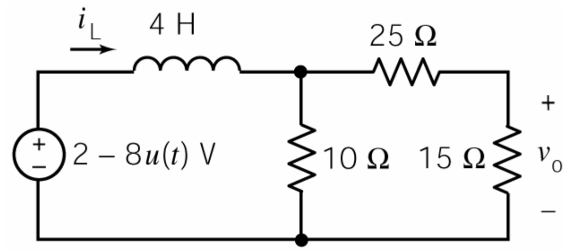
First–Order Circuits

Example 1:
Determine the voltage $v_o(t)$.



Solution:
This is a first order circuit containing an inductor. First, determine $i_L(t)$.

Consider the circuit for time *t* < 0.

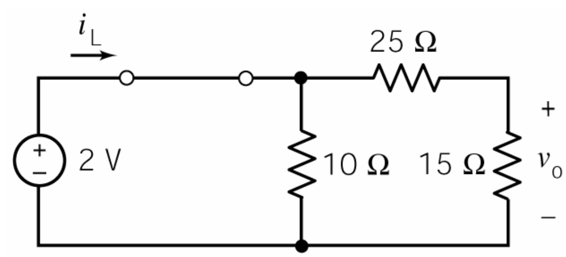
Step 1: Determine the initial inductor current.

The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 2 V, a constant.

The inductor will act like a short circuit.

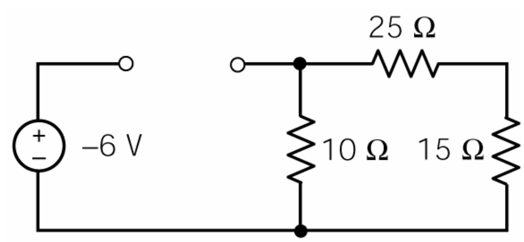
$t < 0$, at steady state:



$$i_L(0) = \frac{2}{10 \| (25+15)} = \frac{2}{8} = 0.25 \text{ A}$$

Consider the circuit for time *t* > 0.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Norton equivalent circuit for the part of the circuit connected to the inductor.

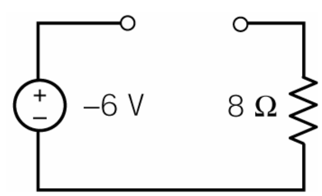


Replacing the resistors by an equivalent resistor, we recognize

$$v_{oc} = -6 \text{ V} \quad \text{and} \quad R_t = 8\ \Omega$$

Consequently

$$i_{sc} = \frac{-6}{8} = -0.75 \text{ A}$$

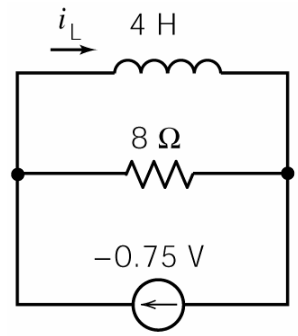


Step 3. The time constant of a first order circuit containing an inductor is given by

$$\tau = \frac{L}{R_t}$$

Consequently

$$\tau = \frac{L}{R_t} = \frac{4}{8} = 0.5 \text{ s} \quad \text{and} \quad a = \frac{1}{\tau} = 2 \ \frac{1}{\text{s}}$$



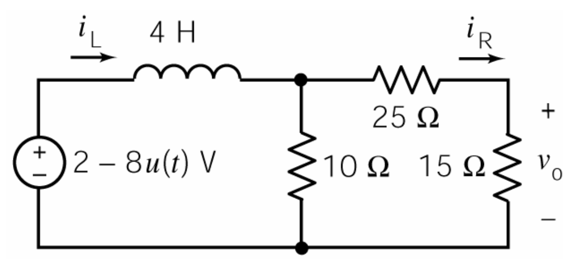
Step 4. The inductor current is given by:

$$i_L(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = -0.75 + \left(0.25 - (-0.75)\right)e^{-2t} = -0.75 + e^{-2t} \quad \text{for } t \ge 0$$

Step 5. Express the output voltage as a function of the source voltage and the inductor current.

Using current division:

$$i_R = \frac{10}{10 + (25 + 15)} i_L = 0.2\, i_L$$



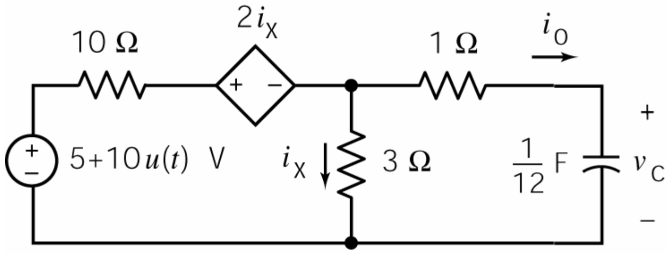
Then Ohm's law gives

$$v_o = 15 i_R = 3 i_L$$

Step 6. The output voltage is given by

$$v_o(t) = -2.25 + 3e^{-2t} \quad \text{for } t \ge 0$$

Example 2:
Determine the current $i_o(t)$.

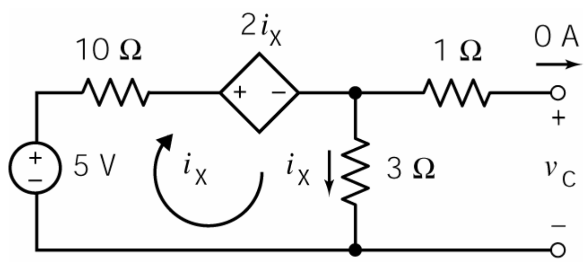


Solution:
This is a first order circuit containing a capacitor. First, determine $v_C(t)$.

Consider the circuit for time *t* < 0.

Step 1: Determine the initial capacitor voltage.

$t < 0$, at steady state:



The circuit will be at steady state before the source voltage changes abruptly at time $t = 0$.

The source voltage will be 5 V, a constant.

The capacitor will act like an open circuit.

Apply KVL to the mesh to get:

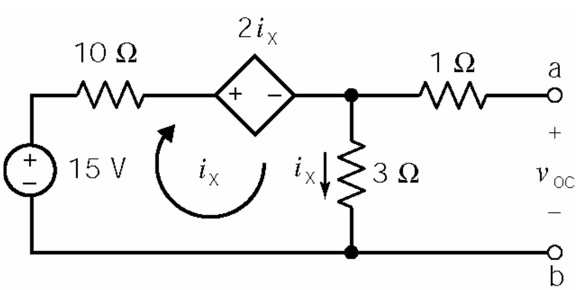
$$(10+2+3)i_x - 5 = 0 \quad \Rightarrow \quad i_x = \frac{1}{3} \text{ A}$$

Then

$$v_C(0) = 3\,i_x = 1 \text{ V}$$

Consider the circuit for time *t* > 0.

Step 2. The circuit will not be at steady state immediately after the source voltage changes abruptly at time $t = 0$. Determine the Thevenin equivalent circuit for the part of the circuit connected to the capacitor. First, determine the open circuit voltage, v_{oc}:



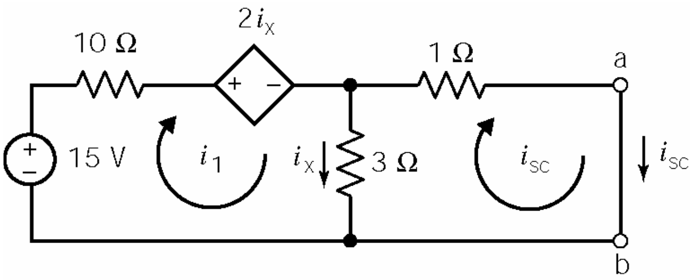
Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\,i_x = 3 \text{ V}$$

Next, determine the short circuit current, i_{sc}:



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\, i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\, i_1 - 5\, i_{sc} = 15$$

And

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\, i_{sc}$$

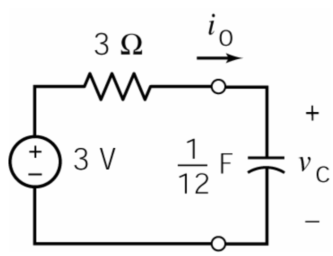
so

$$15\left(\frac{4}{3} i_{sc}\right) - 5\, i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1 \text{ A}$$

The Thevenin resistance is

$$R_t = \frac{3}{1} = 3\ \Omega$$

Step 3. The time constant of a first order circuit containing an capacitor is given by



$$\tau = R_t\, C$$

Consequently

$$\tau = R_t\, C = 3\left(\frac{1}{12}\right) = 0.25 \text{ s and } a = \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

Step 4. The capacitor voltage is given by:

$$v_C(t) = v_{oc} + \left(v_C(0) - v_{oc}\right)e^{-at} = 3 + (1-3)e^{-4t} = 3 - 2\,e^{-4t} \quad \text{for } t \ge 0$$

Step 5. Express the output current as a function of the source voltage and the capacitor voltage.

$$i_o(t) = C\frac{d}{dt}v_C(t) = \frac{1}{12}\frac{d}{dt}v_C(t)$$

Step 6. The output current is given by

$$i_o(t) = \frac{1}{12}\frac{d}{dt}\left(3 - 2\,e^{-4t}\right) = \frac{1}{12}(-2)(-4)e^{-4t} = \frac{2}{3}e^{-4t} \quad \text{for } t \ge 0$$