AE 429 - Aircraft Performance and Flight Mechanics

Range and Endurance

Range and Endurance

- Definitions
  - Range
    - Total ground distance traversed on a full tank of fuel -- Anderson book
    - Distance an airplane can fly on a given amount of fuel -- alternative definition
  - Endurance
    - Total time an airplane stays in the air on a full tank of fuel -- Anderson book
    - Time an airplane can fly on a given amount of fuel -- alternative definition
  - Specific fuel consumption
    - Weight of fuel consumed per unit power per unit time

Range and Endurance

Propeller-Driven Airplane

- Generalized endurance parameters
  - Specific fuel consumption (SFC or c) $\frac{LBS\ FUEL}{BHP-HR}$
  - Weight (W) lbs
  - Time (t) secs or hrs
  - Engine power (P) hp
  - Airplane gross weight (W_0) lbs
    includes fuel and payload
  - Airplane empty weight (W_1) lbs
    weight of airplane without fuel $W = W_1 + W_f$
- Endurance calculation
  - Fuel flow rate $\dot{w}_f = -cP$ (lbs fuel/unit time) = $\frac{dW_f}{dt}$ $c = -\frac{\dot{W}_f}{P}$
  - Differential fuel burned $dW_f = dW = -c\,Pdt$
  - Solving for time and integrating

$$E = \int_0^E dt = -\int_{W_0}^{W_1} \frac{dW_f}{cP} = \int_{W_1}^{W_0} \frac{dW}{cP}$$

Range and Endurance

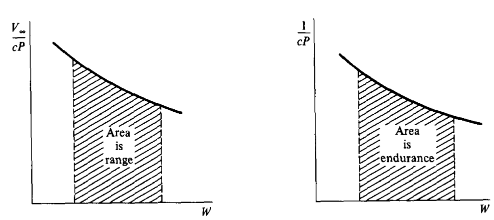
Propeller-Driven Airplane

- Generalized Range Equation
  - Multiply V_∞ by $dt = -\frac{dW}{cP}$ to get differential air distance covered

$$ds = V_\infty dt = -V_\infty \frac{dW}{cP}$$

  - The total distance covered during a range flight is the integral of this expression

$$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty dW}{cP} = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP}$$



Range and Endurance

Propeller-Driven Airplane

- Breguet Range Equation
  - In level flight cruise (no acceleration), the throttle is set to maintain constant airspeed
    - $P_A = P_R = DV_\infty$
    - Shaft HP $= P = \dfrac{P_A}{\eta} = \dfrac{DV_\infty}{\eta}$
  - Substituting into the general range equation for propeller airplanes

$$R = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP} = \int_{W_1}^{W_0} \frac{V_\infty \eta dW}{cDV_\infty} = \int_{W_1}^{W_0} \frac{\eta dW}{cD}$$

  - Multiplying by W / W and using L= W for equilibrium cruise flight (level and unaccelerated)

$$R = \int_{W_1}^{W_0} \frac{\eta}{c}\frac{W}{W}\frac{dW}{D} = \int_{W_1}^{W_0} \frac{\eta}{c}\frac{L}{D}\frac{dW}{W}$$

Range and Endurance

Propeller-Driven Airplane

- Breguet Range Equation (continued)

  Assume

  h is constant throughout flight

  L/D is constant throughout flight

  c is constant throughout flight

  Then, we can integrate

$$R = \frac{\eta}{c}\frac{L}{D}\int_{W_1}^{W_0} \frac{dW}{W} \Rightarrow R = \frac{\eta}{c}\frac{L}{D}\ln\frac{W_0}{W_1}$$

- Because of the assumptions listed, the Breguet approximation is valid only for small dW
  - Angle of attack changes over a weight change as large as the fuel load
  - If weight changes significantly, L/D changes

Range and Endurance

Propeller-Driven Airplane

- Breguet Endurance Equation

Using the same assumptions (L = W and $P_A = P_R = DV_\infty$)

$$E = \int_{W_1}^{W_0} \frac{dW}{cP} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{dW}{DV_\infty} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{L}{DV_\infty} \frac{dW}{W}$$

Since

$$L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L, \quad V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

and

$$E = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{C_L}{C_D} \sqrt{\frac{\rho_\infty S C_L}{2}} \frac{dW}{W^{3/2}}$$

If we assume $C_L,\ C_D,\ \eta,\ c,$ and ρ_∞ are all constant

$$E = \frac{\eta}{c} \frac{C_L^{3/2}}{C_D} \sqrt{2\rho_\infty S} \left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}} \right)$$

in using this equation, E must be in seconds and c must be in ft^{-1}

Importance of the Ratios C_L/C_D , $C_L^{3/2}/C_D$, $C_L^{1/2}/C_D$

$$\left.\frac{C_L}{C_D}\right|_{max}$$ ⇨ Max Range for reciprocating engine/propeller airplanes

$$\left.\frac{C_L^{3/2}}{C_D}\right|_{max}$$ ⇨ Max Endurance for reciprocating engine/propeller

Range and Endurance

Jet-powered airplane

- **General endurance equation**
  - Specific fuel consumption is based on thrust, not power output

$$TSFC \equiv c_t \Rightarrow \frac{lbs\ of\ fuel\ consumed}{lb\ of\ thrust\ produced - sec}$$

  - Again, let dW be the differential change in aircraft weight as fuel is burned in dt

$$c_t = -\frac{\dot{W}_f}{T_A}$$

$$dW = -c_t T_A dt \quad \Rightarrow \quad dt = -\frac{dW}{c_t T_A}$$

  - Integrating: $E = \int_0^E dt = -\int_{W_0}^{W_1} \frac{dW}{c_t T_A} = \int_{W_1}^{W_0} \frac{dW}{c_t T_A}$
  - Recalling that L = W and $T_A = T_R = D$

$$E = \int_{W_1}^{W_0} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W}$$

  - Assuming constant c_t and L/D
  - The Breguet approximation is:

$$E \cong \frac{1}{c_t} \frac{L}{D} \ln \frac{W_0}{W_1}$$

Range and Endurance

Jet-powered airplane

- **To maximize endurance**
  - Reduce thrust specific fuel consumption, TSFC
    - Use a turbofan engine
    - Fly at an altitude where the engine is efficient (high, but not too high)
  - increase fuel fraction, W_f / W_0, to increase W_0 / W_1
    - Carry less payload
    - Use drop tanks
  - maintain L/Dmax
    - Loiter at velocity for L/Dmax (where $C_{D,0} = C_{Di}$)
    - Cruise climb
    - Notice that a jet airplane has best endurance at this velocity for L/Dmax, while a propeller-driven airplane gets best range at this true airspeed

Range and Endurance

Jet-powered airplane

- **general range equation**
  - going back to the differential time expression and multiplying by V

$$ds = V_\infty dt = -\frac{V_\infty dW}{c_t T_A}$$

  - of course, ds is the differential distance traveled by the jet during time dt
    - s = 0 when $w = w_0$
    - s = R when $w = w_1$
  - integrating to get range $\quad R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty dW}{c_t T_A}$
  - for steady, unaccelerated level flight: $T_A = T_R = \frac{W}{L/D}$

$$R = \int_{W_1}^{W_0} \frac{V_\infty}{c_t} \frac{C_L}{C_D} \frac{dW}{W}$$

Range and Endurance

Jet-powered airplane

- **general range equation (continued)**
  - since $V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$, R becomes $\quad R = \int_{W_1}^{W_0} \sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{\sqrt{C_L}}{C_D} \frac{dW}{\sqrt{W}}$
- **Breguet approximation**
  - if we assume constant c_t, $C_{L,}$ C_D, and ρ

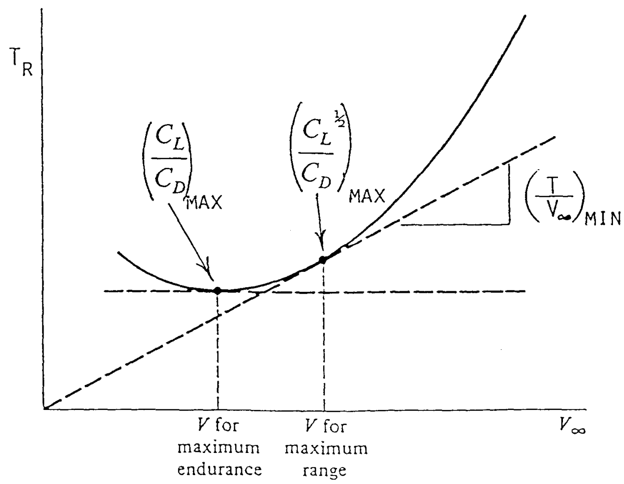
$$R = 2\sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{\sqrt{C_L}}{C_D} \left(\sqrt{W_0} - \sqrt{W_1}\right)$$

  - for maximum range
    - reduce TSFC, c_t
    - fly at low density (high altitude)
    - use a true airspeed to maximize ⟹ $\frac{C_L^{1/2}}{C_D}$

Range and Endurance

Jet-powered airplane

- Cruise for Max range
  - Fly at $\left(\frac{\sqrt{C_L}}{C_D}\right)_{max}$
  - The true air-Speed for This optimum Speed gives $C_{D,0} = 3C_{d_i}$



Importance of the Ratios C_L/C_D, $C_L^{3/2}/C_D$, $C_L^{1/2}/C_D$

$\left.\frac{C_L}{C_D}\right|_{\max}$ ⇨ Max Endurance for jet-propelled airplanes

$\left.\frac{C_L^{1/2}}{C_D}\right|_{\max}$ ⇨ Max Range for jet-propelled airplanes

Range and Endurance

Summary

- Relationship of $C_{D,0}$ and C_{D_i}
  - At L/D_{max}
    - Using $C_D = C_{D,0} + \frac{C_L^2}{\pi e AR} \implies \frac{C_L}{C_D} = \frac{C_L}{C_{D,0} + \frac{C_L^2}{\pi e AR}}$
    - Differentiating with respect to C_L and setting to zero

$$\frac{d\left(\frac{C_L}{C_D}\right)}{dC_L} = \frac{C_{D,0} + \frac{C_L^2}{\pi e AR} - C_L\left(2\frac{C_L}{\pi e AR}\right)}{\left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right)^2} = 0$$

$$C_{D,0} + \frac{C_L^2}{\pi e AR} - 2\left(\frac{C_L^2}{\pi e AR}\right) = 0 \quad \Rightarrow \quad C_{D,0} = \frac{C_L^2}{\pi e AR}$$

$$C_{D,0} = C_{D,i} \quad FOR \quad \left(\frac{C_L}{C_D}\right)_{max}$$

Range and Endurance

Summary

- Relationship of $C_{D,0}$ and C_{D_i}
  - Similar developments show:

$$C_{D,0} = 3C_{D,i} \quad FOR \quad \left(\frac{\sqrt{C_L}}{C_D}\right)_{max}$$

- Applies to maximum range for a jet airplane

$$3C_{D,0} = C_{D,i} \quad FOR \quad \left(\frac{\sqrt{C_L^3}}{C_D}\right)_{max}$$

- Applies to maximum endurance for a propeller-driven airplane

RANGE

PROPELLER/RECIPR. ENGINE

$c = -\dot{W}_f / P_{shaft}$

JET-PROPELLED AIRPLANE

$c_t = -\dot{W}_f / T_A$

$$c_t = \frac{c V_\infty}{\eta_{PR}}$$

since

$$T_A = \frac{P_s \eta_{PR}}{V_\infty} \; ; \quad \text{NOTE } P_A = T_A V_\infty$$

$$V_\infty = \frac{ds}{dt} \Rightarrow ds = V_\infty dt$$

$$c_t = -\frac{dW_f/dt}{T}$$

$$dt = -\frac{dW_f}{c_t T} \Rightarrow ds = \frac{-V_\infty}{c_t T} dW_f$$

$$\boxed{dW_f = dW} \rightarrow ds = -\frac{V_\infty}{c_t T} dW = -\frac{V_\infty W}{c_t T} \frac{dW}{W}$$

$$ds = -\frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W}$$

$$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W}$$

$$\boxed{R = \int_{W_1}^{W_0} \frac{V_\infty}{c_t} \frac{L}{D} \frac{dW}{W} \qquad \text{RANGE}}$$

ENDURANCE

$$\frac{dW_f}{dt} = -c_t T$$

$$dt = -\frac{dW_f}{c_t T} = -\frac{dW_f}{c_t D} = -\frac{dW_f}{W}\frac{L}{D}\frac{1}{c_t}$$

($L = W$, $D = T$)

$$E = -\int_{W_0}^{W_1} \frac{1}{c_t}\frac{L}{D}\frac{dW_f}{W} = \int_{W_1}^{W_0} \frac{1}{c_t}\frac{L}{D}\frac{dW}{W}$$

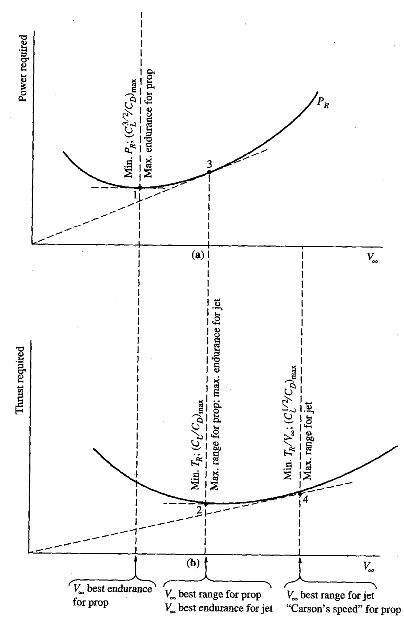


Figure 5.47 Graphical summary of conditions for maximum range and endurance.