# AE 429 - Aircraft Performance and Flight Mechanics

Atmospheric Flight Mechanics

## Atmospheric Flight Mechanics

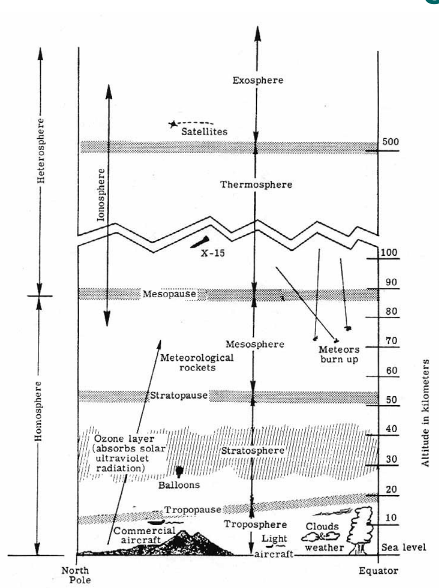
- **Performance**
  - Performance characteristics (range, endurance, rate of climb, take-off and landing distances, flight path optimization)
- **Flight Dynamics**
  - Motion of the aircraft due to disturbances
  - Stability and Control
- **Aeroelasticity**
  - Static and Dynamic Aeroelastic phenomena (control reversal, wing divergence, flutter, aeroelastic response)

**The aerodynamic forces and moment as well as the trust and weight have to be accurately determined**

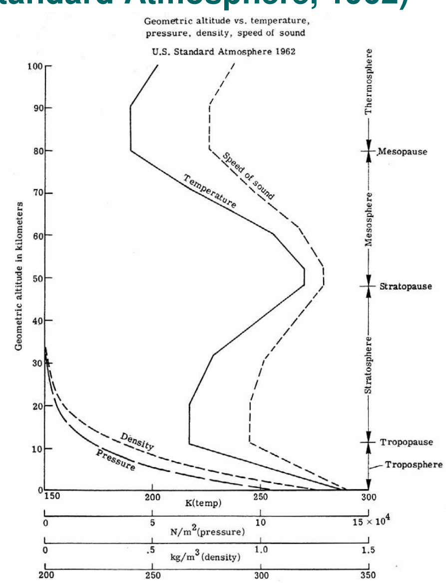
**The aerodynamic forces and moment acting on the aircraft depend on the property of the atmosphere through which is flying**

- Geometric shape
- Attitude to the flow
- Airspeed
- Property of the air mass (pressure, temperature, density, viscosity, speed of sound, etc.)

**Atmospheric structure**




**Atmospheric properties variation. (Based on U.S. Standard Atmosphere, 1962)**



## Overview of Units

- Mass and weight are often confused
- Here are some common units used for mass and weight
  - Kilograms
  - Newtons
  - Pounds
  - Slugs
- Which ones are mass and which ones are weight?
- What is the difference between mass and weight?

## Overview of Units

- Kilograms are a unit of mass (metric)
- Newtons are a unit of weight or force (metric)
- Slugs are a unit of mass (imperial)
- Pounds are a unit of weight or force (imperial)
- Weight = Mass * Gravity
- The weight of an object on the Earth and on the Moon is different
- The mass of an object on the Earth and on the Moon is the same

## Perfect Gas

- A perfect gas is one in which inter-molecular magnetic forces are negligible
- It acts as a continuous material in which the properties are determined by statistical average of the particle effects
  - thermodynamic state equation $P = \rho R T$

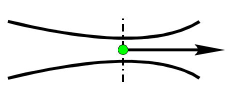
    where $P$ = pressure, $\rho$ = density; $T$ = temperature; $R$ constant for a specific gas
  - for normal air

$$R = 287 \frac{J}{(kg)(^\circ K)} = 1718 \frac{ft - lb_f}{(slug)(^\circ R)}$$

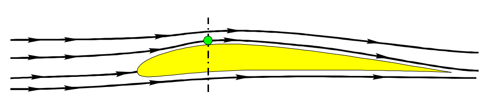
$$R = 287 \frac{m^2}{(\sec)^2 (^\circ K)} = 1718 \frac{ft^2}{(\sec)^2 (^\circ R)}$$

## Velocity/streamlines

- At a fixed point in a fluid/gas
  - the flow velocity is the velocity of an infinitesimally small fluid element as it sweeps through the point along a streamline



*Flow through a Nozzle*



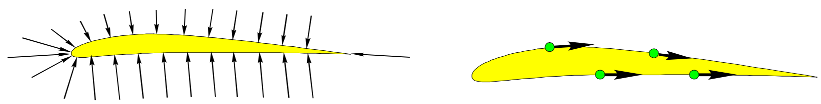
*Streamline Flow about an Airfoil*

Velocity is a vector, having both magnitude and direction

- Each region of gas does not necessarily have the same velocity
- Flow velocity, like pressure, density, and temperature, is a point property

## Aerodynamic forces

- A flow field
  - Is defined using a coordinate frame
  - Is specified using thermodynamic point properties like P, $\rho$, T, and V
    - Pressure = P(x, y, z)
    - Velocity = V(x, y, z)
- Pressure and shear distributions which exist on surfaces are the source of all aerodynamic forces

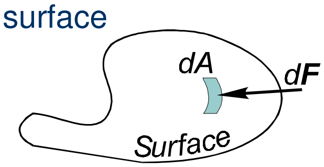


*Pressure* *Shear*

## Pressure (P)

$$P = \rho RT$$

- Pressure is the ***normal force per unit area*** exerted on a surface due to the time rate of change of momentum for gas molecules impacting that surface
  - dA is the incremental area around a point on the surface




  - dF is the force on one side of dA due to pressure, so the pressure at the point on the surface is: $P \equiv \lim_{dA \to 0} \frac{dF}{dA}$ $\qquad P \equiv \frac{F}{A}$
  - units
    - N/m$^2$ - Psf - Psi - Atm - dynes/cm$^2$
    - 1 N/m$^2$ = 1.4504 x $10^{-4}$ lb$_f$/in$^2$ = 2.0886 x $10^{-2}$ lb$_f$/ft$^2$
    - 1 lb$_f$/in$^2$ = 6.8947 x $10^3$ N/m$^2$

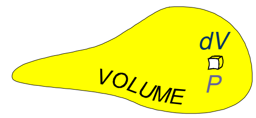
**Ratio of the pressure P at altitude to sea-level standard pressure** $\delta \equiv \frac{P}{P_0}$

## Density (ρ) - Specific Volume (v)

- Density is the mass of a substance per unit volume

  *dV* incremental volume about point *P*
  *dm* the mass of the material (gas) inside *dV*

the density, $\rho$ , at a point *P* is: $\rho = \lim_{dV \to 0} \frac{dm}{dV}$




  - units of density
    - kilograms/cubic meter, $kg/m^3$
    - grams/cubic centimeter, $gm/cm^3$
    - pounds mass/cubic feet, $lb_m/ft^3$
    - slugs/cubic feet, $lb_f\ sec^2/ft^4$

**Ratio of the density $\rho$ at altitude to sea-level standard density** $\sigma = \frac{\rho}{\rho_0}$

- specific volume is volume per unit mass
  - specific volume, v, is the reciprocal of density
  - units for specific volume
    - cubic meters/kilogram, $m^3/kg$
    - cubic feet/slug, $ft^3/slug$
    - cubic feet/slug, $ft^4/lb_f\ sec^2$

## Temperature (T)

- Temperature is a measure of the average kinetic energy of particles making up the gas
  - the temperature, T, of a gas is directly proportional to the average kinetic energy of the particles making up the gas
  - Boltzmann's constant, K, is the constant of proportionality

$$\text{Kinetic Energy} = \frac{3}{2} KT$$

$$K = 1.38 \cdot 10^{-23}\ joules / {}^\circ Kelvin; \qquad 1\ joule = 0.738\ ft - lb$$

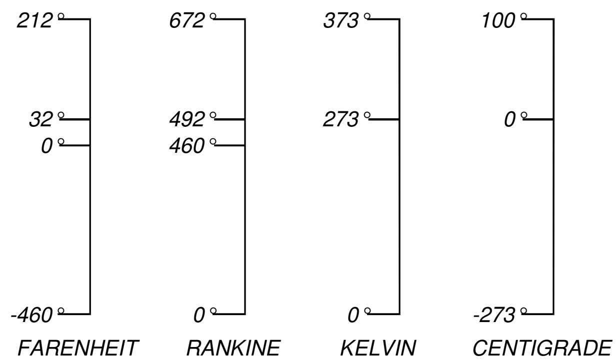
units of temperature:

- Degrees Kelvin (absolute) $^\circ K$
- Degrees Rankine (absolute) $^\circ R$
- Degrees Celsius (not absolute) $^\circ C$
- Degrees Farenheit (not absolute) $^\circ F$

**Ratio of the temperature T at altitude to sea-level standard temperature** $\theta = \frac{T}{T_0}$

Temperature affects the properties of the air such as density and viscosity

## Temperature Scales




– 0 °C = 273.15 °K
– 0 °F = 459.67 °R

## Mach Number (M) and Speed of Sound (a)

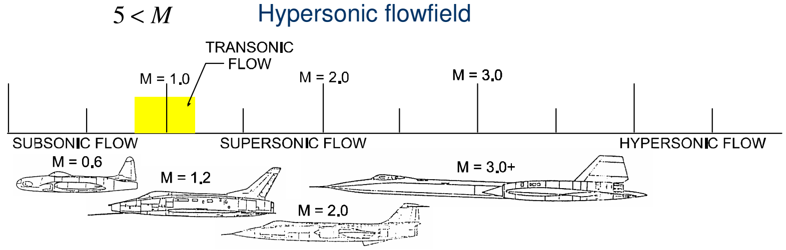
- *V* airplane speed
- *a* speed of sound

$$M = \frac{V}{a}$$

$$a = (\gamma R T)^{1/2}$$

$\gamma$ ratio of specific heats

| | |
|---|---|
| $0 < M < 0.5$ | Incompressible subsonic flowfield |
| $0.5 < M < 0.8$ | Compressible subsonic flowfield |
| $0.8 < M < 1.2$ | Transonic flowfield |
| $1.2 < M < 5$ | Supersonic flowfield |
| $5 < M$ | Hypersonic flowfield |



## Equations Summary

$$A_1V_1 = A_2V_2$$ CONTINUITY EQUATION (INCOMPRESSIBLE)

$$P_1 + \rho\frac{V_1^2}{2} = P_2 + \rho\frac{V_2^2}{2}$$ BERNOULLI'S EQUATION

$$\rho_1 A_1V_1 = \rho_2 A_2V_2$$ CONTINUITY EQUATION (COMPRESSIBLE)

$$dP + \rho V dV = 0$$ EULER EQUATION

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$ ISENTROPIC RELATIONS

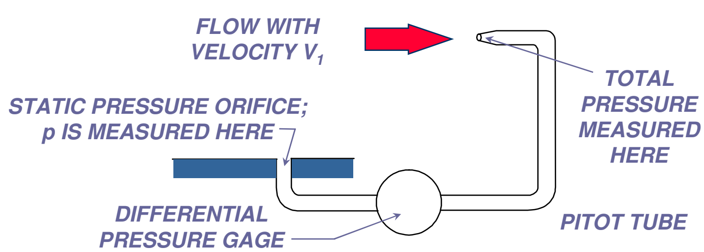
$$c_P T_1 + \frac{V_1^2}{2} = c_P T_2 + \frac{V_2^2}{2}$$ ENERGY

$$P_1 = \rho_1 R T_1; \quad P_2 = \rho_2 R T_2$$ EQUATION OF STATE

**Uniform Streamline flow**

- The continuity, Euler, and Bernoulli Equations all relate point properties in the flow (perhaps on the same streamline)
- If P2 is the same at different streamlines far upstream, these equations can be applied to different streamlines

## Pitot-Static equations




- The total pressure is: (Bernoulli)

$$p_0 = p + \frac{1}{2}\rho_1 V_1^2$$

Total Pressure ($p_0$); Static Pressure ($p$); Dynamic Pressure ($\frac{1}{2}\rho_1 V_1^2$)

  - Solving for $p_0$ - p: $$p_0 - p = \frac{1}{2}\rho V_1^2$$
  - Thus, $V_1$ is: $$V_1 = \sqrt{2\frac{p_0 - p}{\rho}}$$

## Other isentropic relations

$$\frac{p_0}{p_1} = \left(\frac{\rho_0}{\rho_1}\right)^{\gamma} = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

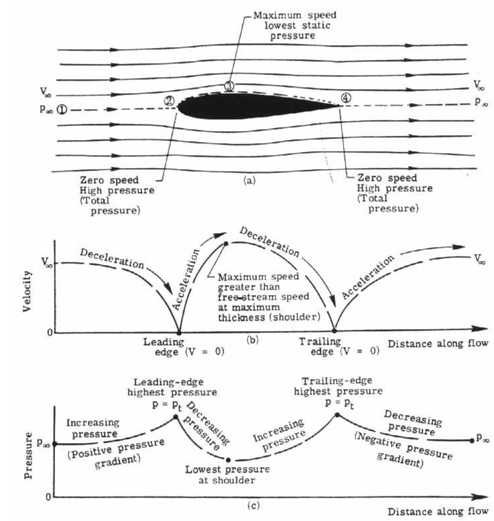
$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2$$

then

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{1}{\gamma-1}}$$

## Ideal fluid flow about an airfoil




$$P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2}$$

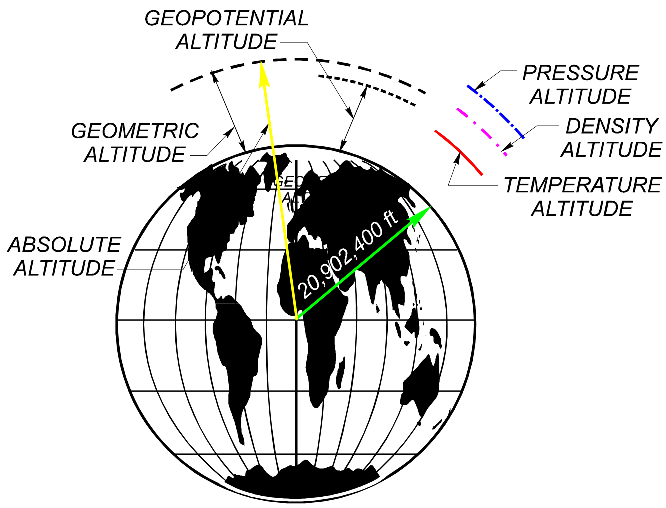
## Standard Atmosphere

- A standard atmosphere is a mathematical model which, on the average, approximates the real atmosphere
  - It provides a basis for performance comparisons
  - It allows experimental data to be generalized

Standard sea level values for pressure, density, temperature

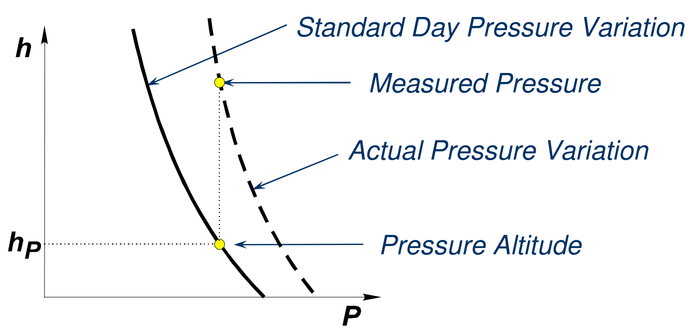
$$p_s = 1.01325 \times 10^5 \text{ N/m}^2 = 2116.2 \text{ lb/ft}^2$$
$$\rho_s = 1.2250 \text{ kg/m}^3 = 0.002377 \text{ slug/ft}^3$$
$$T_s = 288.16 \text{ K} = 518.69°\text{R}$$

## Altitudes



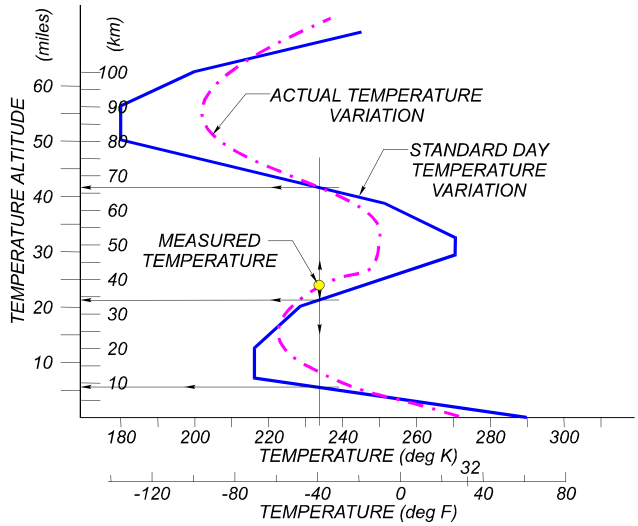

## Altitudes

- There is a unique absolute altitude for each geometric altitude
- The atmospheric model leads to P = P(h)
- Pressure altitude is defined as the reciprocal relationship: $h_P = h(P)$



## Altitudes

- Temperature altitude is not used often
  - $h_T$ is multi-valued for average values of T
  - "Average" values do not match actual values of T

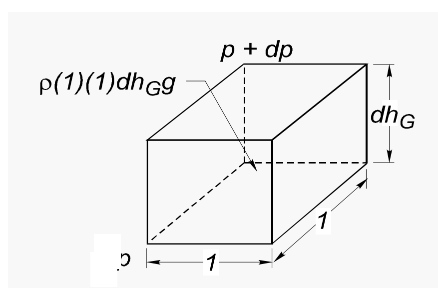



## Hydrostatic Equation

- Derivation of the hydrostatic equation
  - First, sum all vertical forces

$$P = P + dP + \rho g dh_G$$

  - Recalling that $h_G$ is geometric altitude




- Rearranging gives the hydrostatic equation: $dP = -\rho g dh_G$
  - assuming a constant $g = g_0$
  - and taking h as geopotential altitude

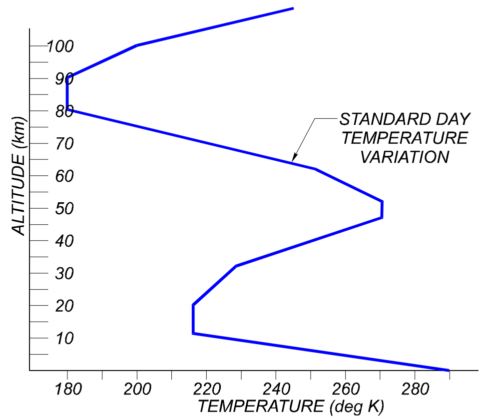
$$\Rightarrow \quad dP = -\rho g_0 dh$$

- Geopotential altitude does not account for changes in gravity as altitude changes

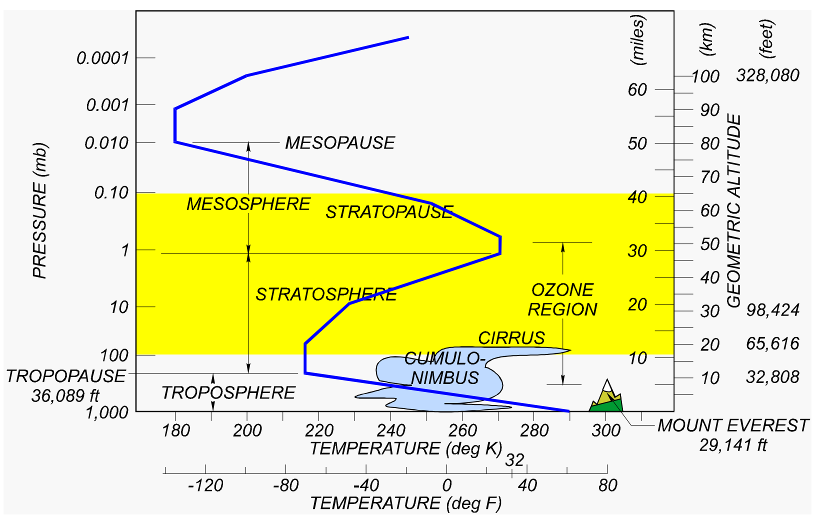
$$h = \left(\frac{r_e}{r_e + h_G}\right) h_G$$

# Standard Atmosphere

- "Standardization" is accomplished using an assumed temperature variation
  - Temperature variations are of two types
    - Constant gradient
    - Isothermal
  - Equations are based upon
    - Hydrostatic equation
    - Definition of Geopotential Altitude
    - Equation of state
    - Defined temperature variation




# Standard Atmosphere



## Standard Atmosphere

- The defining differential equation
  - Is obtained by dividing the geopotential altitude form of the hydrostatic equation by the equation of state for a perfect gas

$$\frac{dP}{P} = \frac{-\rho g_0}{\rho RT} dh = -\frac{g_0}{RT} dh$$

  - Integrates within a constant temperature gradient region, if we define *a* as this gradient or lapse rate

$$a \equiv \frac{dT}{dh} \quad \Rightarrow \quad dh = \frac{1}{a} dT$$

  - By substitution of dh in terms of dT
  - Carrying out the integration

$$\frac{dP}{P} = -\frac{g_0}{aR} \frac{dT}{T}$$

$$\int_{P_1}^{P} \frac{dP}{P} = -\frac{g_0}{aR} \int_{T_1}^{T} \frac{dT}{T} \quad \Rightarrow \quad (\ln P)_{P_1}^{P} = -\frac{g_0}{aR} (\ln T)_{T_1}^{T} \quad \Rightarrow \quad \ln\frac{P}{P_1} = -\frac{g_0}{aR} \ln\frac{T}{T_1}$$

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}}$$

## Standard Atmosphere

- The equation of state allows definition of density in such a gradient region

$$\frac{P}{P_1} = \frac{\rho T}{\rho_1 T_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}} \quad \Rightarrow \quad \frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}$$

  - But the variation of T is linear with h

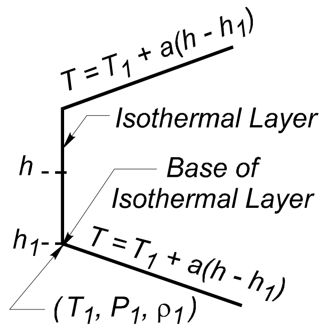
$$T - T_1 = a(h - h_1) \quad \Rightarrow \quad \frac{T}{T_1} = 1 + a\left(\frac{h - h_1}{T_1}\right)$$

  - Substituting this temperature ratio into our integrated equations gives pressure and density ratios as a function of altitude in these constant gradient regions

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{-\frac{g_0}{aR}} = \left(1 + a\left(\frac{h - h_1}{T_1}\right)\right)^{-\frac{g_0}{aR}}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)} = \left(1 + a\left(\frac{h - h_1}{T_1}\right)\right)^{-\left(\frac{g_0}{aR}+1\right)}$$

## Standard Atmosphere




- For isothermal layers, the integration is even easier, since T is constant
  - Going back to our differential equation

$$\frac{dP}{P} = -\frac{g_0}{RT}dh$$

  - Now, we can integrate with no substitution

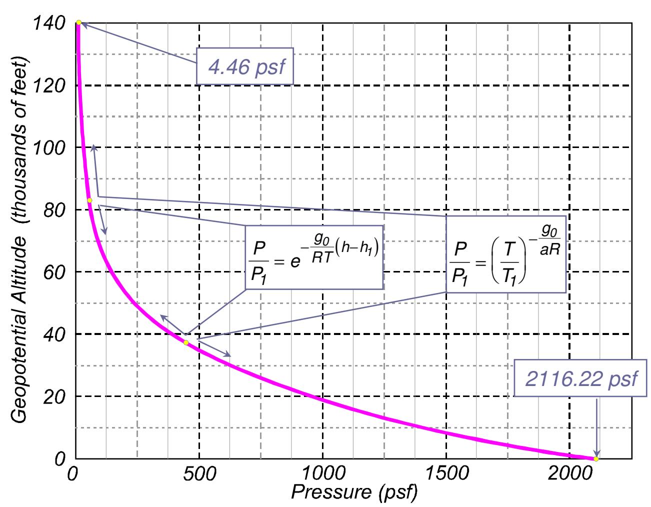
$$\int_{P_1}^{P}\frac{dP}{P} = -\frac{g_0}{RT}\int_{h_1}^{h}dh \quad \Rightarrow \quad (\ln P)_{P_1}^{P} = -\frac{g_0}{RT}(h)_{h_1}^{h} \quad \Rightarrow \quad \ln\frac{P}{P_1} = -\frac{g_0}{RT}(h-h_1)$$

$$\frac{P}{P_1} = e^{-\frac{g_0}{RT}(h-h_1)}$$

  - And, since at constant T, $\frac{P}{P_1} = \frac{\rho}{\rho_1} = e^{-\frac{g_0}{RT}(h-h_1)}$

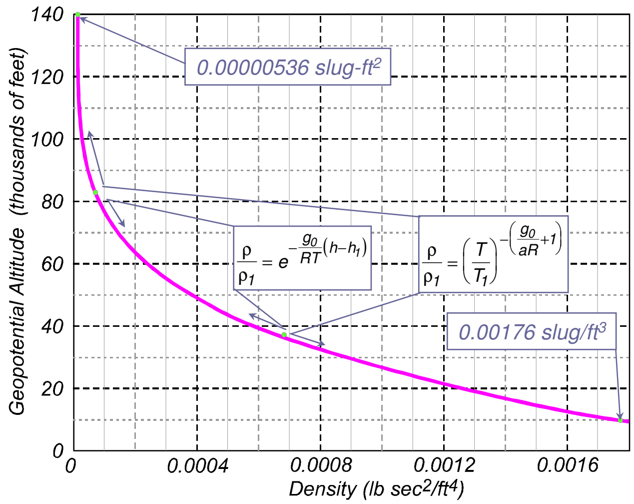
## Standard Atmosphere

- Pressure variation with altitude



## Standard Atmosphere

- Density variation with altitude




## Examples

Calculate the values of pressure, pressure ratio, density, density ratio, and temperature for the standard atmosphere at an altitude of 8000 m. Show the results in SI units.

The standard sea-level values are pressure = 101,325 N/m$^2$, density = 1.2250 kg/m$^3$, and temperature = 288.16 K. The temperature lapse rate $a$ = −0.0065 K/m.

**Solution:** At 8000 m, $T$ = 288.16 − (8000) (0.0065) = 236.16 K. In the gradient region, from equation 5.11,

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/aR}$$

Taking as the initial conditions the sea-level values,

$$\text{pressure ratio} = \frac{p}{101{,}325} = \left(\frac{T}{288.16}\right)^{-g_0/aR}$$

$$= \left(\frac{236.16}{288.16}\right)^{-9.8/(-0.0065)(287.05)}$$

$$= \underline{0.3516}$$

and

$$p = 101{,}325(0.3516) = \underline{35{,}625 \text{ N/m}^2}$$

## Examples

The density is found from the equation of state of a gas:

$$\rho = \frac{p}{RT} = \frac{35{,}625}{(287.05)(236.16)} = \underline{0.52552 \text{ kg/m}^3}$$

$$\sigma = \frac{\rho}{\rho_s} = \frac{0.52554}{1.2250} = \underline{0.4290}$$

## Examples

Calculate the values of pressure, pressure ratio, density, density ratio, and temperature for the standard atmosphere at an altitude of 25,000 ft. Show results in English units.

The standard sea-level values are pressure = 2116 lb/ft$^2$, density = 0.002377 slug/ft$^3$, and temperature = 519°R (59°F). The temperature lapse rate $a$ = −0.00356°R/ft.

**Solution:** At 25,000 ft, $T$ = 519 − (25,000) (0.00356) = 430°R. In the gradient region, from equation 5.11,

$$\frac{p}{p_1} = \left(\frac{T}{T_1}\right)^{-g_0/aR}$$

Taking as the initial conditions the sea-level values,

$$\text{pressure ratio} = \frac{p}{2116} = \left(\frac{T}{519}\right)^{-g_0/aR}$$

$$= \left(\frac{430}{519}\right)^{-32.2/(-0.00356)(1718)}$$

$$= \underline{0.3714}$$

and

$$p = 2116(0.3714) = \underline{785.6 \text{ lb/ft}^2}$$

The density is found from the equation of state of a gas:

$$\rho = \frac{p}{RT} = \frac{785.6}{(1718)(430)} = \underline{0.001063 \text{ slug/ft}^3}$$

$$\sigma = \frac{\rho}{\rho_s} = \frac{0.001063}{0.002377} = \underline{0.4472}$$

# Examples

Air flowing at high speed in the working section of a wind tunnel has pressure and temperature values equal to 0.6 atm at sea level and −40°C, respectively. Calculate, in English engineering units:

(a) Air density.
(b) Density ratio.
(c) Specific volume.

**Solution:**

(a) Pressure = 0.6 × 2116 = 1269.6 lb/ft². Temperature = $(-40 \times \frac{9}{5}) + 32 =$ −40°F, or, since we must work with absolute temperature,

$$T = -40 + 460 = 420°\text{R}$$

From the equation of state,

$$\rho = \frac{p}{RT} = \frac{1269.6}{(1718)(420)} = \underline{0.001760 \text{ slug/ft}^3}$$

$$\text{(b) } \sigma = \frac{0.001760}{\rho_{\text{sea level std. day}}} = \frac{0.001760}{0.002377} = \underline{0.740.}$$

$$\text{(c) Specific volume} = \frac{1}{\rho} = \underline{568.18 \text{ ft}^3/\text{slug.}}$$

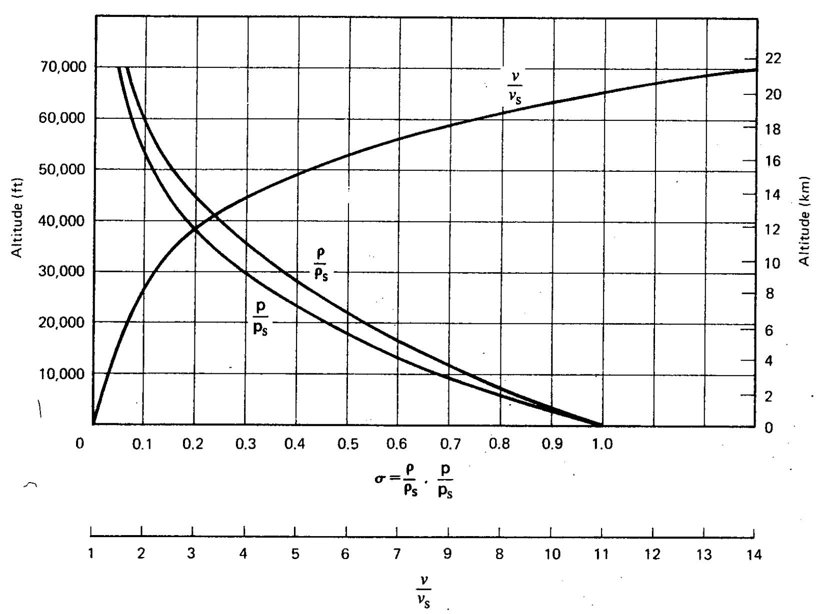



**Figure 5.3** Variation of density, pressure, and kinematic viscosity ratios for the U.S. standard atmosphere.