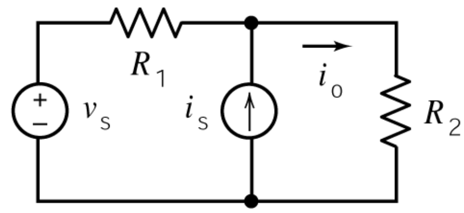
Another Sample ES 250 Second Midterm Exam

1. This circuit has two inputs, v_s and i_s, and one output i_o. The output is related to the inputs by the equation



$$i_o = a\,i_s + b\,v_s$$

Given the following two facts:

The output is i_o = 0.45 A when the inputs are i_s = 0.25 A and v_s = 15 V.

and

The output is i_o = 0.30 A when the inputs are i_s = 0.50 A and v_s = 0 V.

Determine the following:

The values of the constants a and b are a = ____0.6_____ and b = __0.02___ A/V.

The values of the resistances are R_1 = ____30____ Ω and R_2 = ____20___ Ω.

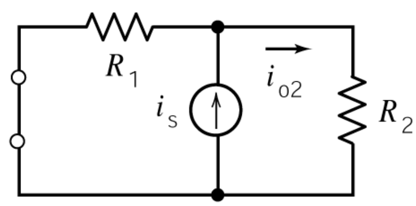
From the 1st fact: $$0.45 = a(0.25) + b(15)$$

From the 2nd fact: $$0.30 = a(0.50) + b(0) \quad \Rightarrow \quad a = \frac{0.30}{0.50} = 0.60$$

Substituting gives $$0.45 = (0.60)(0.25) + b(15) \quad \Rightarrow \quad b = \frac{0.45 - (0.60)(0.25)}{15} = 0.02$$

Next, consider the circuit:

$$a\,i_s = i_{o1} = i_o\big|_{v_s=0} = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

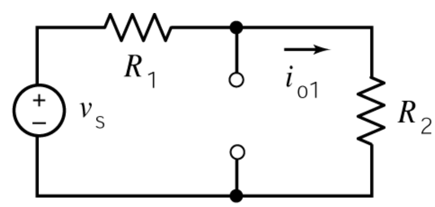


so

$$0.60 = \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad 2R_1 = 3R_2$$

and

$$b\,v_s = i_{o2} = i_o\big|_{i_s=0} = \frac{v_s}{R_1 + R_2}$$



so

$$0.02 = \frac{1}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = \frac{1}{0.02} = 50\ \Omega$$

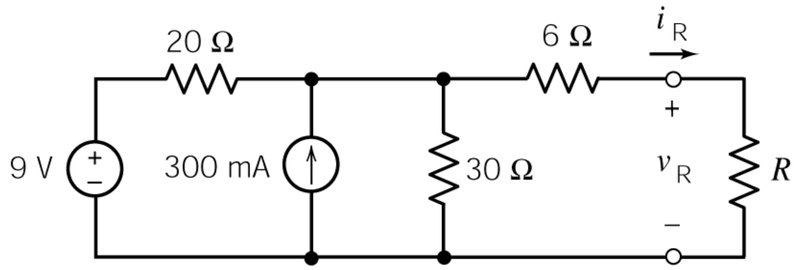
Solving these equations gives R_1 = 30 Ω and R_2 = 20 Ω.

2. Fill in the blanks in the following statements:

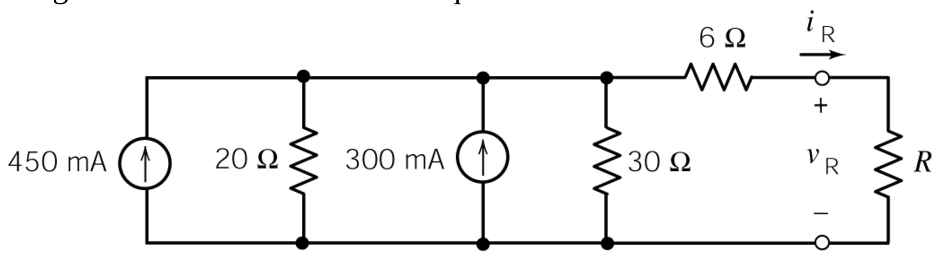
When $R = 9\ \Omega$ then v_R =_____3____V.

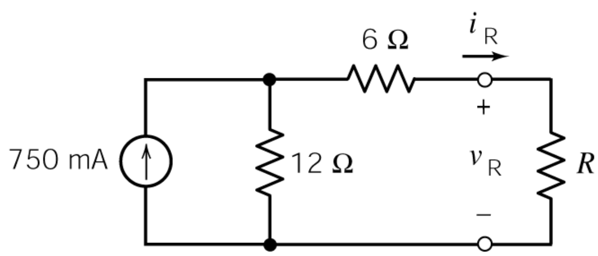
When R = ____27____Ω then $v_R = 5.4$ V.

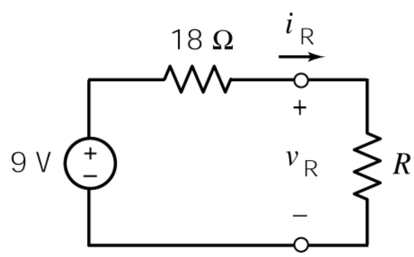
When R = ____12____Ω then $i_R = 300$ mA.



Reduce this circuit using source transformations and equivalent resistance:





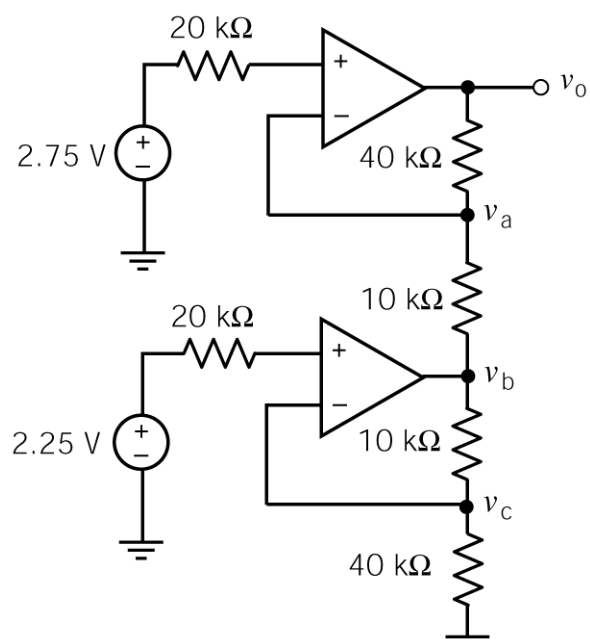


Now $v_R = \left(\dfrac{R}{R+18}\right)9$ and $i_R = \dfrac{9}{R+18}$ so the questions can be easily answered.

3. Determine the values of the node voltages v_a, v_b, v_c and v_o:

v_a = ___2.75___ V, v_b = __2.8125__ V,

v_c = ___2.25___ V, and v_o = ___2.50___ V.



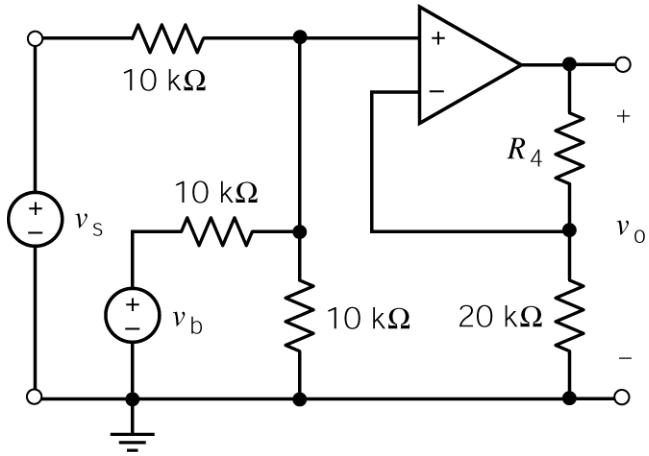
Due to the properties of the ideal op amp, $v_a = 2.75$ V and $v_c = 2.25$ V. The node equation at node c is

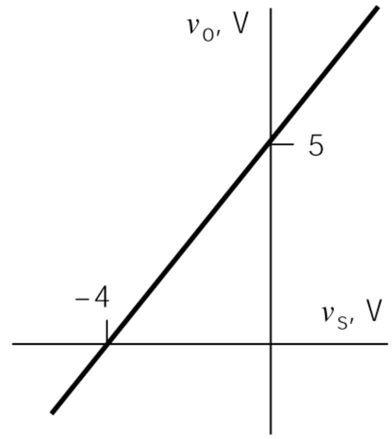
$$\frac{v_b - v_c}{10\times10^3} = \frac{v_c}{40\times10^3} \quad\Rightarrow\quad v_b = \frac{5}{4}v_c = 2.8125\ \text{V}$$

The node equation at node c is

$$\frac{v_o - v_a}{40\times10^3} = \frac{v_a - v_b}{10\times10^3} \quad\Rightarrow\quad v_o = 5v_a - 4v_b = 2.5\ \text{V}$$

4.

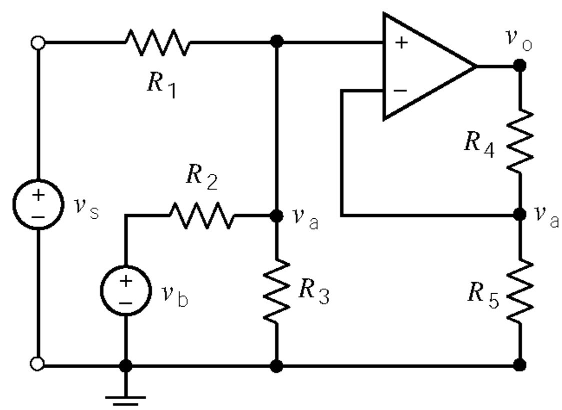




The input to this circuit is the voltage, v_s. The output is the voltage v_o. The voltage v_b is used to adjust the relationship between the input and output. Determine values of R_4 and v_b that cause the circuit input and output have the relationship specified by the graph

v_b = ____4_____ V and R_4 = ____55____ kΩ.

(a) Label the node voltages as shown. The node equations are



$$\frac{v_s - v_a}{R_1} + \frac{v_b - v_a}{R_2} = \frac{v_a}{R_3}$$

and

$$\frac{v_a}{R_5} = \frac{v_o - v_a}{R_4} \quad \Rightarrow \quad v_a = \left(\frac{R_5}{R_4 + R_5}\right) v_o$$

Solving these equations gives

$$\frac{v_s}{R_1} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_a - \frac{v_b}{R_2} = \left(\frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1 R_2 R_3} \times \frac{R_5}{R_4 + R_5}\right) v_o - \frac{v_b}{R_2}$$

So
$$v_o = \left(\frac{R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5}\right) v_s + \frac{R_1 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3} \times \frac{R_4 + R_5}{R_5} \times v_b$$

When $R_1 = R_2 = R_3 = 10\ \text{k}\Omega$:
$$v_o = \left(\frac{R_4 + R_5}{3 R_5}\right) v_s + \frac{R_4 + R_5}{3 R_5} \times v_b$$

So
$$a = \frac{R_4 + R_5}{3 R_5} \quad \text{and} \quad b = \frac{R_4 + R_5}{3 R_5} \times v_b$$

(b) The equation of the straight line is
$$v_o = \frac{5}{4} v_s + 5$$

We require
$$\frac{R_4 + R_5}{3 R_5} = \frac{5}{4}$$

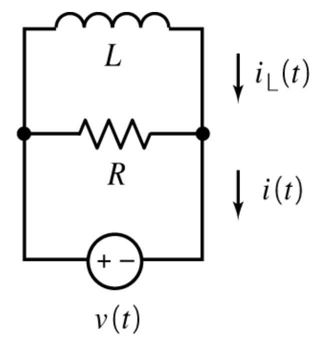
When $R_5 = 20\ \text{k}\Omega$ then $R_4 = 55\ \text{k}\Omega$. Next we require

$$5 = \frac{R_4 + R_5}{3R_5} \times v_\text{b} = \frac{5}{4} v_\text{b}$$

i.e.

$$v_\text{b} = 4\ \text{V}$$

5. The input to this circuit is the voltage: $v(t) = 4e^{-20t}$ V for $t > 0$



The output is the current: $i(t) = -1.2e^{-20t} - 1.5$ A for $t > 0$

The initial condition is $i_\text{L}(0) = -3.5$ A . Determine the values of the resistance and inductance:

R = ___5___ Ω and L = ___0.1____ H.

Solution: Apply KCL at either node to get

$$i(t) = \frac{v(t)}{R} + i_\text{L}(t) = \frac{v(t)}{R} + \left[\frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)\right]$$

That is

$$-1.2e^{-20t} - 1.5 = \frac{4e^{-20t}}{R} + \frac{1}{L}\int_0^t 4e^{-20\tau}\,d\tau - 3.5 = \frac{4e^{-20t}}{R} + \frac{4}{L(-20)}\left(e^{-20t} - 1\right) - 3.5$$

$$= \left(\frac{4}{R} - \frac{1}{5L}\right)e^{-20t} + \frac{1}{5L} - 3.5$$

Equating coefficients gives $$-1.5 = \frac{1}{5L} - 3.5 \quad \Rightarrow \quad L = 0.1\ \text{H}$$

And $$-1.2 = \frac{4}{R} - \frac{1}{5L} = \frac{4}{R} - \frac{1}{5(0.1)} = \frac{4}{R} - 2 \quad \Rightarrow \quad R = 5\ \Omega$$

6. The initial inductor current is $i(0)$ = 25 mA.

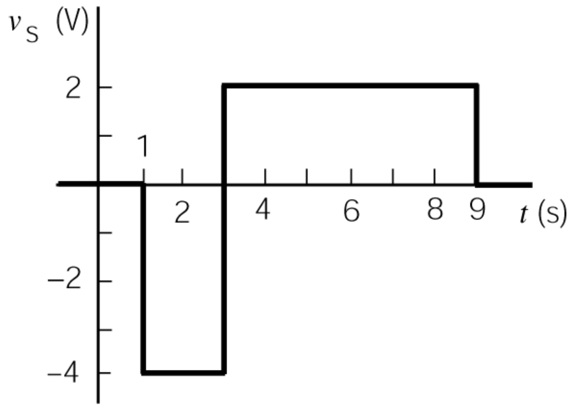
Determine the values of the inductor current at 2, 3, 6 and 9 seconds:

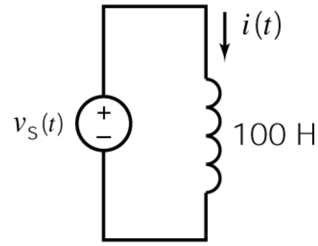
$i(2)$ = ___–15___ mA,

$i(3)$ = ___–55____ mA,

$i(6)$ = _____5____ mA,

$i(9)$ = ____65____ mA.





$$i(t)=\frac{1}{100}\int_0^t 0\ dt+0.025=0.025 \qquad \text{for} \qquad 0<t<1$$

so $i(1)=0.025$ A

$$i(t)=\frac{1}{100}\int_1^t -4\ d\tau+0.025=\frac{-4(t-1)}{100} \qquad \text{for} \qquad 1<t<3$$

so $i(2)$ = –0.015 A, and $i(3)=-0.055$ A

$$i(t)=\frac{1}{100}\int_3^t 2\ d\tau-0.055=\frac{2(t-3)}{100}-0.055 \qquad \text{for} \qquad 3<t<9$$

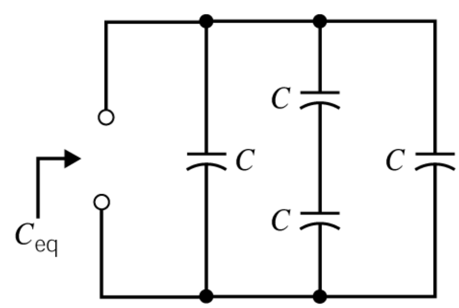
so $i(9)=0.065$ A

$$i(t)=\frac{1}{100}\int_9^t 0\ d\tau+0.065=0.065 \qquad \text{for} \qquad t>9$$

7.

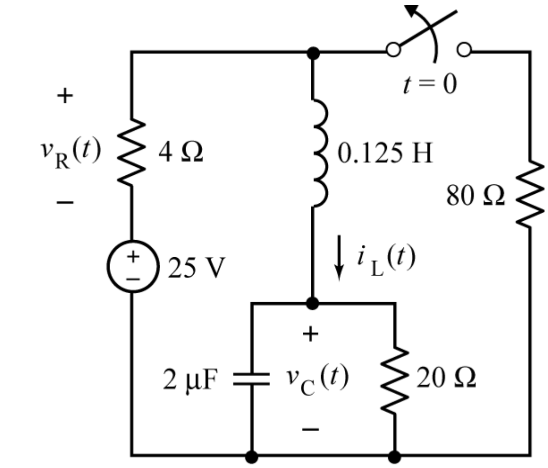
a. When C = 10 F then C_{eq} = ___25__F.

b. When C = ___3.2___F then C_{eq} = 8 F.



$$C_{eq}=C+\frac{C\times C}{C+C}+C=\frac{5}{2}C$$

8. This circuit has reached steady state before the switch opens at time t = 0. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens:



$i_L(0-)$=____1____A, $v_C(0-)$ =____20____V

and

$v_R(0-)$=_____–5_____V

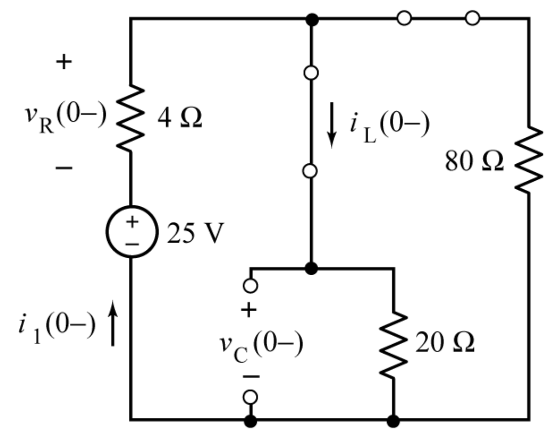
Determine the value of $v_R(t)$ immediately after the switch opens:

$v_R(0+)$=_____–4_____V

Solution: Because

- This **circuit has reached steady state** before the switch opens at time t = 0.
- The only source is a **constant** voltage **source**.

At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**. From the circuit



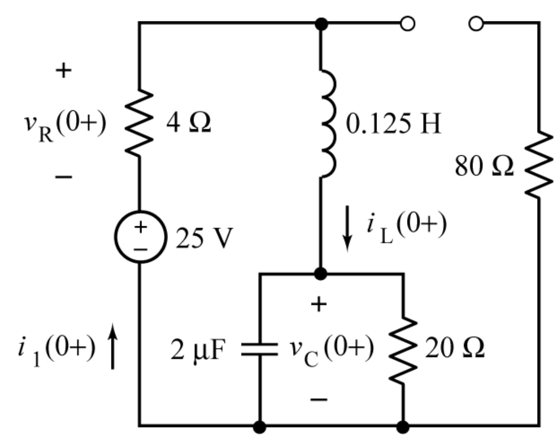
$$i_1(0-) = \frac{25}{4+(20\|80)} = \frac{25}{4+16} = 1.25 \text{ A},$$

$$i_L(0-) = \left(\frac{80}{20+80}\right) i_1(0-) = 1 \text{ A}, \; v_C(0-) = 20\, i_L(0-) = 20 \text{ V}$$

and

$$v_R(0-) = -4\, i_1(0-) = -5 \text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so



$$v_C(0+) = v_C(0-) = 20 \text{ V} \text{ and } i_L(0+) = i_L(0-) = 1 \text{ A}$$

Apply KCL at the top node to see that

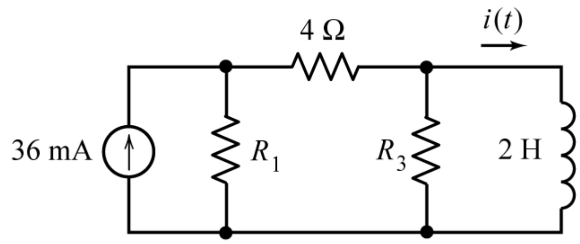
$$i_1(0+) = i_L(0+) = 1 \text{ A}$$

From Ohm's law

$$v_R(0+) = -4\, i_1(0+) = -4 \text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

9. After time t = 0, a given circuit is represented by this circuit diagram.



a. Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t} \text{ mA} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_3: R_1 = ___6____ Ω and R_3 = ___40____ Ω.

b. Suppose instead that R_1 = 16 Ω , R_3 = 20 Ω, the initial condition is $i(0)$ = 10 mA, and the inductor current is $i(t) = A - Be^{-at}$ for $t \geq 0$. Determine the values of the constants A, B, and a:

A = ___28.8__ mA, B = __–18.8___ mA and a = ____5___ s.

Solution:

The inductor current is given by $i(t) = i_{sc} + (i(0) - i_{sc})e^{-at}$ for $t \geq 0$ where $a = \frac{1}{\tau} = \frac{R_t}{L}$.

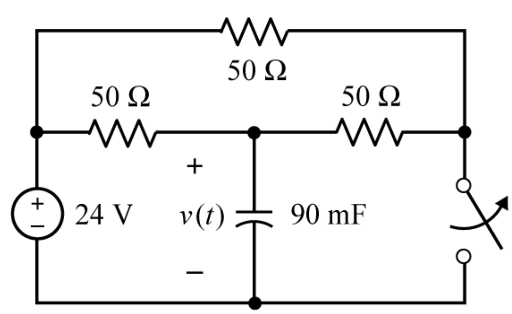
a. Comparing this to the given equation gives $21.6 = i_{sc} = \frac{R_1}{R_1 + 4}(36) \Rightarrow R_1 = 6\ \Omega$ and

$4 = \frac{R_t}{2} \Rightarrow R_t = 8\ \Omega$. Next $8 = R_t = (R_1 + 4) \| R_3 = 10 \| R_3 \Rightarrow R_3 = 40\ \Omega$.

b. $R_t = (16 + 4) \| 20 = 10\ \Omega$ so $a = \frac{1}{\tau} = \frac{10}{2} = 5$ s. also $i_{sc} = \frac{16}{16+4}(36) = 28.8$ mA. Then

$i(t) = i_{sc} + (i(0) - i_{sc})e^{-at} = 28.8 + (10 - 28.8)e^{-5t} = 28.2 - 18.8e^{-5t}$.

10. a) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **open**:

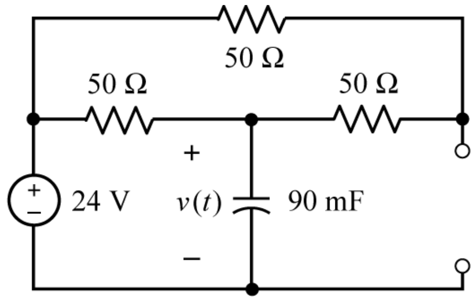


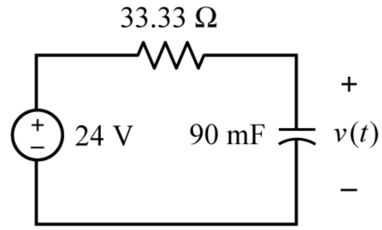
τ = _____3____ s and $v(\infty)$ = ____24____ V

b) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **closed**:

τ = __2.25__ s and $v(\infty)$ = ____12____ V

Solution:
a.) When the switch is open we have



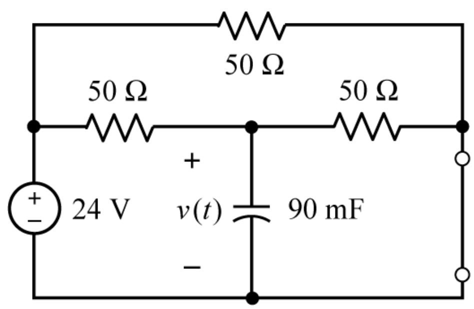


After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is

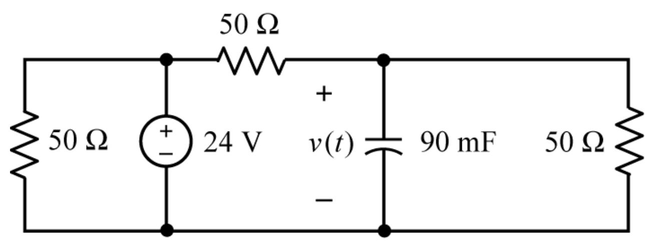
$\tau = R_t\, C = 33.33(0.090) = 3\text{ s}$.

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty) = 24$ V.

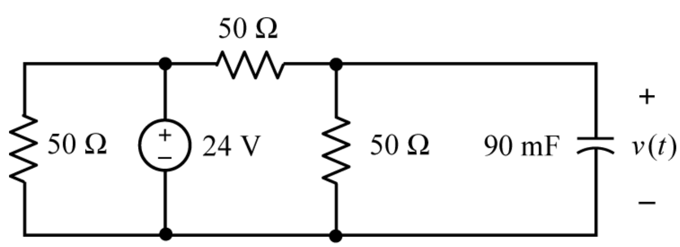
b.) When the switch is closed we have

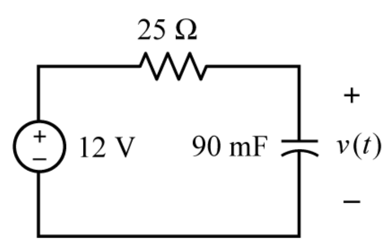


This circuit can be redrawn as



Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:





So $R_t = 25\ \Omega$ and

$$\tau = R_t\, C = 25(0.090) = 2.25\text{ s}$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.