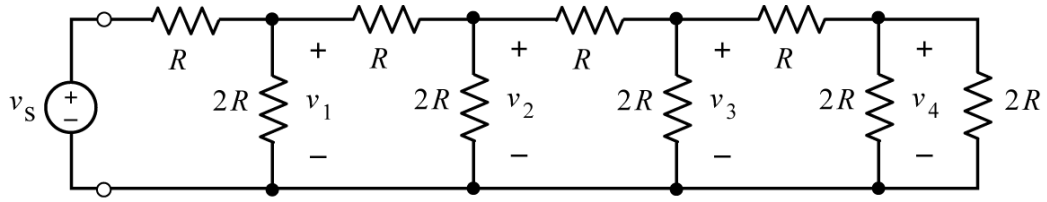
R-2*R* Ladder Networks

Example

Consider the *R*-2*R* ladder network:

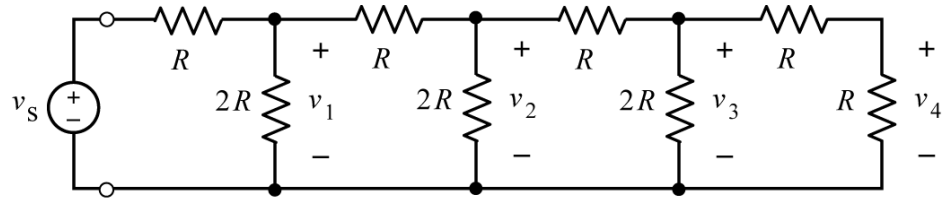


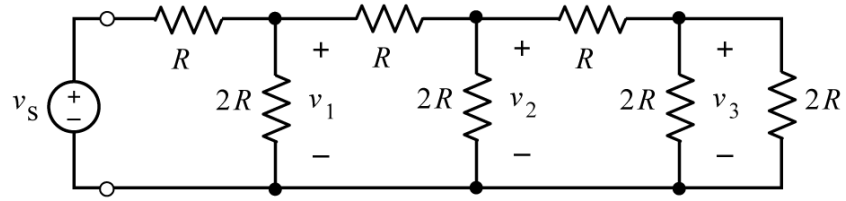
Show that

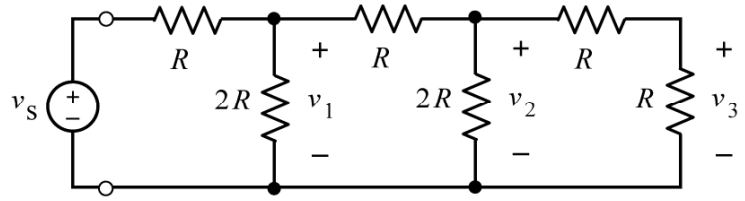
$$v_1 = \frac{1}{2^1}v_s = \frac{1}{2}v_s,\ v_2 = \frac{1}{2^2}v_s = \frac{1}{4}v_s,\ v_3 = \frac{1}{2^3}v_s = \frac{1}{8}v_s \text{ and } v_4 = \frac{1}{2^4}v_s = \frac{1}{16}v_s$$

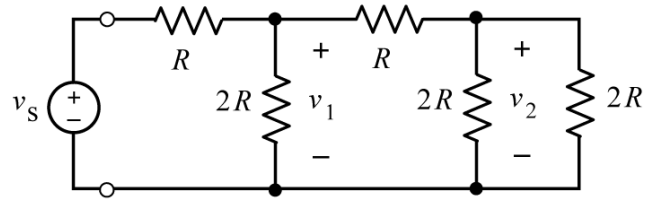
Solution

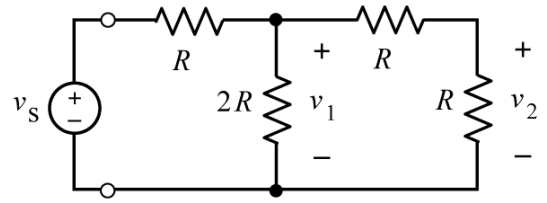
Reduce the circuit using equivalent resistances as follows:

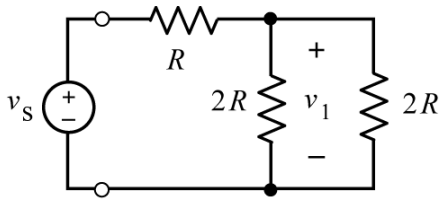


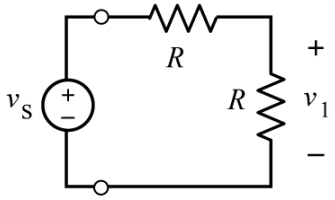












Using voltage division, we see that

$$v_4 = \frac{1}{2}v_3,\ v_3 = \frac{1}{2}v_2,\ v_2 = \frac{1}{2}v_1,\ \text{and}\ v_1 = \frac{1}{2}v_s$$

Consequently

$$v_1 = \frac{1}{2^1}v_s = \frac{1}{2}v_s,\ v_2 = \frac{1}{2^2}v_s = \frac{1}{4}v_s,\ v_3 = \frac{1}{2^3}v_s = \frac{1}{8}v_s\ \text{and}\ v_4 = \frac{1}{2^4}v_s = \frac{1}{16}v_s$$