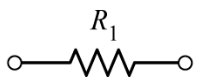
Monte Carlo Analysis



$R_{1\text{ nom}} =$ nominal resistance

$rand =$ random number $\in (0,1]$

$2*rand - 1 =$ random number $\in (-1,1]$

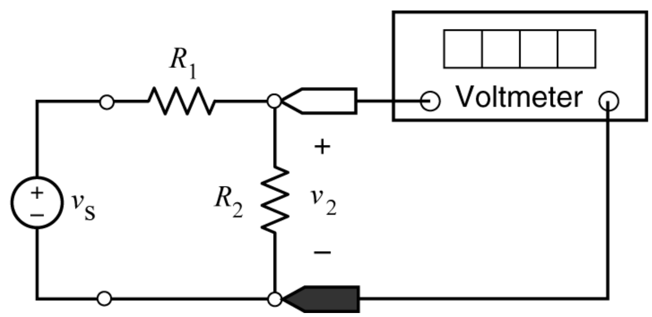
$t_1 =$ tolerance in %

$$R_1 = R_{1\text{ nom}}\left(1 + (2*rand - 1)\left(\frac{t_1}{100}\right)\right) = \text{ actual resistance}$$

Notice that $$R_{1\text{ nom}}\left(1 - \frac{t_1}{100}\right) < R_1 \le R_{1\text{ nom}}\left(1 + \frac{t_1}{100}\right)$$

Voltage Divider:

$$v_2 = \left(\frac{R_2}{R_1 + R_2}\right) v_s \triangleq G\, v_s$$



$$G = \frac{R_2}{R_1 + R_2} = \frac{R_{2\text{ nom}}\left(1 + (2*rand_2 - 1)\left(\frac{t_2}{100}\right)\right)}{R_{1\text{ nom}}\left(1 + (2*rand_1 - 1)\left(\frac{t_1}{100}\right)\right) + R_{1\text{ nom}}\left(1 + (2*rand_2 - 1)\left(\frac{t_2}{100}\right)\right)}$$

Example: Consider a voltage divider with nominal resistances $R_{1\text{ nom}} = 100\ \Omega$ and $R_{2\text{ nom}} = 400\ \Omega$ and tolerances $t_1 = t_2 = 5\%$. Suppose the first two calls to *rand* return $rand_1 = 0.21347$ and $rand_2 = 0.83721$.

Determine the actual values of the resistances: R_1 = __________Ω and R_2 = __________Ω .

$$R_1 = R_{1\text{ nom}}\left(1 + (2*rand - 1)\left(\frac{t_1}{100}\right)\right) = 100\left(1 + (2*0.21347 - 1)\left(\frac{5}{100}\right)\right)$$
$$= 100\left(1 - 0.57306(0.05)\right) = 97.1347\ \Omega$$

and

$$R_2 = R_{2\text{ nom}}\left(1+\left(2*rand_2-1\right)\left(\frac{t_2}{100}\right)\right) = 400\left(1+\left(2*0.83721-1\right)\left(\frac{5}{100}\right)\right)$$

$$= 400\left(1+0.67442\left(0.05\right)\right) = 413.4884\ \Omega$$

Determine the nominal and actual values of gain of the voltage divider:

G_{nom} = ___________V/V and G_{act} = ___________V/V

$$G_{\text{nom}} = \frac{R_{2\text{ nom}}}{R_{1\text{ nom}}+R_{2\text{ nom}}} = \frac{400}{100+400} = 0.8\text{ V/V}$$

and

$$G_{\text{act}} = \frac{R_2}{R_1+R_2} = \frac{413.4884}{97.1374+413.4884} = 0.809772\text{ V/V}$$

Here's a MATLAB script that performs a Monte Carlo analysis of a voltage divider:

```
%vdivMonte.m
R1nom=750;  %Nominal resistance values
R2nom=250;
t1=2;       %Resistor tolerances in %
t2=2;
Gnom=R2nom/(R1nom+R2nom);  %nominal gain
tg=1.5;     %Gain tolerance
n=15;       %Number of trials
m=0;        %

fprintf('\n')
fprintf('       R1          R2        vo/vs \n')
fprintf('------------------------------------\n')
for i=1:n
    R1=(1+(2*rand-1)*t1/100)*R1nom; %actual resistance values
    R2=(1+(2*rand-1)*t1/100)*R2nom;
    G = R2/(R1+R2);  %voltage divider gain
    if abs(G-Gnom)/Gnom<tg/100
        m=m+1;
    end
    fprintf('    %6.3f    %6.3f    %6.4f \n',R1,R2,G)
end
fprintf('\n%4.1f%% of the voltage dividers have a ',100*m/n)
fprintf('gain\nthat is within %3.1f%% of %4.2f.\n',tg,Gnom)
```

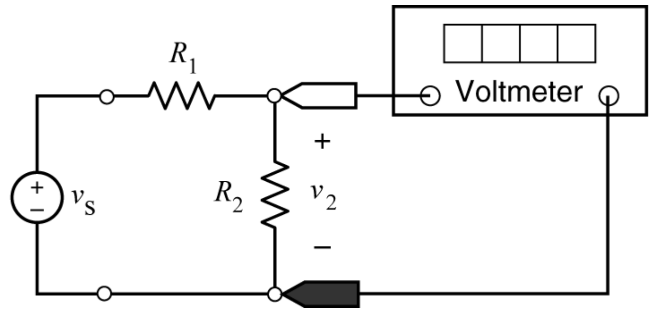
Here's what happens when we execute that MATLAB script:

```
>> vdivMonte

       R1          R2       vo/vs
------------------------------------
    763.504    247.311    0.2447
    753.205    249.860    0.2491
    761.739    252.621    0.2490
    748.694    245.185    0.2467
    759.642    249.447    0.2472
    753.463    252.919    0.2513
    762.654    252.382    0.2486
    740.288    249.057    0.2517
    763.064    254.169    0.2499
    747.308    253.936    0.2536
    736.737    248.529    0.2522
    759.395    245.099    0.2440
    739.167    247.028    0.2505
    740.962    251.038    0.2531
    743.166    246.988    0.2494

86.7% of the voltage dividers have a gain
that is within 1.5% of 0.25.
>>
```

"Worst-case" Analysis

Voltage Divider:

$$v_2 = \left(\frac{R_2}{R_1 + R_2} \right) v_s \triangleq G\, v_s$$



It can be shown that:

$$\frac{R_{2\ \text{nom}}\left(1-\dfrac{t_2}{100}\right)}{R_{1\ \text{nom}}\left(1+\dfrac{t_1}{100}\right)+R_{2\ \text{nom}}\left(1-\dfrac{t_2}{100}\right)} < G \le \frac{R_{2\ \text{nom}}\left(1+\dfrac{t_2}{100}\right)}{R_{1\ \text{nom}}\left(1-\dfrac{t_1}{100}\right)+R_{2\ \text{nom}}\left(1+\dfrac{t_2}{100}\right)}$$

Example: Consider a voltage divider with nominal resistances $R_{1\ \text{nom}} = 100\ \Omega$, $R_{2\ \text{nom}} = 400\ \Omega$ and tolerances $t_1 = t_2 = 5\%$.

Determine the range values of gain of the voltage divider:

$G_{\min}$ = ___________ < G ≤ $G_{\max}$ = ___________

Determine appropriate tolerance for the gain of the voltage divider: t_G = _________%

Example: Consider a voltage divider with nominal resistances $R_{1\ \text{nom}} = 80\ \Omega$, $R_{2\ \text{nom}} = 20\ \Omega$ and tolerances $t_1 = t_2 = 2\%$.

Suppose we measure $v_2 = 4.8696$ V when $v_s = 24$ V. Is this measurement explained by the resistor tolerances?

$$0.1937 = \frac{20(0.98)}{80(1.02)+20(0.98)} < G \le \frac{20(1.02)}{80(0.98)+20(1.02)} = 0.2065$$

Since $$0.1937 < \frac{4.8696}{24} = 0.2029 < 0.2065$$

The error could be entirely due to the resistor tolerances.