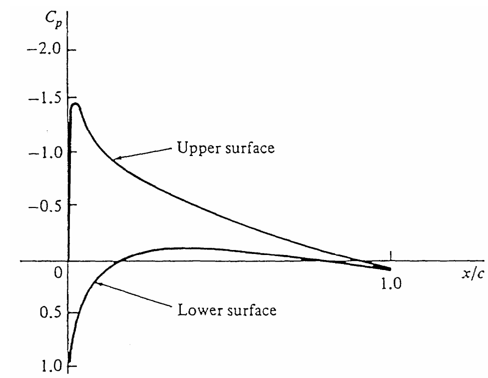


Pressure coefficient

- Pressure coefficient
  - Let us define another coefficient that describes the pressure distribution over an airfoil surface

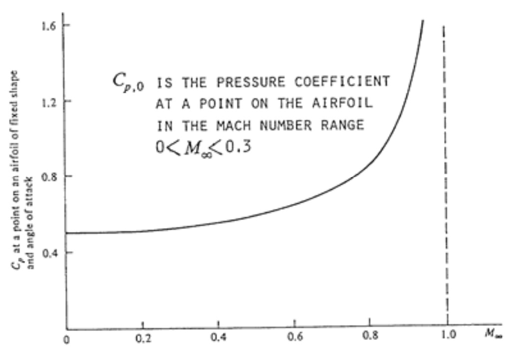
$$C_p \equiv \frac{p - p_\infty}{q_\infty} = \frac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

  - The sketch shows how c_p varies over both upper and lower surfaces
  - c_p can be measured experimentally in the wind tunnel



Pressure coefficient

- Pressure coefficient versus mach

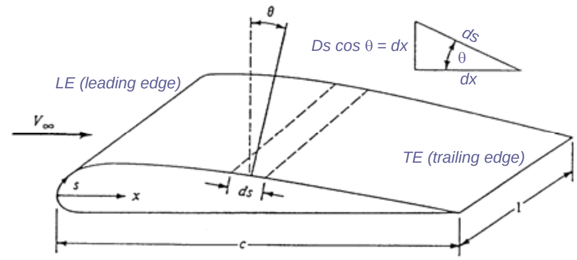


  - Prandtl-Glauert rule

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

Obtaining c_l from c_p

- Lift per unit span



  - Lift is the upward force due to the pressure distribution on the lower surface minus the down-ward force due to the pressure distribution on the upper surface

$$L = \int_{LE}^{TE} p_l \cos\theta ds - \int_{LE}^{TE} p_u \cos\theta ds$$

  - Or, since ds cos θ = dx

$$L = \int_0^c p_l dx - \int_0^c p_u dx$$

Obtaining c_l from c_p

- Lift per unit span
  - Adding and subtracting p_∞ $$L = \int_0^c (p_l - p_\infty) dx - \int_0^c (p_u - p_\infty) dx$$
  - Recalling the definition of lift coefficient

$$c_l \equiv \frac{L}{q_\infty S} = \frac{L}{q_\infty c(1)} = \frac{L}{q_\infty c}$$

  - Combining these two equations

$$c_l = \frac{1}{c}\int_0^c \frac{p_l - p_\infty}{q_\infty} dx - \frac{1}{c}\int_0^c \frac{p_u - p_\infty}{q_\infty} dx$$

Note:

$$C_{p,l} \equiv \frac{p_l - p_\infty}{q_\infty} \quad \text{and} \quad C_{p,u} \equiv \frac{p_u - p_\infty}{q_\infty}$$

  - It follows that:

$$c_l = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right) dx$$

Obtaining c_l from c_p

- Summary
  - If plots of pressure coefficient data over the upper and lower surfaces vs. chordwise distance (x) are available
    - The lift coefficient can be found as the net area between the upper and lower pressure coefficient curves divided by the chord length
    - The section lift coefficient (c_l) equation is a good approximation only for small angles of attack

$$c_l = \frac{1}{c}\int_0^c \left(C_{p,l} - C_{p,u}\right)dx \quad IF\ \alpha \approx 0$$

  - Pressure coefficients defined

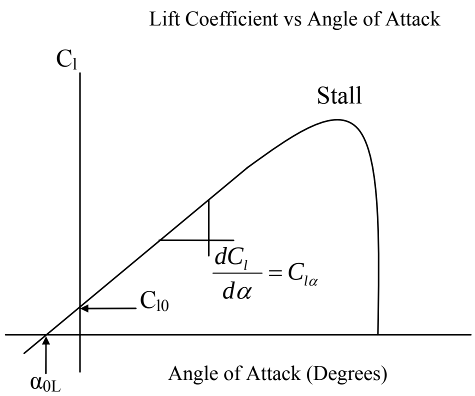
$C_{p,l} \equiv$ Pressure coefficient on the lower surface

$C_{p,u} \equiv$ Pressure coefficient on the upper surface

Lift, Drag and Moment Coefficients

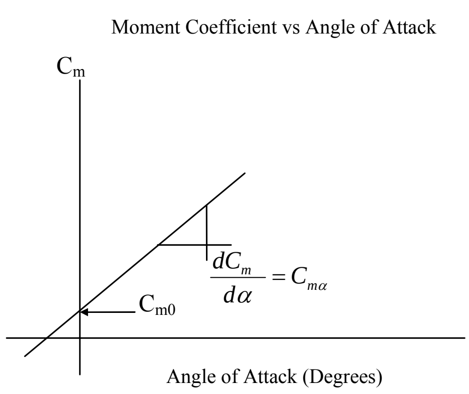
- Lift, Wing and Drag coefficients are written as $\mathbf{C_{ab}}$
  - Where **a** describes the type of coefficient (lift, drag or moment)
  - and **b** describes what the coefficient is a function of (reference coefficient, angle of attack, sideslip)
  - **b** also describes the units of the coefficient
  - For example if b is 0 (reference coefficient) then there are no units. If b is α (angle of attack) then the units are per degree or per radian

Lift Coefficients



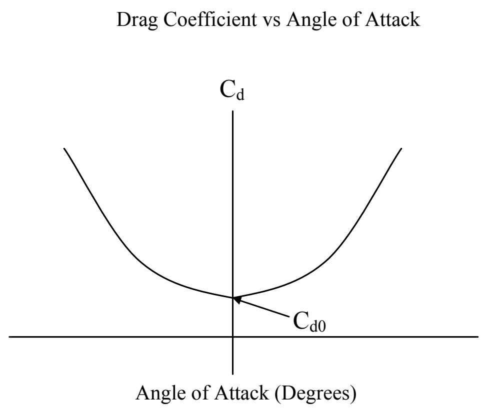
- α_{0L} = zero lift angle of attack
- Slope of the linear region gives the infinite wing lift coefficient
- The reference lift coefficient is given by the point where the angle of attack is zero

Moment Coefficients



- Slope of the linear region gives the infinite wing moment coefficient
- The reference moment coefficient is given by the point where the angle of attack is zero

Drag Coefficients

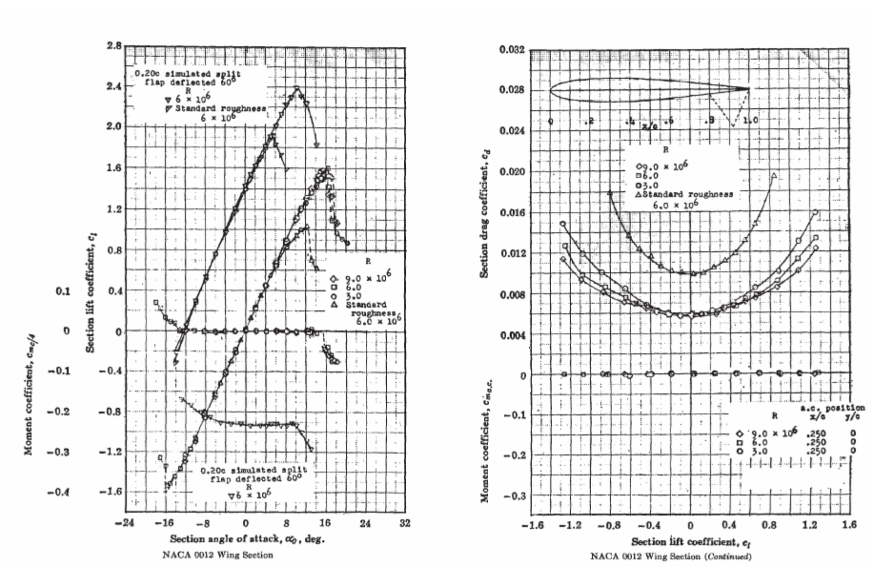


- The reference drag coefficient is given by the point where the angle of attack is zero
- The other drag coefficients can be determined using Excel, Matlab etc to perform a quadratic regression

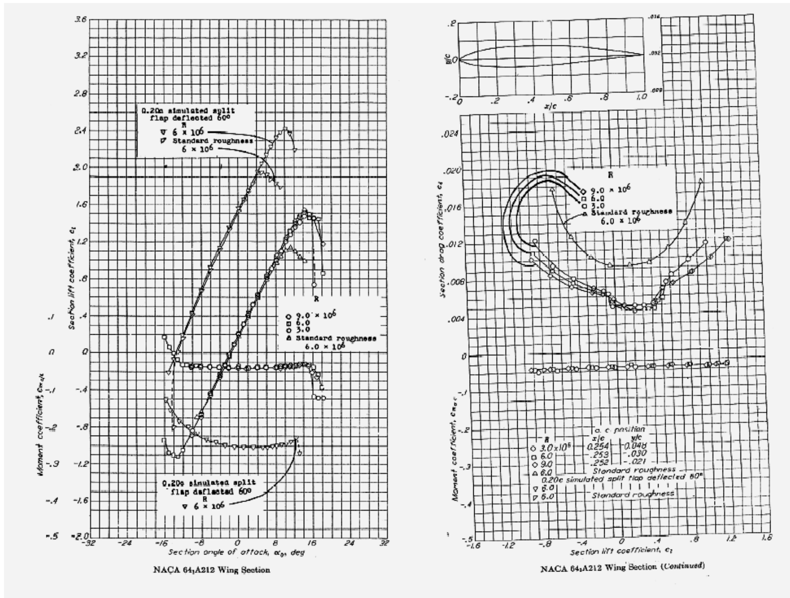
References:

Pinkerton, R. M. and Greenberg, H, "Aerodynamic Characteristics of a Large Number of Airfoils Tested in the Variable-Density Wind Tunnel", NACA Report No. 628

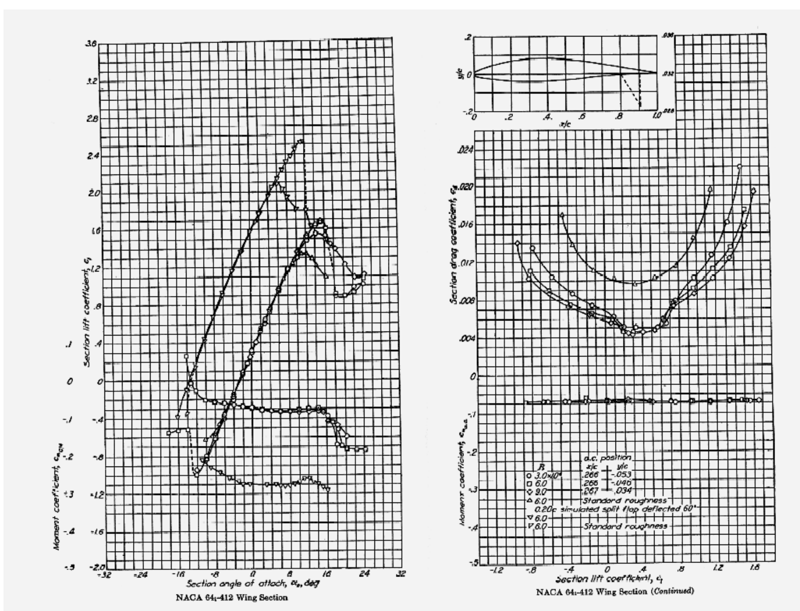
Abbott, I. H. and Von Doenhoff, A. E., "Theory of Wing Sections",Dover Publications, New York, 1959



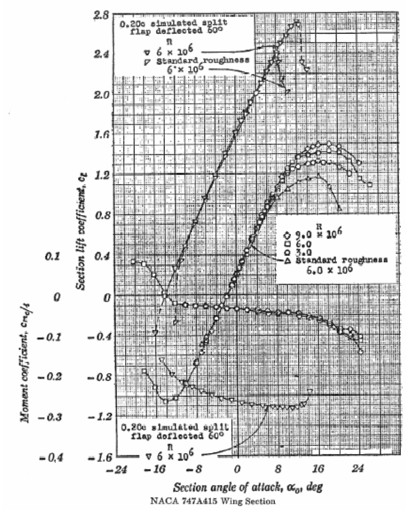
NACA Airfoils

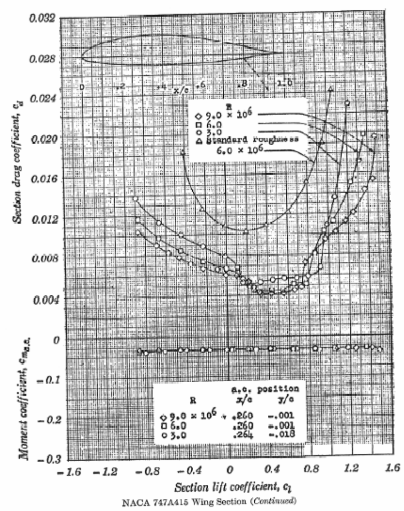


NACA Airfoils

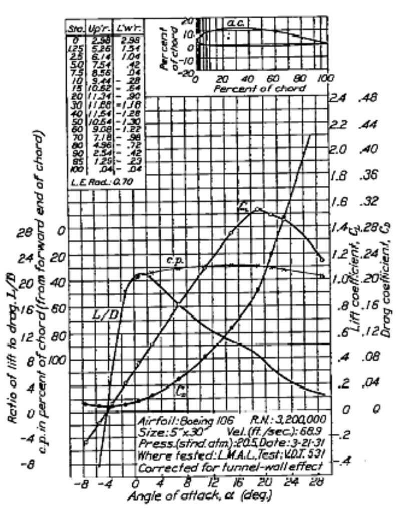


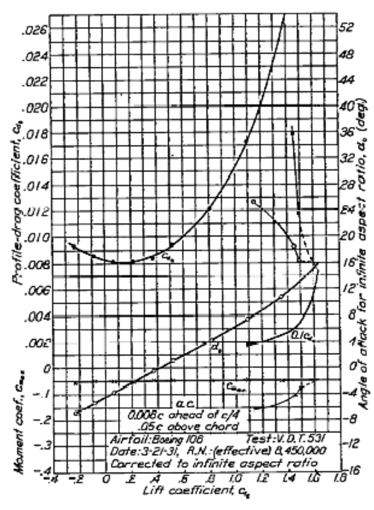
NACA Airfoils





Boeing Airfoils

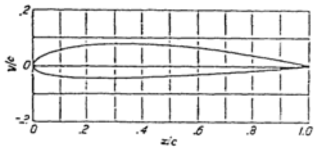




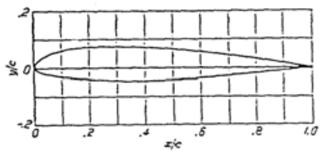
http://www.engr.utk.edu/~rbond/airfoil.html

NACA Airfoil data

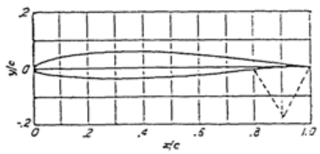
- AIRFOIL CLASSIFICATIONS
  - NACA 2412



  - NACA 23012



  - NACA 63-210



Some NACA Airfoils

NACA 0012: four digit airfoil. First two digits indicate no camber. Last two digits indicate max t/c=12 percent

NACA 6412: This airfoil combines a 0012 thickness (four digit) with a two-digit 64 camber line. A 64 camber line has 6% max camber at 40% chord.

NACA 16-015: This airfoil is identical to a 0015-45 airfoil. The modified four-digit thickness has a leading edge index of 4 and the maximum thickness is at 50% chord

NACA 23012: This airfoil combines a 230 mean line (three-digit) with a 0012 thickness (four digit). A 230 mean line has CL=0.3 and maximum camber at 15% chord.

NACA 63A010: First three characters: 63A series thickness. Fourth character: no camber (CL design=0) Last two digits: 10 percent t/c

NACA 63A409: First three characters: 63A series thickness. Fourth character: CL design=0.4 (63A airfoils always use 6-series modified mean line) Last two digits: 9 percent t/c