AE 429 - Aircraft Performance and Flight Mechanics

Rate of Climb

Time to Climb

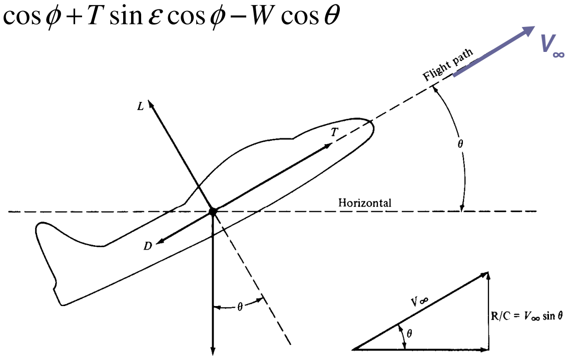
Rate of Climb R/C

- Now let's analyze a steady climb
  - Forces include a gravity component now

$$m\frac{dV_\infty}{dt} = T\cos\varepsilon - D - W\sin\theta$$

$$m\frac{V_\infty^2}{r_1} = L\cos\phi + T\sin\varepsilon\cos\phi - W\cos\theta$$

$$\Longrightarrow \quad T = D + W\sin\theta \qquad L = W\cos\theta$$



The rate of climb (**R**/**C**) is the vertical component of velocity

Rate of Climb R/C

- For low climb angles (up to about 20°)
  - We can assume $\cos\theta \approx 1$ to calculate C_L
  - So we work on the drag equation, multiplying by V_∞

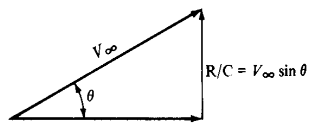
$$TV_\infty = DV_\infty + WV_\infty \sin\theta$$

Excess Power

$$\frac{TV_\infty - DV_\infty}{W} = \frac{\boxed{(T-D)V_\infty}}{W} = V_\infty \sin\theta$$

  - In climb, the rate of climb is the vertical component of velocity

$$R/C = V_\infty \sin\theta$$



- Maximum R/C
  - Maximize excess power
  - Minimize weight
- Maximum angle of climb
  - Maximize excess thrust
  - Minimize weight

$$R/C = \frac{(T-D)V_\infty}{W}$$

$$\theta = \sin^{-1}\frac{(T-D)}{W} \approx \frac{(T-D)}{W}$$

Rate of Climb R/C $\equiv \frac{(T-D)V_\infty}{W}$

$$C_L = \frac{L}{q_\infty S} = \frac{W\cos\theta}{q_\infty S}$$

$$D = q_\infty S C_D = q_\infty S\left(C_{D,0} + KC_L^2\right) = q_\infty S\left(C_{D,0} + K\left(\frac{W\cos\theta}{q_\infty S}\right)^2\right) =$$

$$= q_\infty S C_{D,0} + \frac{KW^2\cos^2\theta}{q_\infty S}$$

$$R/C = V_\infty \sin\theta = \frac{(T-D)V_\infty}{W} = V_\infty\left(\frac{T}{W} - \frac{D}{W}\right)$$

$$R/C = V_\infty \sin\theta = V_\infty\left[\frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2\left(\frac{W}{S}\right)^{-1} C_{D,0} - \frac{2K}{\rho_\infty V_\infty^2}\frac{W}{S}\cos^2\theta\right]$$

Preliminary design $\cos\theta \approx 1$ OK for $\theta \le 50^\circ$

Maximum Climb Angle

θmax

$$\sin\theta = \frac{T}{W} - \frac{D}{W} = \frac{T}{W} - \frac{D}{L/\cos\theta} \overset{\cos\theta \approx 1}{=} \frac{T}{W} - \frac{1}{L/D}$$

$$\sin\theta_{\max} = \frac{T}{W} - \frac{1}{(L/D)|_{\max}} = \frac{T}{W} - \sqrt{4C_{D,0}K}$$

$$L = W\cos\theta = \frac{1}{2}\rho_\infty V_\infty^2 S C_L \qquad C_L = \sqrt{\frac{C_{D,0}}{K}} \quad \text{from L/D max}$$

$$L = W\cos\theta_{\max} = \frac{1}{2}\rho_\infty V_{\theta\max}^2 S\sqrt{\frac{C_{D,0}}{K}}$$

Speed at max θ for jet propelled airplane

$$V_{\theta\max}^2 = \frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}}\frac{W}{S}\cos\theta_{\max}$$

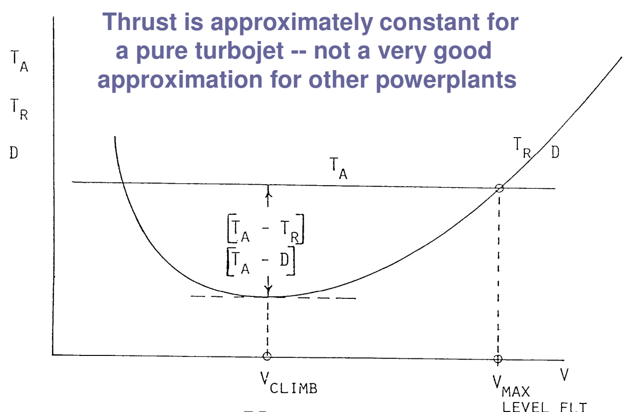
Rate of climb at max θ

$$(R/C)_{\theta_{\max}} = V_{\theta_{\max}}\sin\theta_{\max}$$

Max Angle to Climb

- If thrust available is constant with V
  - Maximum climb angle occurs at minimum drag
  - This speed also gives best acceleration in level flight

$$\theta = \frac{(T_A - D)}{W}$$



$V_{\theta\max}$

Rate of Climb R/C

- Typically, climbs are not flown at constant V_∞

$$T = D + W\sin\theta + \frac{W}{g}\frac{dV_\infty}{dt} = D + W\sin\theta + \frac{W}{g}\frac{dV_\infty}{dh}\frac{dh}{dt}$$

- Recognizing

$$\frac{dh}{dt} = R/C = V_\infty \sin\theta$$

- and rearranging

$$\frac{T-D}{W} = \sin\theta + \frac{1}{g}\frac{dV_\infty}{dh}V_\infty \sin\theta \Rightarrow \sin\theta = \frac{\frac{T-D}{W}}{1+\frac{V_\infty}{g}\frac{dV_\infty}{dh}} = \frac{T-D}{W}\frac{1}{k}$$

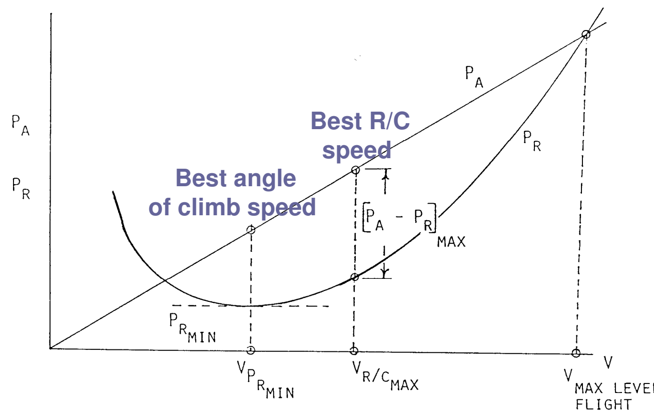
$$k = 1 + \frac{V_\infty}{g}\frac{dV_\infty}{dh}$$

Rate of Climb R/C

- For our idealized jet airplane, best rate of climb does not occur at minimum power required
  - Maximum rate of climb occurs at the velocity where excess power is greatest
  - The velocity for maximum rate of climb is determined for any aircraft by
    - Plotting the power required versus true airspeed
    - Overplotting the power available versus true airspeed
    - Choosing the velocity where the distance between the two curves is greatest

Rate of Climb R/C

- The chart below illustrates this procedure



Rate of Climb R/C

Max Rate of Climb

$$R/C = V_\infty\left[\frac{T}{W} - \frac{1}{2}\rho_\infty V_\infty^2\left(\frac{W}{S}\right)^{-1}C_{D,0} - \frac{2K}{\rho_\infty V_\infty^2}\frac{W}{S}\right]$$

$$\frac{d(R/C)}{dV_\infty} = \left[\frac{T}{W} - \frac{3}{2}\rho_\infty V_\infty^2\left(\frac{W}{S}\right)^{-1}C_{D,0} + \frac{2K}{\rho_\infty V_\infty^2}\frac{W}{S}\right] = 0$$

$$V_{(R/C)_{\max}} = \left\{\frac{(T/W)(W/S)}{3\rho_\infty C_{D,0}}\left[1 + \sqrt{1 + \frac{3}{(L/D)_{\max}^2 (T/W)^2}}\right]\right\}^{1/2}$$

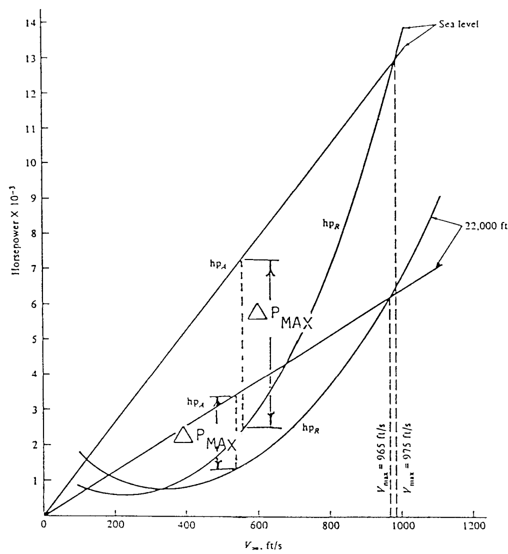
$$(R/C)_{\max} = \left[\frac{(W/S)Z}{3\rho_\infty C_{D,0}}\right]^{1/2}\left(\frac{T}{W}\right)^{3/2}\left[1 - \frac{Z}{6} - \frac{3}{2(T/W)^2 (L/D)_{\max}^2 Z}\right]$$

$$Z \equiv 1 + \sqrt{1 + \frac{3}{(L/D)_{\max}^2 (T/W)^2}}$$

Rate of Climb R/C: Effect of the Altitude

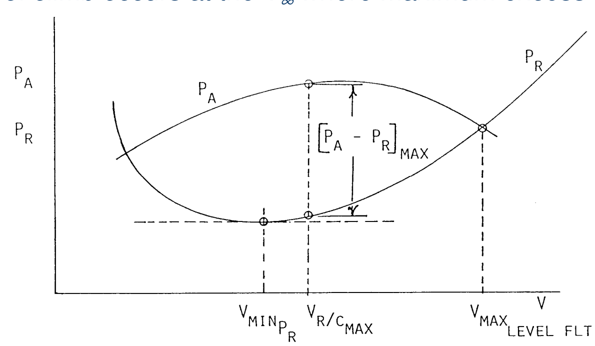
- This chart shows the effect of altitude
  - At higher true airspeeds, P_R decreases

    with altitude
  - However, P_A falls off faster than P_R
  - The best climb speed usually decreases slightly with altitude



Rate of Climb R/C

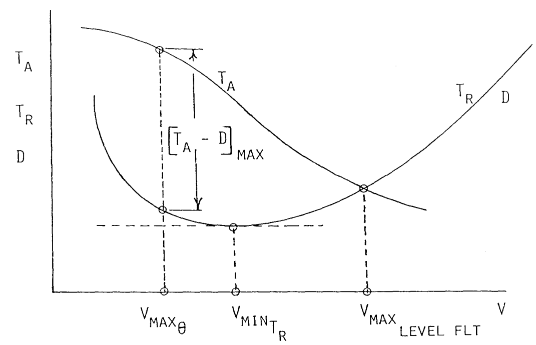
- For a propeller aircraft
  - *Maximum rate of climb occurs at the V_∞ where maximum excess power occurs*



- *Maximum R/C does not occur at V_{minPR}*
- *However, if P_R is assumed constant with V, R/C_{max} does occur at V_{minPR}*
- *DV_∞ is approximately the power required in the climb*

Rate of Climb R/C

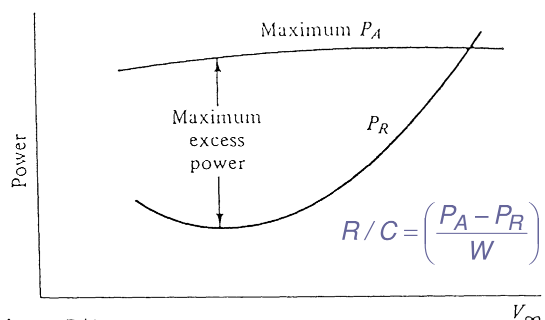
- For propeller aircraft
  - Maximum angle of climb occurs at the V_∞ for which maximum excess thrust occurs



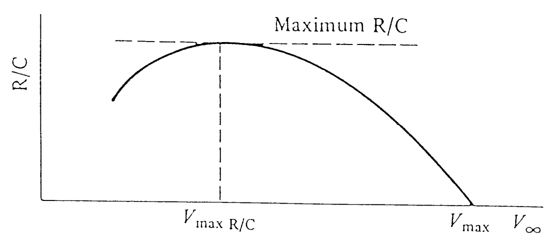
- Maximum climb angle (which is used to clear obstacles on takeoff) occurs at a velocity < V_{minTR}

Rate of Climb R/C

- For a given altitude
  - For any type of airplane, excess power determines R/C
  - Induced drag changes P_R



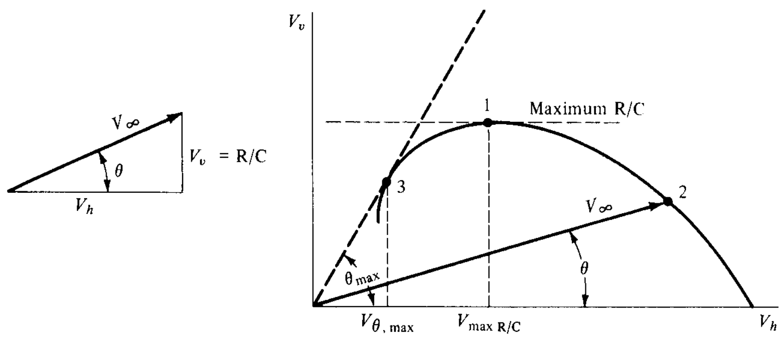
$$R/C = \left(\frac{P_A - P_R}{W}\right)$$



- R/C changes with velocity
- Plots like the one on left allow determination of best R/C speed

Rate of Climb R/C

- Climb performance hodograph
  - Vertical velocity versus horizontal velocity

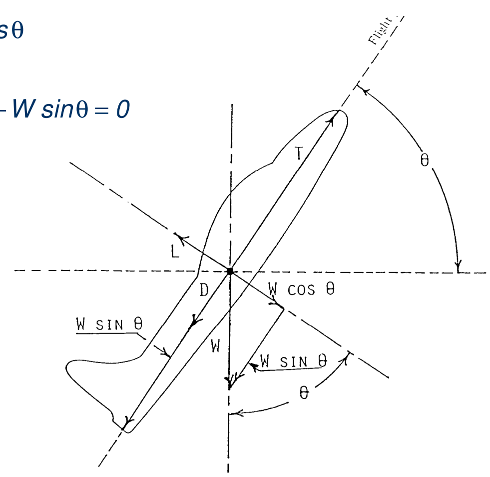


  - Notice the difference in velocity for θ_{max} and the velocity for $(R/C)_{max}$

Rate of Climb R/C: High performance climb

  - Lift forces Eq. $L = W\cos\theta$
  - Drag forces Eq. $T - D - W\sin\theta = 0$
  - Solving the two equations for θ
    - solve each for qS
    - set $qS = qS$
    - solve the resulting quadratic for θ

$$\sin\theta = \frac{C_L^2 T}{W\left(C_L^2 + C_D^2\right)} - \sqrt{\left(\frac{C_L^2 T}{W\left(C_L^2 + C_D^2\right)}\right)^2 - \frac{C_L^2 T^2 - C_D^2 W^2}{W\left(C_L^2 + C_D^2\right)}}$$



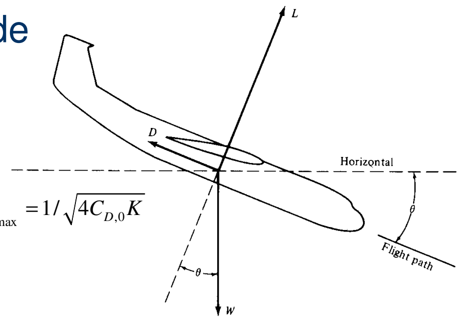
Gliding flight

- Forces in a power-off glide

$D = W \sin\theta \qquad L = W \cos\theta$

  - Dividing drag by lift

$$\tan\theta = \frac{1}{L/D} \qquad \tan\theta_{\min} = \frac{1}{L/D|_{\max}} \qquad (L/D)|_{\max} = 1/\sqrt{4C_{D,0}K}$$



  - Once again, an important performance parameter is set by L/D
    - the smallest θ gives maximum gliding range
    - this maximum range occurs when L/D is maximum for V_∞ constant
    - airplanes with good aerodynamic efficiency (high L/D) can glide 20-50 times as far as their altitude
    - Does not depend on wing loading or altitude.

$$L = \frac{1}{2}\rho_\infty V_\infty^2 S C_L = W\cos\theta$$

$$V_\infty = \sqrt{\frac{2\cos\theta}{\rho_\infty C_L}\frac{W}{S}}$$

Equilibrium glide velocity

$$V_\infty = \sqrt{\frac{2\cos\theta}{\rho_\infty C_L}\frac{W}{S}}$$

Equilibrium glide velocity

$$V_V = V_\infty \sin\theta$$
$$V_H = V_\infty \cos\theta$$

$$D = W\sin\theta$$
$$DV_\infty = W\sin\theta V_\infty = WV_V$$
$$V_V = \frac{DV_\infty}{W} = \frac{P_R}{W}$$

$$(L/D)|_{\max} = 1/\sqrt{4C_{D,0}K}$$

$$\cos\theta \approx 1$$

$$\boxed{\left(\frac{L}{D}\right)_{\max} = \left(\frac{C_L}{C_D}\right)_{\max} = \sqrt{\frac{1}{4C_{D,0}K}}}$$

$$L = W = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$\boxed{V_{(L/D)_{\max}} = \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{C_{D,0}}}\frac{W}{S}\right)^{1/2}}$$

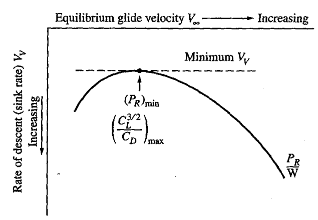


Figure 5.42 Rate of descent versus equilibrium guide velocity.

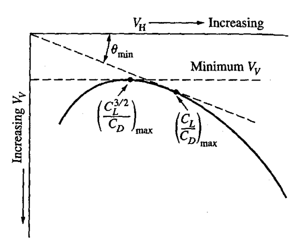


Figure 5.43 Hodograph for unpowered flight.

or

$$V_\infty = \sqrt{\frac{2W\cos\theta}{\rho_\infty S C_L}} \qquad \textbf{[5.130]}$$

Substituting Eq. (5.130) into Eq. (5.128), we have

$$V_V = V_\infty \sin\theta = (\sin\theta)\sqrt{\frac{2\cos\theta}{\rho_\infty C_L}\frac{W}{S}} \qquad \textbf{[5.131]}$$

Dividing Eq. (5.124) by Eq. (5.123), we obtain

$$\sin\theta = \frac{D}{L}\cos\theta = \frac{C_D}{C_L}\cos\theta \qquad \textbf{[5.132]}$$

Inserting Eq. (5.132) into Eq. (5.131), we have

$$V_V = \sqrt{\frac{2\cos^3\theta}{\rho_\infty (C_L^3/C_D^2)}\frac{W}{S}} \qquad \textbf{[5.133]}$$

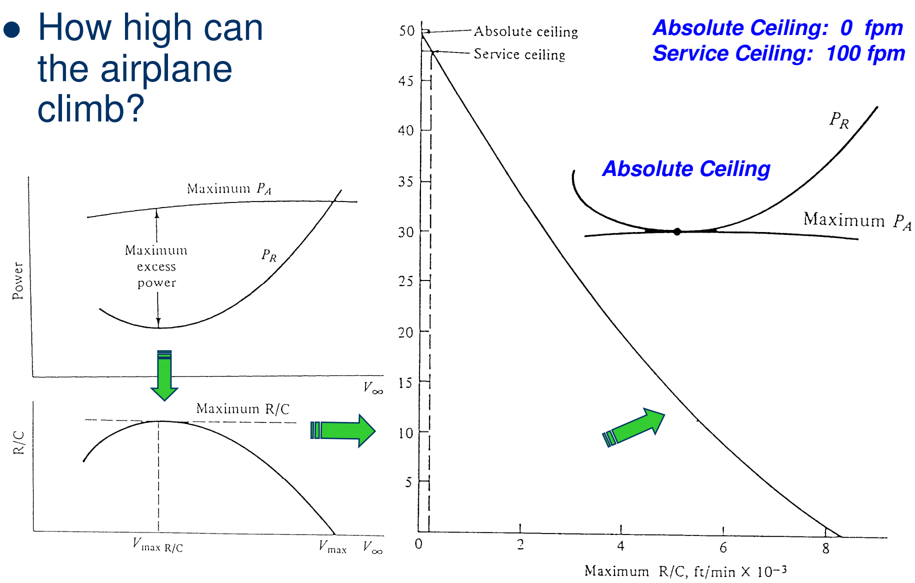
By making the assumption that $\cos\theta = 1$, Eq. (5.133) is written as

$$\boxed{V_V = \sqrt{\frac{2}{\rho_\infty (C_L^3/C_D^2)}\frac{W}{S}}} \qquad \textbf{[5.134]}$$

Equation (5.134) explicitly shows that $(V_V)_{\min}$ occurs at $(C_L^{3/2}/C_D)_{\max}$. It also shows that the sink rate decreases with decreasing altitude and increases as the square root of the wing loading.

Ceilings

- How high can the airplane climb?



Ceilings

- The ceiling is the altitude at which R/C has reached some minimum value
  - Absolute ceiling
    - Is defined as the altitude at which the $R/C = 0$
    - Is dictated when P_A is just tangent to the P_R curve
  - Service ceiling
    - is defined as that altitude where $R/C_{max} = 100\ ft/min$
    - is the practical upper limit for steady, level flight
  - Procedure
    - calculate values of R/C_{max} for different altitudes
    - plot R/C_{max} versus altitude
    - extrapolate this latter curve to 100 fpm and 0 fpm to get the service and absolute ceilings

Time to Climb

- Time to climb
  - Needs to be short
  - Calculating R/C

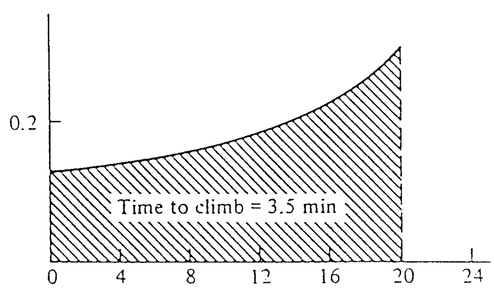
$$R/C = V_\infty \sin\theta = \frac{dh}{dt}$$

$$dt = \frac{dh}{R/C}$$

  - Integrating

$$\int_{t_1}^{t_2} dt = \int_{h_1}^{h_2} \frac{dh}{R/C} \approx \sum_{i=1}^{n} \left(\frac{\Delta h}{R/C}\right)_i$$

  - Calculating time-to-climb graphically
    - plot $(R/C)^{-1}$ versus h
    - *Approximate the area under the curve*
    - *Subtract time to climb to the starting altitude*



$$(R/C)_{\max} = a + bh$$

$$t_{\min} = \int_0^{t_{\min}} dt = \int_0^{h_2} \frac{dh}{a+bh} = \frac{1}{b}\left[\ln(a+bh_2) - \ln(a)\right]$$