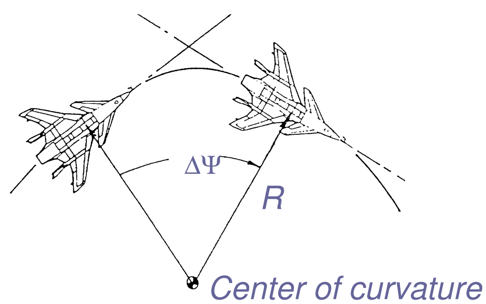
AE 429 - Aircraft Performance and Flight Mechanics

Level Turn, Pull Up
and Pull Down

Turning Performance

- What is a turn?

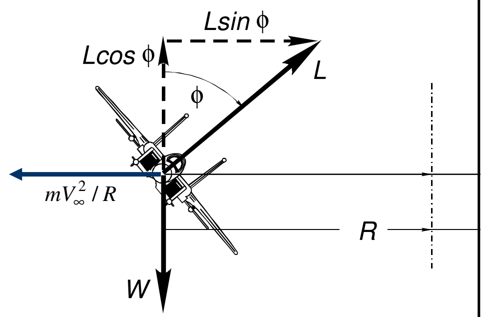


- a turn is a change in flight path direction
- turn rate is the time rate of change in heading

$$\dot{\Psi} \equiv \lim_{\Delta t \to 0} \frac{\Delta\Psi}{\Delta t}$$

Turning Performance

- More definitions
  - Turn radius, R, is the distance between the flight path and the instantaneous center of curvature



- Load factor and turn radius
  - Load factor n is defined as

$$n \equiv \frac{L}{W}$$

  - In a level, un-accelerated turn

$$W = L\cos\phi$$

$$n \equiv \frac{L}{W} = \frac{1}{\cos\phi}$$

  - N is a function of ϕ (bank angle) only in a steady, level turn

$$\cos\phi = W/L = 1/(L/W) = 1/n \qquad \phi = \arccos(1/n)$$

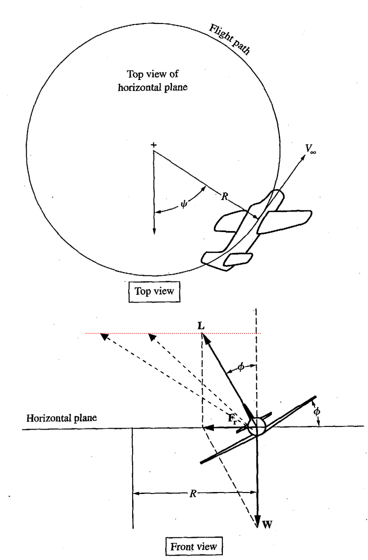


Figure 6.1 An airplane in a level turn.

$$L\cos\phi = W$$ for level turn, constant altitude

Perpendicular to flight path in the horizontal plane $\quad r_2 = R$

$$m\frac{(V_\infty \overset{1}{\cancel{\cos\theta}})^2}{r_2} = L\sin\phi + T\underset{0}{\cancel{\sin\varepsilon}}\sin\phi$$

Performance parameters:

Turn radius R

Turn rate $\omega = d\psi/dt$

ψ local angular velocity along the curved flight path

Larger the magnitude of F_r : tighter and faster will be the turn

Note: L and ϕ are not independent in level turn

Turn Radius

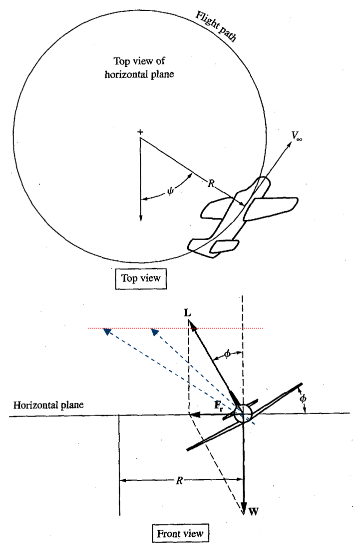


Figure 6.1 An airplane in a level turn.

$$m\frac{(V_\infty)^2}{R} = L\sin\phi \quad \Rightarrow$$

$$R = m\frac{V_\infty^2}{L\sin\phi} = \frac{W}{L}\frac{V_\infty^2}{g\sin\phi} = \frac{1}{n}\frac{V_\infty^2}{g\sin\phi}$$

$$\cos\phi = 1/n \qquad \cos^2\phi + \sin^2\phi = 1$$

$$1/n^2 + \sin^2\phi = 1 \qquad \sin^2\phi = 1 - 1/n^2$$

$$\Rightarrow \quad \sin\phi = \sqrt{1-1/n^2}$$

$$\Rightarrow \quad R = \frac{1}{n}\frac{V_\infty^2}{g\sqrt{1-1/n^2}} = \frac{V_\infty^2}{g\sqrt{n^2-1}}$$

Small R ⇨ high n (large L/W)
⇨ low Velocity

Turn Rate

$$\omega = d\psi/dt = V_\infty/R$$

$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}} \quad \Rightarrow \quad \omega = \frac{d\psi}{dt} = \frac{V_\infty}{R} = \frac{g\sqrt{n^2-1}}{V_\infty}$$

high ω ⇨ high n (large L/W)
⇨ low Velocity

High Performance: smallest R and largest ω for **largest n ; lowers speed V**

what is the higher possible n ?

R and ω are function of n and V ⇨ Do not depend on W/S, T/W, k, C_{d0}, ρ

L ↑ ⇨ ϕ ↑ ⇨ D ↑ ⇨ T_R ↑ but T < $T_{max\,A}$ implying that for $T_{max\,A}$ ⇨ $\phi_{Tmax\,A}$

$$n = \frac{1}{\cos\phi} \quad \Rightarrow \quad n_{max} = \frac{1}{\cos\phi_{max}} = \frac{1}{\cos\phi_{T_A\,max}}$$

Level turn: D = T; L = n W = ½ ρ V² S C_L
L/D = n W/T

$$T = \frac{1}{2}\rho V^2 S\left[C_{D_0} + K\left(\frac{2nW}{\rho V^2 S}\right)^2\right]$$

$$n_{max} = \frac{\frac{1}{2}\rho V^2 S}{K(W/S)}\left[\left.\frac{T}{W}\right|_{max} - \frac{1}{2}\rho V^2\frac{C_{D_0}}{W/S}\right]^{1/2} = \frac{L}{D}\left.\frac{T}{W}\right|_{max} \qquad 1 \le n \le n_{max} \qquad n_{max} = \frac{1}{2}\rho V^2\frac{C_{L\max}}{W/S}$$

Minimum Turn Radius

- Minimum turn radius
  - Stall speed in straight and level flight (L = W) is

$$V_s = \sqrt{\frac{2W}{\rho_\infty S C_{L_{max}}}}$$

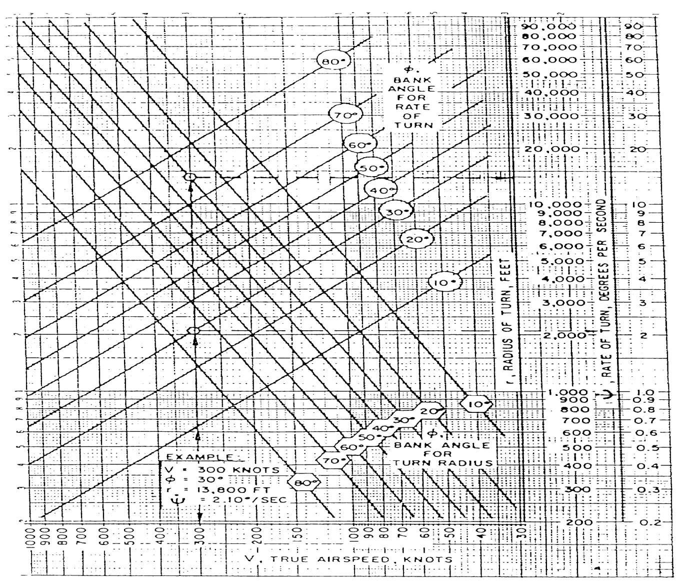
  - In a level turn, stall speed becomes (L=*n*W)

$$V_{s_{turn}} = \sqrt{\frac{2nW}{\rho_\infty S C_{L_{max}}}} \qquad V_{s_{turn}} = V_s\sqrt{n}$$

  - Which suggests that
  - Replacing V_s with $V_{s_{turn}}$ in the turn radius equation gives the aerodynamic limit on minimum turn radius

$$R_{min} = \frac{V_{s_{turn}}^2}{g\sqrt{n^2-1}} = \frac{V_s^2 n}{g\sqrt{n^2-1}} = \frac{V_s^2}{g\sqrt{1-\frac{1}{n^2}}}$$

Level Turn Chart



Pull-Up

- Consider a turn in the vertical plane: wing-level Pull-Up (instantaneous turn)
  - Different from level turn (constant flight properties)
  - The radial forces are:

$$\text{at } t=0;\ \theta=0$$

$$F_r = L - W = W(n-1)$$

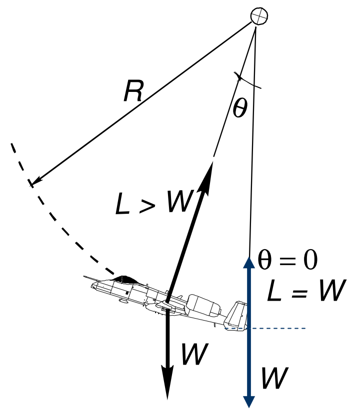
$$= m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

  - Solving for R:

$$R = \frac{V_\infty^2}{g(n-1)}$$

  - And for turn rate:

$$\omega = \frac{V_\infty}{R} = \frac{g(n-1)}{V_\infty}$$



Pull-Down

- Now, look at another instantaneous turning maneuver in the vertical plane -- *a "split s"*
  - Using the same approach as for a Pull-Up

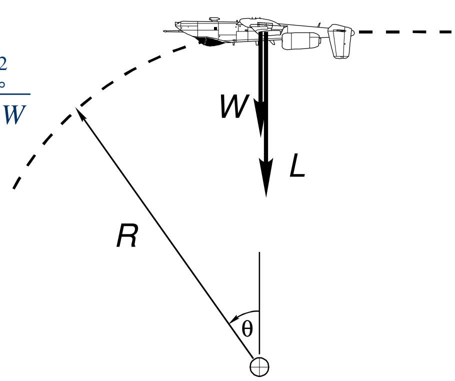
$$\text{at } t=0;\ \theta=0$$

$$m\frac{V_\infty^2}{R} = L + W \Rightarrow R = m\frac{V_\infty^2}{L+W}$$

$$R = \frac{V_\infty^2}{g(n+1)}$$

$$\omega = \frac{g(n+1)}{V_\infty}$$

  - The rate is improved and the radius is enlarged over pull-ups



Limiting cases: n large

- Effect of W/S (wing loading) and $C_{l_{max}}$
  - *When n is large,*

$$n+1 \approx n-1 \approx n \quad \Rightarrow \quad R \approx \frac{V_\infty^2}{gn}, \quad \omega \approx \frac{gn}{V_\infty}$$

  - *Recalling that* $V_\infty^2 = \dfrac{2L}{\rho_\infty S C_L}$
  - *Substituting, we obtain radius and rate of turn*

$$R = \frac{2L}{\rho_\infty S C_L g (L/W)} \quad \Rightarrow \quad R = \frac{2}{\rho_\infty C_L g} \frac{W}{S}$$

$$\omega = \frac{gn}{\sqrt{2L / \rho_\infty S C_L}} \quad \Rightarrow \quad \omega = g \sqrt{\frac{\rho_\infty C_L n}{2 (W/S)}}$$

- For minimum turn radius and maximum turn rate
  - *Maximize both C_L and load factor*

$$R_{min} = \frac{2}{\rho_\infty g C_{L_{max}}} \frac{W}{S} \qquad \omega_{max} = g \sqrt{\frac{\rho_\infty C_{L_{max}} n_{max}}{2 (W/S)}}$$

  - *Practical constraints on load factor*
    - n_{max} is a function of C_{Lmax};
      - at low speeds it will be limited by the aerodynamic lifting capability (stall) of the lifting surfaces

$$n_{max} = \frac{1}{2} \rho_\infty V_\infty^2 \frac{C_{L_{max}}}{W/S}$$

      - at high speeds, structural loads on the airframe may also limit n_{max}
    - for many airplanes, the other force balance (T = D) governs the minimum turn radius and the maximum turn rate -- turn performance is limited by available thrust

- *constraints on V*
  - *V as small as possible for R_{min} and ω_{max}*

L = n W = ½ ρ V² S C_L

$$V_\infty = \sqrt{\frac{2nW}{\rho_\infty S C_L}} \quad \Rightarrow \quad C_L = C_{L\max} \quad \Rightarrow \quad V_{Stall} = \sqrt{\frac{2nW}{\rho_\infty S C_{L\max}}}$$

  - *R_{min} does not necessarily correspond to n_{max}*

$$1 \le n \le n_{\max}$$

$$R = \frac{V_\infty^2}{g\sqrt{n^2-1}} = \frac{2q_\infty}{g\rho_\infty\sqrt{n^2-1}} \quad \Rightarrow \frac{\partial R}{\partial q_\infty} = 0$$

$$R_{\min} = \frac{4k(W/S)}{g\rho_\infty (T/W)\sqrt{1-4kC_{Do}/(T/W)^2}}$$

$$\omega_{\max} = q_\infty\sqrt{\rho_\infty/(W/S)[(T/W)/(2k)-(C_{Do}/k)^{1/2}]}$$

$$n_{R\min} = \sqrt{2-4kC_{Do}/(T/W)^2}$$

$$V_{\infty R\min} = \sqrt{4k(W/S)/(\rho_\infty (T/W))}$$

V-n diagram

- the *V-n* diagram illustrates 2 of these constraints

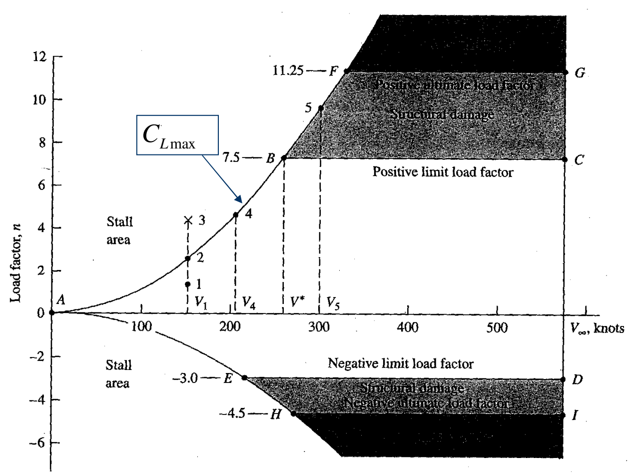
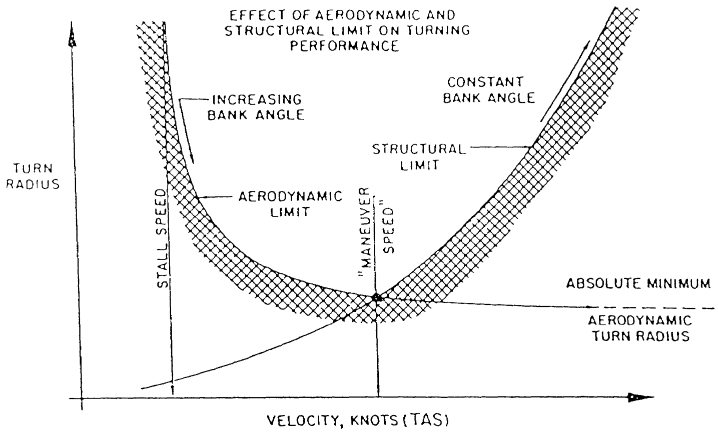


Figure 6.7 The V–n diagram for a typical jet trainer aircraft. Free-stream velocity V_∞ is given in knots. 1 knot (kn) = 1.15 mi/h.

Corner Velocity or maneuver velocity

$$V^* = \sqrt{\frac{2n_{\max}}{\rho_\infty C_{L\max}}\frac{W}{S}}$$

- Aerodynamic and structural limits on turn performance



- Aerodynamic and thrust limits on turn performance

Aerodynamics Wing Design

$$n_{max} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L_{max}}}{W/S}$$

Thrust Available Drag

Structural (Materials/ Wing Size)