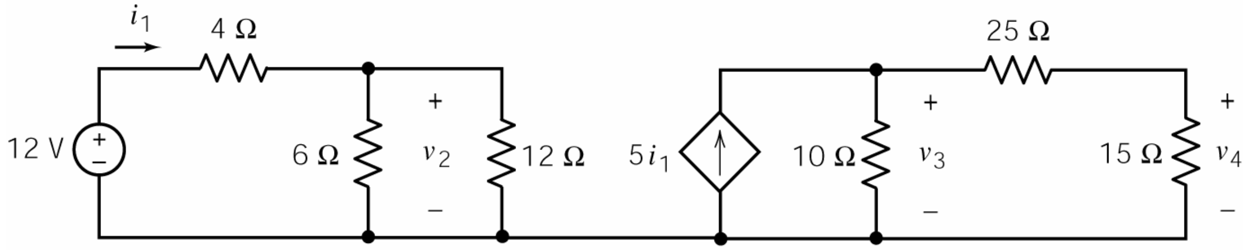
Example



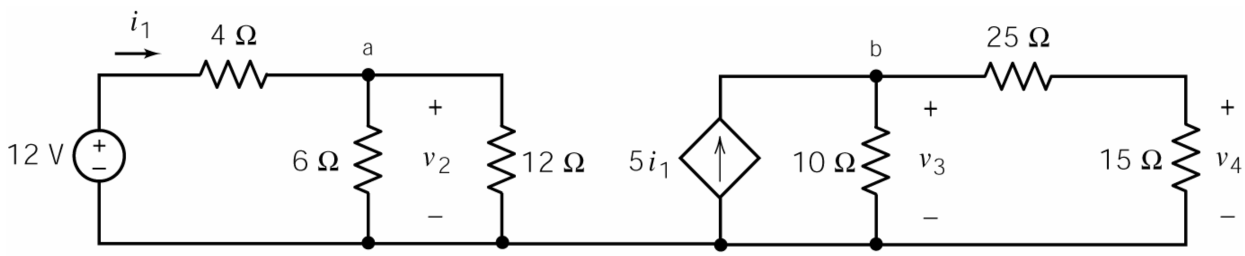
Computer analysis of this circuit gives

$$i_1 = 1.5 \text{ A}, \quad v_2 = 6 \text{ V}, \quad v_3 = 60 \text{ V} \quad \text{and} \quad v_4 = 22.5 \text{ V}$$

How can we check that these values are correct?

Solution:

1. Check KCL.
Label nodes a and b:



Apply KCL at node a to get

$$i_1 = \frac{v_2}{6} + \frac{v_2}{12}$$

Substituting the given values we get

$$1.5 = \frac{6}{6} + \frac{6}{12}$$

Hence KCL is satisfied at node a. Next, apply KCL at node b to get

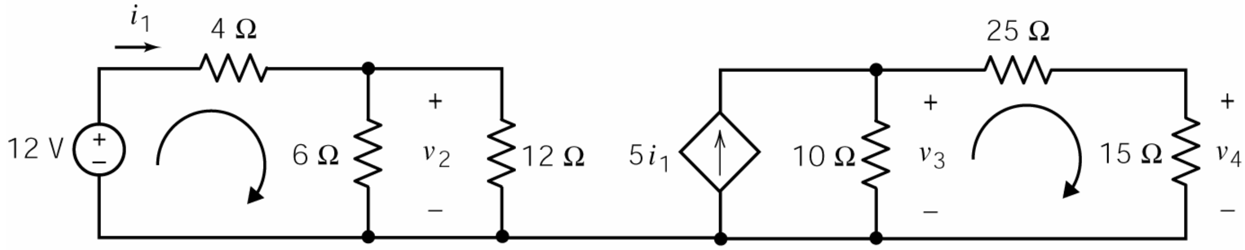
$$5i_1 = \frac{v_3}{10} + \frac{v_4}{15}$$

Substituting the given values we get

$$5 \times 1.5 = \frac{60}{10} + \frac{22.5}{15} \quad \Rightarrow \quad 7.5 = 6 + 1.5$$

Hence KCL is satisfied at node b.

2. Check KVL.



Apply KVL to the left mesh to get

$$4i_1 + v_2 - 12 = 0$$

Substituting the given values we get

$$4(1.5) + 6 - 12 = 0$$

Hence KVL is satisfied for the left mesh. Next, apply KVL to the right mesh to get

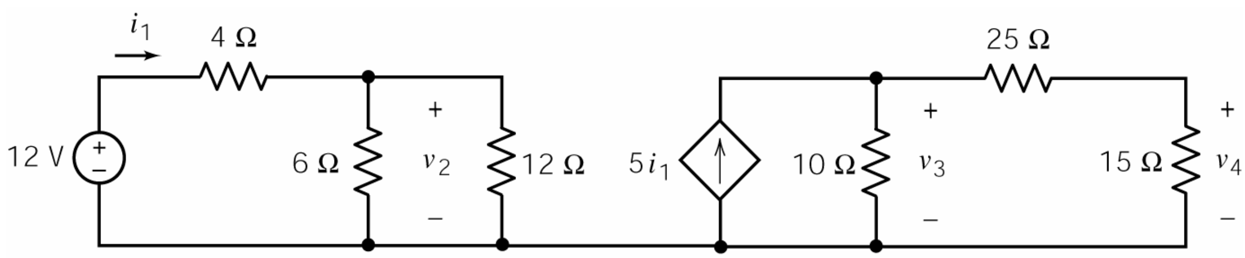
$$25\left(\frac{v_4}{15}\right) + v_4 - v_3 = 0$$

Substituting the given values we get

$$25\left(\frac{22.5}{15}\right) + 22.5 - 60 = 0$$

Hence KVL is satisfied for the right mesh.

3. Check Ohm's law for equivalent resistances.



The parallel 6 Ω and 12 Ω resistors are equivalent to a single $\frac{6 \times 12}{6+12} = 4\ \Omega$ resistor. Apply Ohm's law to this equivalent resistor to get

$$v_2 = 4i_1$$

Substituting the given values we get

$$6 = 4(1.5)$$

Hence Ohm's law is satisfied. The equivalent 4 Ω resistor is in series with the 4 Ω in the given circuit. These series resistors are equivalent to a single 8 Ω resistor. Apply Ohm's law to this equivalent resistor to get

$$12 = 8i_1$$

Substituting the given values we get

$$12 = 8(1.5)$$

Hence Ohm's law is satisfied. The series 25 Ω and 15 Ω resistors are equivalent to a single 40 Ω resistor. That equivalent resistor is connected in parallel with the 10 Ω in the given circuit. These parallel resistors are equivalent to a single $\frac{10\times 40}{10+40} = 8\ \Omega$. Apply Ohm's law to this equivalent resistor to get

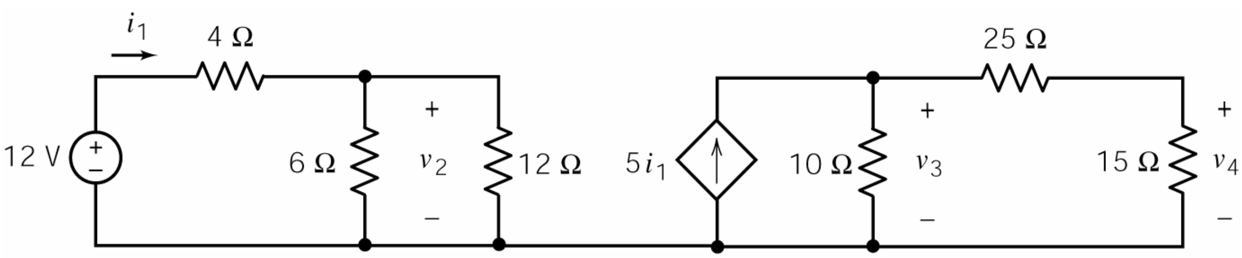
$$v_3 = 8(5i_1) = 40i_1$$

Substituting the given values we get

$$60 = 40(1.5)$$

Hence Ohm's law is satisfied.

4. Check voltage division.



Consider the series 25 Ω and 15 Ω resistors. Voltage division gives

$$v_4 = \left(\frac{15}{15+25}\right)v_3 = \left(\frac{3}{8}\right)v_3$$

Substituting the given values we get

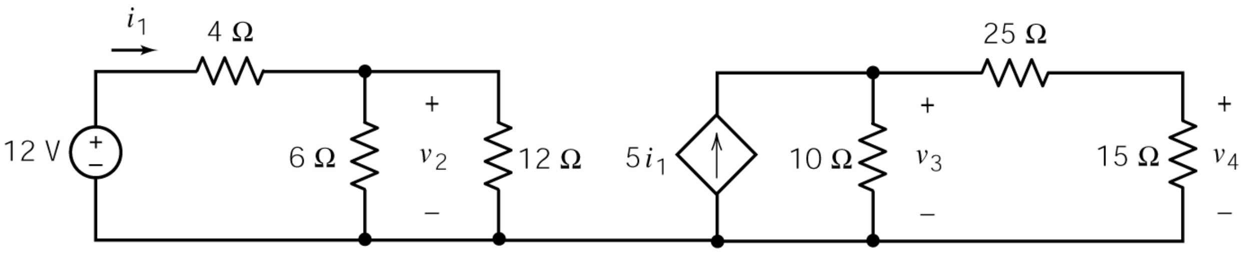
$$22.5 = \left(\frac{3}{8}\right)60$$

Hence voltage division is satisfied for the series 25 Ω and 15 Ω resistors. Next, notice that the parallel 6 Ω and 12 Ω resistors are equivalent to a single 4 Ω resistor. That equivalent resistor is in series with the 4 Ω in the given circuit. Voltage division gives

$$v_2 = \left(\frac{4}{4+4}\right)12 = 6\ \text{V}$$

Hence voltage division is satisfied.

5. Check conservation of power.



The power supplied by the sources must be equal to the power received by the resistors. That is

$$12i_1 + v_3(5i_1) = 4i_1^2 + \frac{v_2^2}{6} + \frac{v_2^2}{12} + \frac{v_3^2}{10} + 25\left(\frac{v_4}{15}\right)^2 + \frac{v_4^2}{15}$$

Substituting the given values we get

$$12(1.5) + v_3(5\times1.5) = 4(1.5^2) + \frac{6^2}{6} + \frac{6^2}{12} + \frac{60^2}{10} + 25\left(\frac{22.5}{15}\right)^2 + \frac{22.5^2}{15}$$

$$18 + 60(7.5) = 4(2.25) + 6 + 3 + 360 + 25(2.25) + 33.75$$

$$468 = 468$$

The power supplied by the sources is indeed equal to the power received by the resistors.