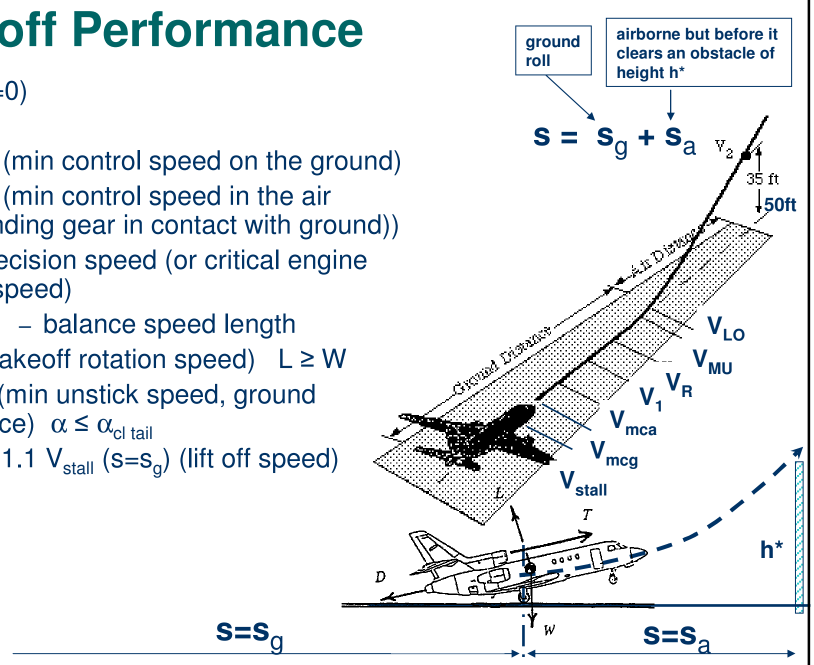
AE 429 - Aircraft Performance and Flight Mechanics

Takeoff and Landing

Takeoff Performance

- V=0 (s=0)
- $V=V_{stall}$
- $V=V_{mcg}$ (min control speed on the ground)
- $V=V_{mca}$ (min control speed in the air (w/o landing gear in contact with ground))
- $V=V_1$ decision speed (or critical engine failure speed)
  - balance speed length
- $V=V_R$ (takeoff rotation speed) $L \geq W$
- $V=V_{MU}$ (min unstick speed, ground clearance) $\alpha \leq \alpha_{cl\ tail}$
- $V=V_{LO}=1.1\ V_{stall}$ ($s=s_g$) (lift off speed)



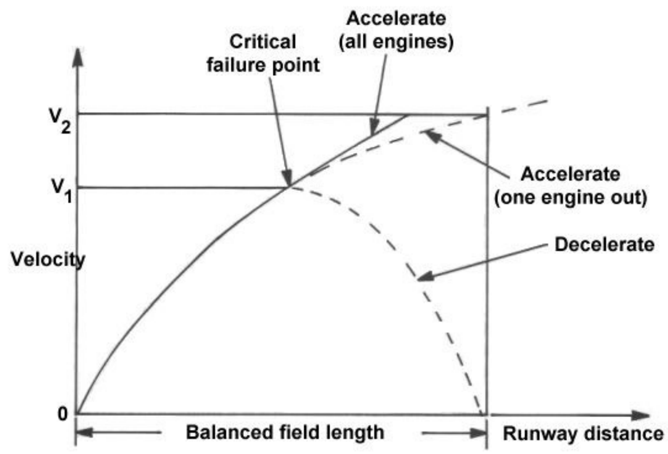
Takeoff Speed and FAR 25 requirements

Speed	Description	FAR 25 Requirement
V_s	stall speed in takeoff configuration	-
V_{mc}	minimum control speed with one engine inoperative (OEI)	-
V_1	OEI decision speed	= or > V_{mc}
V_r	rotation speed	5% > V_{mc}
V_{mu}	minimum unstick speed for safe flight	= or > V_s
V_{lof}	liftoff speed	10% > V_{mu} 5% > V_{mu} (OEI)
V_2	takeoff climb speed at 35 ft	20% > V_s 10% > V_{mc}

Examples of Takeoff Speeds

Aircraft	Takeoff Weight	Takeoff Speed
Boeing 737	100,000 lb 45,360 kg	150 mph 250 km/h 130 kts
Boeing 757	240,000 lb 108,860 kg	160 mph 260 km/h 140 kts
Airbus A320	155,000 lb 70,305 kg	170 mph 275 km/h 150 kts
Airbus A340	571,000 lb 259,000 kg	180 mph 290 km/h 155 kts
Boeing 747	800,000 lb 362,870 kg	180 mph 290 km/h 155 kts
Concorde	400,000 lb 181,435 kg	225 mph 360 km/h 195 kts

- The critical engine speed defines the point on the runway at which the distance needed to stop is exactly the same as the that required to reach takeoff speed. The resulting total takeoff distance is correspondingly known as the balanced field length.



Definition of critical engine-failure speed and balanced field length

EOM Airplane During Takeoff

This equation gives instantaneous forces during the acceleration

$$F = T - D - R = T - D - \mu_r \left(W - L\right) = m\frac{dV}{dt}$$

$$R \equiv \mu_r \left(W - L\right)$$

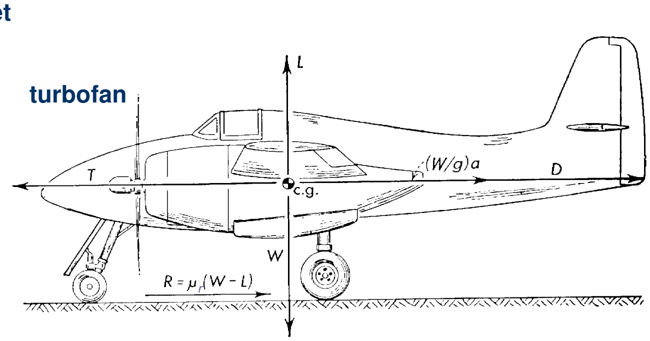
rolling friction coeff.

$P = V_\infty T \qquad T = \dfrac{\text{const}}{V_\infty}$ **reciprocating engine/propeller**

$T = \text{const}$ **turbojet**

$T = k_1^* - k_2^* V_\infty + k_3^* V_\infty^2$ **turbofan**

$W = \text{const}$ **Weight**



EOM Airplane During Takeoff

– Both L and D vary with V

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad D = \frac{1}{2}\rho_\infty V_\infty^2 S C_D$$

$C_D = C_{D0} + \Delta C_{D0} + \left(k_1 + G k_3\right) C_L^2$ **Drag Polar** $k_2 = 0$ **Wave Drag**

$\Delta C_{D0} = \frac{W}{S} K_{uc} m^{-0.219}$ W/S **[N/m²]** m **max mass aircraft [kg]**

$K_{uc} = 5.81 \cdot 10^{-5}; K_{uc} = 3.16 \cdot 10^{-5}$

zero flap; max flap down

– "Ground Effect"

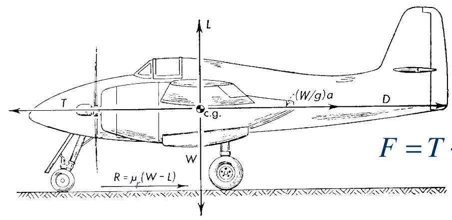
$$\frac{C_{Di}(\text{in ground effect})}{C_{Di}(\text{out-of-ground effect})} = G \qquad G = \frac{\left(16h/b\right)^2}{1+\left(16h/b\right)^2}$$

h = height of wing above the ground, b = wingspan

EOM Airplane During Takeoff

$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L$ $C_L \leq 0.1$

limited and determined by features of the geometric design configuration of the airplane rolling along the ground



$$F = T - D - R = T - D - \mu_r\left(W - L\right) = m\frac{dV_\infty}{dt}$$

$$V_\infty\left(t=0\right) = 0; V_\infty\left(t = t_{LO}\right) = V_{LO} = 1.1V_{MU} \approx 1.1V_{stall}$$

$V\left(t\right) \Rightarrow$ **solved numerically**

Ground Roll Distance

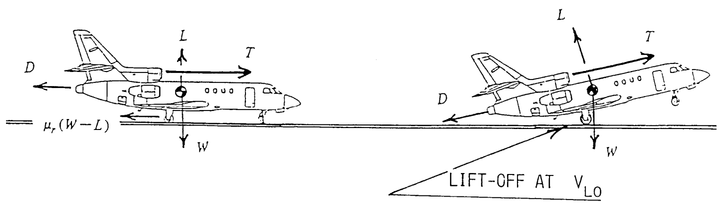
$$ds = \frac{ds}{dt}dt = V_\infty dt$$

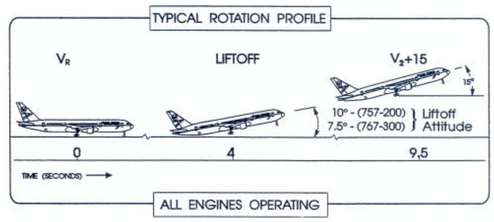
$$\int_0^{s_g} ds = \int_0^{t_{LO}} V_\infty dt \quad \Rightarrow \quad s_{sg} = \int_0^{t_{LO}} V_\infty dt$$

Average Forces Acting During Takeoff

$$F_{eff} = T - [D + \mu_r (W - L)]_{average} \approx CONSTANT$$

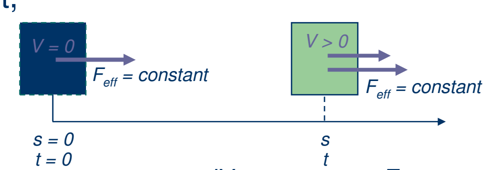
– Why consider "average" force during the takeoff roll?
  - average 0.7 V_{LO}





Takeoff Roll

- We are no longer considering a "Statics" problem
  – Finite (even large) accelerations are present
  – we apply Newton's second law to any body initially at rest,



$$F = ma = m\frac{dV}{dt} \quad \Rightarrow \quad dV = \frac{F}{m}dt$$

  – Integrating

$$\int_0^V dV = \frac{F_{eff}}{m}\int_0^t dt \quad \Rightarrow \quad V = \frac{F_{eff}}{m}t \quad OR \quad t = \frac{Vm}{F_{eff}}$$

$$\int_0^s ds = \int_0^t V dt = \int_0^t \frac{F}{m} t dt = \frac{F_{eff}}{m}\int_0^t t dt \quad \Rightarrow \quad s = \frac{F_{eff}}{m}\frac{t^2}{2}$$

Approximate Analysis Ground Roll

$$ds = \frac{ds}{dt}dt = V_\infty dt = V_\infty \frac{dt}{dV_\infty}dV_\infty \qquad ds = V_\infty \frac{dV_\infty}{\left(dV_\infty / dt\right)} = \frac{1}{2}\frac{d\left(V_\infty^2\right)}{\left(dV_\infty / dt\right)}$$

$$\frac{dV_\infty}{dt} = \frac{1}{m}\left(T - D - \mu_r\left(W - L\right)\right) = \frac{g}{W}\left(T - D - \mu_r\left(W - L\right)\right)$$

$$s_g = \int_0^{V_{LO}} \frac{d\left(V_\infty^2\right)}{2g\left(K_T - K_A V_\infty^2\right)} \underset{\text{approx}}{=} \int_0^{V_{LO}} \frac{W}{2g} \underbrace{\frac{d\left(V_\infty^2\right)}{\left(T - D - \mu_r\left(W - L\right)\right)}}_{\sim \text{ constant , value at } V = 0.7\, V_{LO}} + NV_{LO}$$

$$K_T = \frac{T}{W} - \mu_r$$

$$K_A = -\frac{\rho_\infty}{2\left(W/S\right)}\left(C_{D0} + \Delta C_{D0} + \left(k_1 + \frac{G}{\pi e AR}\right)C_L^2 - \mu_r C_L\right)$$

$N = 3$ large aircraft; $N = 1$ small aircraft;

NV_{LO} distance covered during rotation

Takeoff Roll

- Lift-off distance for a jet airplane
  - Using the average Force we have postulated
    - The sum

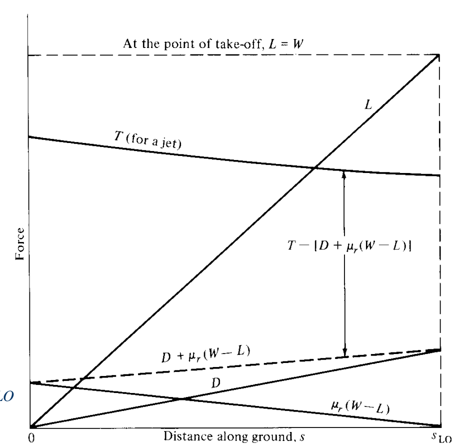
$$F_{eff} = T - \left[D + \mu_r\left(W - L\right)\right]_{average}$$

    - Is fairly constant
    - Thrust is also fairly constant
  - Substituting into

$$D + \mu_r\left(W - L\right)$$

$$s = \frac{V^2 m}{2F_{eff}} \quad \Rightarrow \quad s_{LO} = \frac{V_{LO}^2 W}{2gF_{eff}}$$

$$s_{LO} = \frac{V_{LO}^2 W}{2g\left\{T - \left[D - \mu_r\left(W - L\right)\right]_{average}\right\}} + NV_{LO}$$



Takeoff Roll

- Lift-off distance (continued)
  - Generally we use a safety margin at lift-off by 10%

$$V_{LO} = 1.1V_{stall} = 1.1\sqrt{\frac{2W}{\rho_\infty S C_{L_{\max}}}}$$

  - Substituting into the expression for s_{LO}

$$s_{LO} = \frac{1.21(W/S)}{g\rho_\infty C_{L_{\max}}\left\{T/W - \left[D/W + \mu_r(1 - L/W)\right]_{average}\right\}} + 1.1N\sqrt{\frac{2}{\rho_\infty}\frac{1}{C_{L_{\max}}}\frac{W}{S}}$$

$$\left[D + \mu_r(W - L)\right]_{average} \approx \left[D + \mu_r(W - L)\right]_{0.7V_{LO}}$$

  - One method of estimating quickly the average drag and rolling resistance force is to use $0.7V_{LO}$ in calculating the aerodynamic forces

Takeoff Ground Roll Distance – Simplified Equation

$$T >> \left[D + \mu_r(W - L)\right]_{0.7V_{LO}}$$

$$s_{LO} = \frac{1.21(W/S)}{g\rho_\infty C_{L_{\max}}\{T/W\}}$$

$$T \propto \rho_\infty \Rightarrow \quad s_{LO} \propto 1/\rho_\infty^2$$

$$s_{LO} \propto W^2$$

hot days, low air density
cold days, high air density

s_{LO} at altitude is > s_{LO} at sea level

Distance While Airborne to Clear an Obstacle

$V \approx 1.15 V_{stall}$

$C_L \approx 0.9 C_{L\max}$

$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho_\infty S\left(1.15V_{stall}\right)^2 0.9C_{L\max}}{W}$$

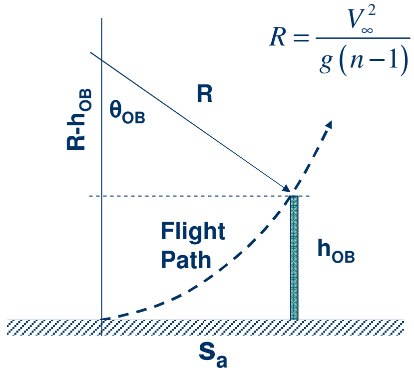
$$= \frac{\frac{1}{2}\rho_\infty S\left(1.15V_{stall}\right)^2 0.9C_{L\max}}{\frac{1}{2}\rho_\infty S\left(V_{stall}\right)^2 C_{L\max}} = 1.19$$

$$R = \frac{1.15V_{stall}^2}{g\left(1.19-1\right)} = \frac{6.95V_{stall}^2}{g}$$

$$\cos\theta_{OB} = \frac{R-h_{OB}}{R} = 1 - \frac{h_{OB}}{R}$$

$$s_a = R\sin\theta_{OB}$$

$$R = \frac{V_\infty^2}{g\left(n-1\right)}$$



$V \approx 1.1V_{stall}$ $\quad$ $V \approx 1.2V_{stall}$

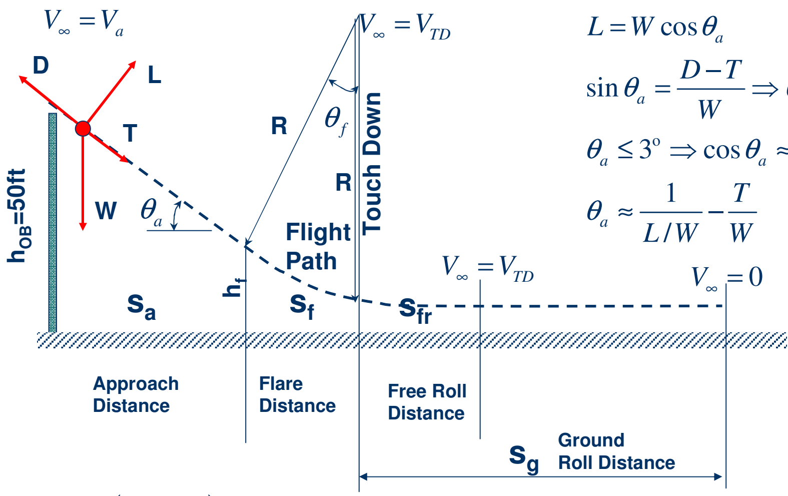
ground roll

airborne but before it clears an obstacle of height h*

s = s$_g$ + s$_a$

Landing Performance

$$L = W\cos\theta_a$$
$$D = T + W\sin\theta_a$$
$$L = W\cos\theta_a$$
$$\sin\theta_a = \frac{D-T}{W} \Rightarrow \theta_a = \frac{D}{W} - \frac{T}{W}$$
$$\theta_a \le 3^\circ \Rightarrow \cos\theta_a \approx 1 \Rightarrow L \approx W$$
$$\theta_a \approx \frac{1}{L/W} - \frac{T}{W}$$



$$s_a = \frac{(50 - h_f)}{\tan\theta_a}$$

$$h_f = R - R\cos\theta_f = R(1-\cos\theta_f) = R(1-\cos\theta_a)$$

since $\theta_f = \theta_a$

Landing Performance

$$R = \frac{V_\infty^2}{g(n-1)}$$

from $V_a = 1.3V_{stall}$ to $V_{TD} = 1.15V_{stall}$ commercial airplanes

from $V_a = 1.2V_{stall}$ to $V_{TD} = 1.1V_{stall}$ military airplanes

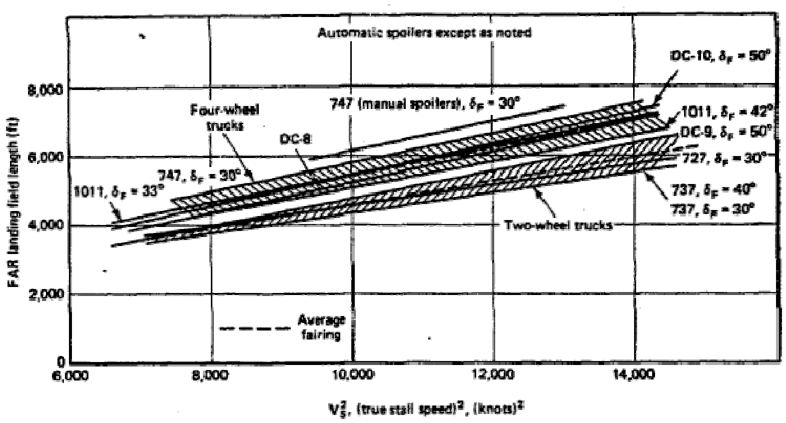
$V_{average} = V_{flare} = 1.23V_{stall}$ commercial airplanes

$V_{average} = V_{flare} = 1.15V_{stall}$ military airplanes

Load factor $n = 1.2 \Rightarrow R = \frac{V_{flare}^2}{0.2g}$ **Radius of turn**

$$s_a = \frac{(50-h_f)}{\tan\theta_a} \qquad \theta_a = \theta_f \qquad s_f = s_{flare} = R\sin\theta_f$$

Landing data



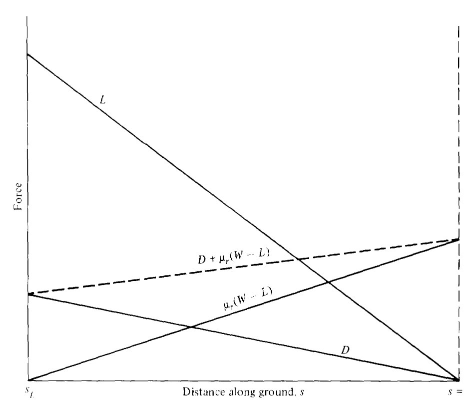
The figure shows the landing data for two different flap angles for some airplanes.
The FAR landing field length is defined as the actual demonstrated distance from a 50 ft. height to a full stop increased by the factor 1/0.60, a 67% increase.

Landing Roll – Ground Roll

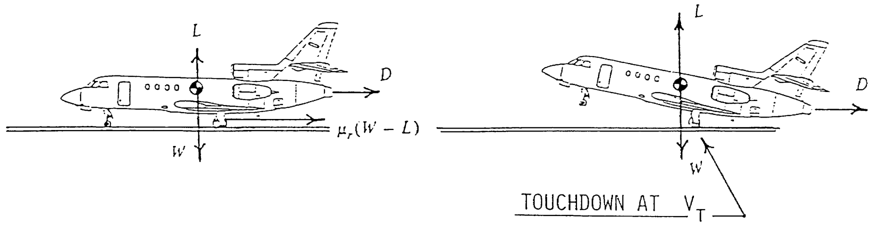
- The landing roll is very similar to the takeoff ground roll, except:
  - Thrust = 0 (or near zero)
  - The sign of the acceleration is negative

$$-[D+\mu_r(W-L)]_{average} = m\frac{dV}{dt}$$

  - We seek an approximate expression like S_{TO} for landing using an average Force again



Landing Roll



- Following this logic

$$F \approx -[D+\mu_r(W-L)]_{average} \approx -[D+\mu_r(W-L)]_{0.7V_T}$$

  - Notice that this assumption is less accurate for landing than for takeoff
  - Nonetheless, let's do the integration again

$$\int_{s_L}^{0} ds = \frac{F}{m}\int_0^t dt \quad \Rightarrow \quad s_L = -\frac{F}{m}\frac{t^2}{2} \quad \text{or} \quad s_L = -\frac{V^2 m}{2F}$$

Landing Roll

- Combining the previous expression for s_L and using the approximately constant retarding force

$$s_L \approx \frac{V_{TD}^2 W}{2g\left[D+\mu_r(W-L)\right]_{0.7V_{TD}}}$$

  - again, adding a safety factor (10% in this case) to touch down at a speed above the stall speed

$$V_{TD} \geq 1.15V_{stall} \quad \text{commercial airplanes}$$
$$V_{TD} \geq 1.10V_{stall} \quad \text{military airplanes}$$
$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L_{max}}}}$$

  - Substituting into the expression above for s_L

$$s_L \approx \frac{2.645W^2}{g\rho_\infty S C_{L_{max}}\left[D+\mu_r(W-L)\right]_{0.7V_{TD}}} \quad \text{(commercial airplane)}$$

Landing Roll

- Landing roll can be reduced if a thrust reverser is installed

Thrust reverser: 40-50% max T for Jet airplane;
40% max T for Reciprocating engine/propeller; 60% max T for Turbofan

$$-T_R - D - \mu_r\left(W - L\right) = m\frac{dV}{dt} \Rightarrow \text{ where } \; T_R = \text{Reverse Trust}$$

D can be increased with: Spoilers, speed brakes, drogue chutes

- If the thrust reverser produces constant thrust,

$$s_L \approx \frac{W^2}{g\rho_\infty S C_{L_{\max}}\left\{ T + \left[D + \mu_r\left(W - L\right)\right]_{0.7V_T} \right\}}$$

- Lift and drag forces are calculated, accounting for ground effect as we did for takeoff roll

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad D = \frac{1}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} + G\frac{C_L^2}{\pi e AR}\right)$$

Where $G = \dfrac{\left(16h/b\right)^2}{1 + \left(16h/b\right)^2}$

Landing Roll - sg

$$m\frac{dV}{dt} < 0$$

$$ds = \frac{m}{2}\frac{d\left(V_\infty^2\right)}{-T_R - D - \mu_r\left(W - L\right)}$$

$$s_g - s_{fr} = \frac{W}{2g}\int_{V_{TD}}^{0}\frac{d\left(V_\infty^2\right)}{-T_R - D - \mu_r\left(W - L\right)}; \qquad s_{fr} = NV_{TD} \quad \text{N=1 (mil); 3 (comm)}$$

$$s_g = NV_{TD} + \frac{W}{2g}\int_0^{V_{TD}}\frac{d\left(V_\infty^2\right)}{T_R + D + \mu_r\left(W - L\right)} = NV_{TD} + \frac{WV_{TD}^2}{2g}\frac{1}{T_R + \left[D + \mu_r\left(W - L\right)\right]_{0.7V_{TD}}}$$

$$V_{TD} \geq 1.15V_{stall} = \text{j}V_{stall} \quad \text{commercial airplanes}$$
$$V_{TD} \geq 1.10V_{stall} = \text{j}V_{stall} \quad \text{military airplanes}$$

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S C_{L_{\max}}}}$$

$$s_g = jN\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{C_{L\max}}} + j^2\frac{W/S}{\rho_\infty g C_{L\max}}\frac{1}{T_R/W + \left[D/W + \mu_r\left(1 - L/W\right)\right]_{0.7V_{TD}}}$$