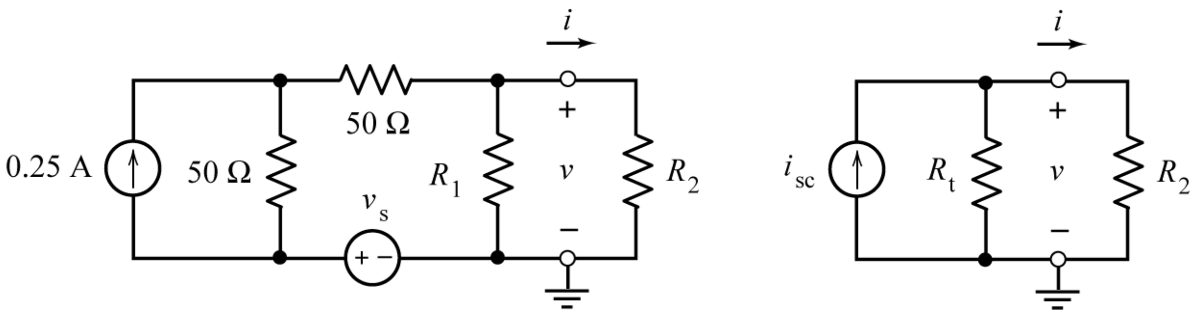
# Sample ES 250 Second Midterm Exam

**1.**




The part of the first circuit to the left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit will be characterized by the parameters:

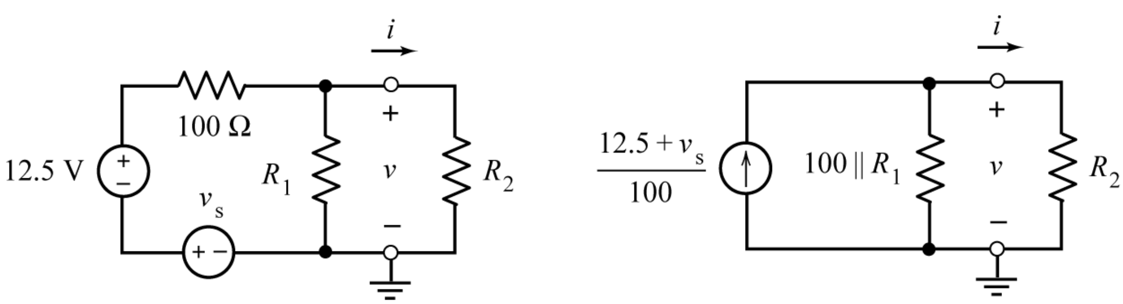
$$i_{sc} = 0.5\text{ A} \quad \text{and} \quad R_t = 20\ \Omega$$

Determine the values of $v_s$ and $R_1$: $v_s$ =____37.5___V and $R_1$ =___25_____Ω

Given that $0 \le R_2 \le \infty$, determine the maximum values of the voltage, *v*, and of the power, *p* = *vi*:

max *v* =____10_____V and max *p* =____1.25____W

Two source transformations reduce the circuit as follows:



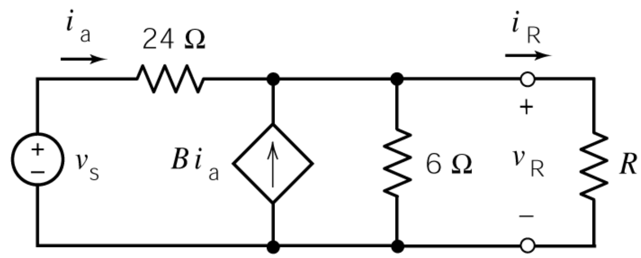

Recognizing the parameters of the Norton equivalent circuit gives:

$$0.5 = i_{sc} = \frac{12.5 + v_s}{100} \quad \Rightarrow \quad v_s = 37.5\text{ V} \;\text{ and }\; 20 = R_t = 100 \,\|\, R_1 = \frac{100\,R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 25\ \Omega$$

Next, the voltage across resistor $R_2$ is given by $v = i_{sc}\left(R_t \,\|\, R_2\right) = \dfrac{R_t\,R_2\,i_{sc}}{R_t + R_2} = \dfrac{R_t\,i_{sc}}{\dfrac{R_t}{R_2} + 1}$ so this voltage is maximum when $R_2 = \infty$ and max $v = R_t\,i_{sc} = 10$ V. The power *p* = *vi* will be maximum when $R_2 = R_t = 20\ \Omega$.

Then $v = \dfrac{R_t\,i_{sc}}{2} = \dfrac{20(0.5)}{2} = 5\text{ V}$, $i = \dfrac{v}{R_2} = \dfrac{5}{20} = 0.25\text{ A}$ and $p = v\,i = 5(0.25) = 1.25\text{ W}$.

**2.** Given that $0 \le R \le \infty$ in this circuit, consider these two observations:




When $R = 2\ \Omega$ then $v_R = 4$ V and $i_R = 2$ A.

When $R = 6\ \Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Fill in the blanks in the following statements:

a. The maximum value of $i_R$ is ____4_____A.

b. The maximum value of $v_R$ is ____8_____V.

c. The maximum value of $p_R = i_R v_R$ occurs when $R$ = _____2____Ω.

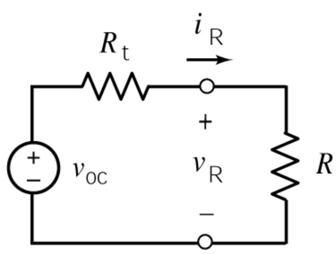
d. The maximum value of $p_R = i_R v_R$ is ____8____W.

e. When $R = 5\ \Omega$ then $v_R =$ __5.714__V.

f. When $R$ = _____8____Ω then $v_R = 6.4$ V.

g. When $R$ = _____14____Ω then $i_R = 500$ mA.

We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:




Using voltage division $v_R = \frac{R}{R+R_t} v_{oc}$ and using Ohm's law $i_R = \frac{v_{oc}}{R+R_t}$.

By inspection , $v_R = \frac{R}{R+R_t} v_{oc} = \frac{v_{oc}}{1+\frac{R_t}{R}}$ will be maximum when $R = \infty$. The maximum value of $v_R$ will be $v_{oc}$. Similarly, $i_R = \frac{v_{oc}}{R+R_t}$ will be maximum when $R = 0$. The maximum value of $i_R$ will be $\frac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells use that $p_R = i_R v_R$ will be maximum when $R = R_t$. Then

$$p_R = i_R v_R = \left(\frac{v_{oc}}{R+R_t}\right)\left(\frac{R}{R+R_t} v_{oc}\right) = R\left(\frac{v_{oc}}{R+R_t}\right)^2.$$

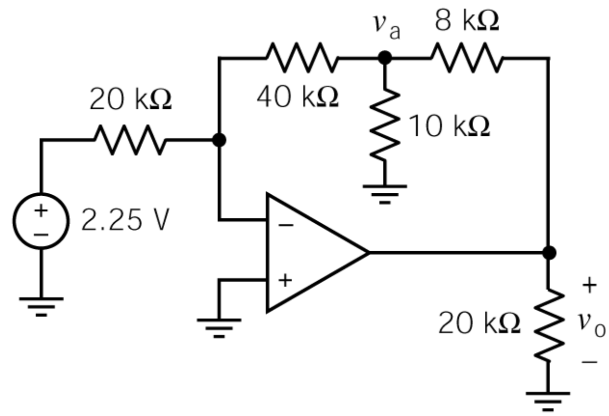
Let's substitute the given data into the equation $i_R = \frac{v_{oc}}{R+R_t}$.

When $R = 2\ \Omega$ we get $2 = \frac{v_{oc}}{2+R_t} \Rightarrow 4 + 2R_t = v_{oc}$. When $R = 6\ \Omega$ we get $1 = \frac{v_{oc}}{6+R_t} \Rightarrow 6 + R_t = v_{oc}$.

So $6 + R_t = 4 + 2R_t \Rightarrow R_t = 2\ \Omega$ and $v_{oc} = 4 + 2R_t = 8$ V. Also $i_{sc} = \frac{v_{oc}}{R_t} = \frac{8}{2} = 4$ A.

Now the blanks can be easily filled-in.

**3.** Determine the values of the node voltages $v_a$ and $v_o$:




$v_a$ = ____–4.5____ V and $v_o$ = ____–9___ V.

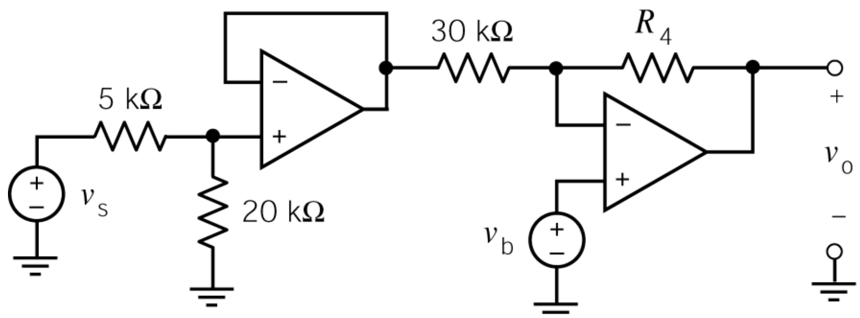
Writing node equations:

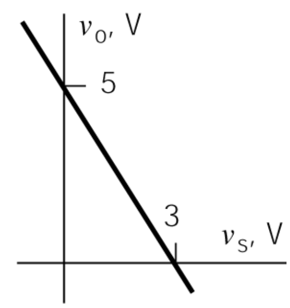
$$\frac{2.25}{20\times10^3}+\frac{v_a}{40\times10^3}=0 \quad\Rightarrow\quad v_a=-\left(40\times10^3\right)\frac{2.25}{20\times10^3}=-4.5\text{ V}$$

and

$$\frac{v_a}{40\times10^3}+\frac{v_a}{10\times10^3}+\frac{v_a-v_o}{8\times10^3}=0 \quad\Rightarrow\quad v_o=\left(\frac{8}{40}+\frac{8}{10}+\frac{8}{8}\right)v_a=2(-4.5)=-9\text{ V}$$

**4.**




The input to this circuit is the voltage, $v_s$. The output is the voltage $v_o$. The voltage $v_b$ is used to adjust the relationship between the input and output. Determine values of $R_4$ and $v_b$ that cause the circuit input and output have the relationship specified by the graph

$v_b$ = ___1.62__ V and $R_4$ = ___62.5___ kΩ.

Recognize the voltage divider, voltage follower and noninverting amplifier to write

$$v_o=\left(\frac{20\times10^3}{20\times10^3+5\times10^3}\right)\left(-\frac{R_4}{30\times10^3}\right)v_s+\left(1+\frac{R_4}{30\times10^3}\right)v_b=\left(-\frac{2R_4}{75\times10^3}\right)v_s+\left(1+\frac{R_4}{30\times10^3}\right)v_b$$

(Alternately, this equation can be obtained by writing two node equations: one at the noninverting node of the left op amp and the other at the inverting node of the right op amp.)

The equation of the straight line is $\quad v_o=-\frac{5}{3}v_s+5$

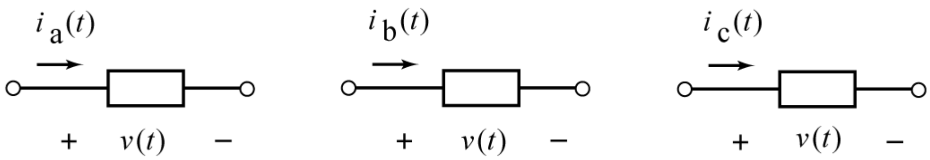
Comparing coefficients gives

$$-\frac{2R_4}{75\times10^3}=-\frac{5}{3} \quad\Rightarrow\quad R_4=\frac{5}{3}\times\frac{75\times10^3}{2}=62.5\times10^3=62.5\text{ k}\Omega$$

and

$$5=\left(1+\frac{R_4}{30\times10^3}\right)v_\text{b}=\left(1+\frac{62.5\times10^3}{30\times10^3}\right)v_\text{b}=3.08333\,v_\text{b}\quad\Rightarrow\quad v_\text{b}=\frac{5}{3.08333}=1.62\text{ V}$$

**5.** One of these three elements is a resistor, one is a capacitor and one is an inductor:




Given $v(t)=24\cos(5t)\ \text{V}$,

And $i_\text{a}(t)=3\cos(5t)\ \text{A}$, $i_\text{b}(t)=12\sin(5t)\ \text{A}$ and $i_\text{c}(t)=-1.8\sin(5t)\ \text{A}$

Determine the resistance of the resistor, the capacitance of the capacitor and the inductance of the inductor. (We require positive values of resistor, capacitance and inductance.)

resistance = ___8___Ω, capacitance = ___0.015___F and inductance = ___0.4__H

First, the current of element a is proportional to the voltage and the constant of proportionality is positive. Consequently, element a is the resistor and $R=\dfrac{v(t)}{i_\text{a}(t)}=\dfrac{24\cos(5t)}{3\cos(5t)}=8\ \Omega$.

Next $$\frac{d}{dt}v(t)=\frac{d}{dt}\big(24\cos(5t)\big)=-(24)(5)\sin(5t)=-120\sin(5t)$$

The current of a capacitor is proportional to the derivative of the voltage. The constant of proportionality is the capacitance. We see that $i_\text{c}(t)$ is proportional to $\dfrac{d}{dt}v(t)$ and the constant of proportionality is positive. Consequently, element c is the capacitor. Then

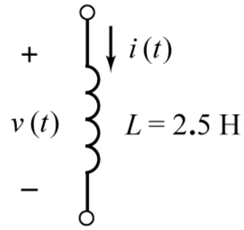
$$C=\frac{i_\text{c}(t)}{\dfrac{d}{dt}i(t)}=\frac{-1.8\sin(5t)}{-120\sin(5t)}=0.015\text{ F}=15\text{ mF}$$

Next $$\int_{-\infty}^{t}v(\tau)\,d\tau=\int_{-\infty}^{t}24\cos(5\tau)\,d\tau=\frac{24\sin(5\tau)}{5}=4.8\sin(5\tau)$$

The voltage of an inductor is proportional to the integral of the current. The constant of proportionality is the reciprocal of the inductance. We see that $i_\text{b}(t)$ is proportional to $\int_{-\infty}^{t}v(\tau)\,d\tau$ and the constant of proportionality is positive. Consequently, element b is the inductor. Then

$$\frac{1}{L}=\frac{i_\text{b}(t)}{\int_{-\infty}^{t}v(\tau)\,d\tau}=\frac{12\sin(5t)}{4.8\sin(5t)}=2.5\quad\Rightarrow\quad L=\frac{1}{2.5}=0.4\ \text{H}$$

**6.** Consider this inductor. The current and voltage are given by




$$i(t) = \begin{cases} 5t - 4.6 & 0 \le t \le 0.2 \\ at + b & 0.2 \le t \le 0.5 \\ c & t \ge 0.5 \end{cases} \quad \text{and} \quad v(t) = \begin{cases} 12.5 & 0 < t < 0.2 \\ 25 & 0.2 < t < 0.5 \\ 0 & t > 0.5 \end{cases}$$

where a, b and c are real constants. (The current is given in Amps, the voltage in Volts and the time in seconds.) Determine the values of the constants:

$a$ = ___10___A/s, $b$ = ___–5.6___A and $c$ = ___–0.6__A

At $t$ = 0.2 s

$$i(0.2) = 5(0.2) - 4.6 = -3.6 \text{ A}$$

For $0.2 \le t \le 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.2}^{t} 25\, d\tau - 3.6 = 10\tau \Big|_{0.2}^{t} - 3.6 = 10(t - 0.2) - 3.6 = 10t - 5.6 \text{ A}$$

At $t$ = 0.5 s

$$i(0.5) = 10(0.5) - 5.6 = -0.6 \text{ A}$$

For $t \ge 0.5$

$$i(t) = \frac{1}{2.5}\int_{0.5}^{t} 0\, d\tau - 0.6 = -0.6$$

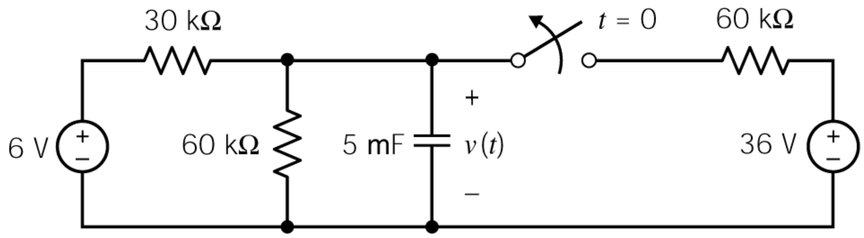
Checks:

At $t$ = 0.2 s $\quad i(0.2) = 10(0.2) - 5.6 = -3.6 \text{ A} \quad \surd$

For $0.2 \le t \le 0.5 \quad v(t) = 2.5\frac{d}{dt}i(t) = 2.5\frac{d}{dt}(10t - 5.6) = 2.5(10) = 25 \text{ V} \quad \surd$

$$-0.6 - (-3.6) = i(0.5) - i(0.2) = \frac{1}{2.5}\int_{0.2}^{0.5} 25\, d\tau = 10(0.5 - 0.2) = 3 \text{ A} \quad \surd$$

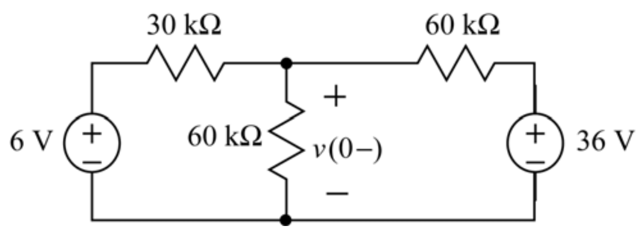
**7.** This circuit is at steady state when the switch opens at time $t = 0$.




The capacitor voltage is $v(t) = A - Be^{-at}$ for $t \geq 0$. Determine the values of the constants *A*, *B*, and *a*:

*A* = ____4__ V, *B* = __8___ V and *a* = ___0.01___ s.

**Solution:**
Before $t = 0$, with the switch closed and the circuit at the steady state, the capacitor acts like an open circuit so we have

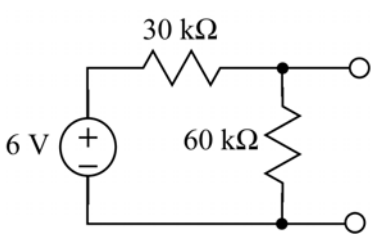



Using superposition

$$v(0-) = \frac{60\,||\,60}{30 + (60\,||\,60)} 6 + \frac{60\,||\,30}{60 + (60\,||\,30)} 36 = \left(\frac{1}{2}\right) 6 + \left(\frac{1}{4}\right) 36 = 12 \text{ V}$$

The capacitor voltage is continuous so $v(0+) = v(0-) = 12 \text{ V}$.

After $t = 0$ the switch is open. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor:




$$v_{oc} = \frac{60}{60 + 30} 6 = 4 \text{ V}$$

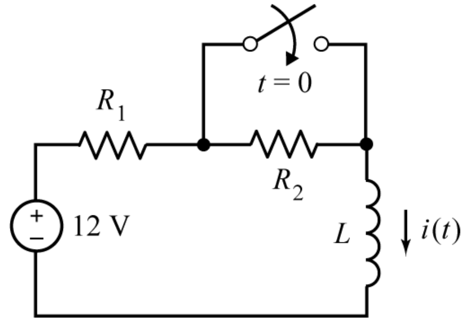
$$R_t = 30\,||\,60 = 20 \text{ k}\Omega$$

The time constant is $\tau = R_t C = (20 \times 10^3)(5 \times 10^{-3}) = 100$ s so $a = \frac{1}{\tau} = 0.01 \ \frac{1}{\text{s}}$.
The capacitor voltage is given by

$$v(t) = (v(0+) - v_{oc}) e^{-t/\tau} + v_{oc} = (12 - 4) e^{-t/100} + 4 = 4 + 8e^{-0.01t} \text{ V} \quad \text{for } t \geq 0$$

**8.** This circuit is at steady state before the switch closes at time t = 0. After the switch closes, the inductor current is given by

$$i(t) = 0.6 - 0.2e^{-5t} \text{ A} \quad \text{for } t \ge 0$$




Determine the values of $R_1$, $R_2$ and $L$:

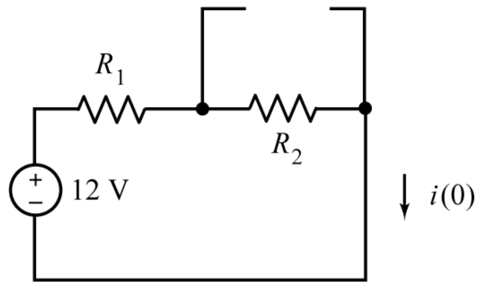
$R_1$ = ___20____ Ω , $R_2$ = ___10____ Ω

and

$L$ = ___4____ H

**Solution:**

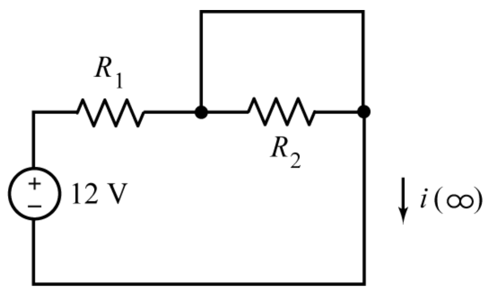
The steady state current before the switch closes is equal to $i(0) = 0.6 - 0.2e^{-5(0)} = 0.4$ A .




The inductor will act like a short circuit when this circuit is at steady state so

$$0.4 = i(0) = \frac{12}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 = 30\ \Omega$$

After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be $i(\infty) = 0.6 - 0.2e^{-5(\infty)} = 0.6$ A




The inductor will act like a short circuit when this circuit is at steady state so

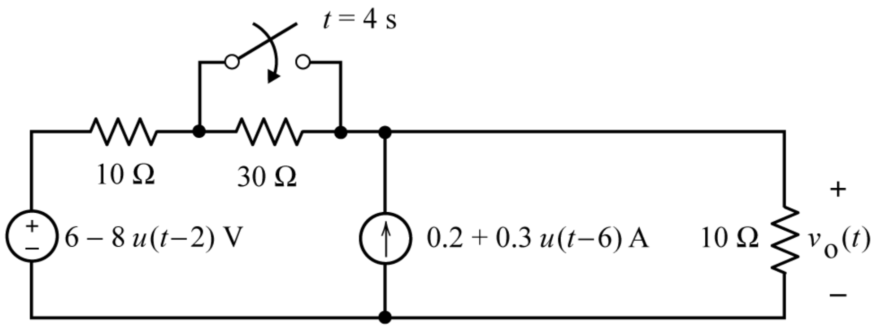
$$0.6 = i(\infty) = \frac{12}{R_1} \quad \Rightarrow \quad R_1 = 20\ \Omega$$

Then $R_2 = 10$ Ω.

After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t = R_1$. Then

$$5 = \frac{1}{\tau} = \frac{R_t}{L} = \frac{R_1}{L} = \frac{20}{L} \quad \Rightarrow \quad L = 4 \text{ H}$$

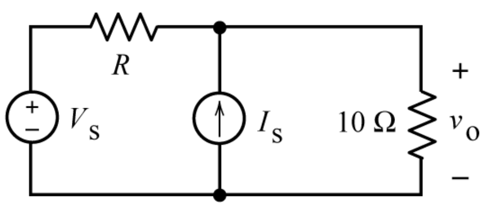
**9.**




Determine $v_o(1)$, $v_o(3)$, $v_o(5)$, and $v_o(7)$; the values of the voltage $v_o(t)$ at times $t$ = 1, 3, 5 and 7 seconds.

$v_o(1)$ = ____2.8___ V, $v_o(3)$ = ____1.2___ V, $v_o(5)$ = ____0_____ V, and $v_o(7)$ = ___1.5____ V

**Solution:**




$$v_o = \left(\frac{10}{10+R}\right)V_s + \left(R \parallel 10\right) I_s$$

At $t$ = 1 s:

$V_S$ = 6 V, $R$ = 10 + 30 = 40 Ω, $I_S$ = 0.2 A and $v_o = \left(\frac{10}{10+40}\right)6 + \left(40 \parallel 10\right)0.2 = \frac{6}{5} + 8\left(0.2\right) = 2.8\text{ V}$

At $t$ = 3 s:

$V_S$ = –2 V, $R$ = 10 + 30 = 40 Ω, $I_S$ = 0.2 A and $v_o = \left(\frac{10}{10+40}\right)(-2) + \left(40 \parallel 10\right)0.2 = \frac{-2}{5} + 8\left(0.2\right) = 1.2\text{ V}$

At $t$ = 5 s:

$V_S$ = –2 V, $R$ = 10 + 0 = 10 Ω, $I_S$ = 0.2 A and $v_o = \left(\frac{10}{10+10}\right)(-2) + \left(10 \parallel 10\right)0.2 = \frac{-2}{2} + 5\left(0.2\right) = 0\text{ V}$

At $t$ = 7 s:

$V_S$ = –2 V, $R$ = 10 + 0 = 10 Ω, $I_S$ = 0.5 A and $v_o = \left(\frac{10}{10+10}\right)(-2) + \left(10 \parallel 10\right)0.5 = \frac{-2}{2} + 5\left(0.5\right) = 1.5\text{ V}$