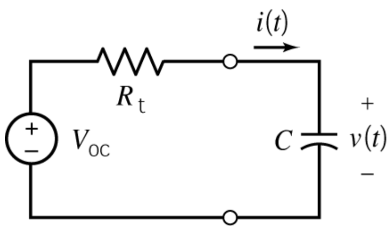
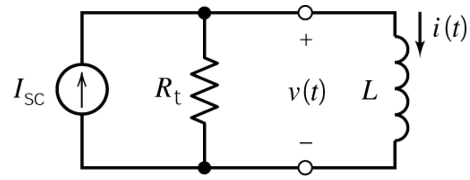
First-Order Dynamic Circuits

A Table at the end of Chapter 8 of Introduction to Electric Circuits indicates that the response of a first-order circuit can be obtained using Thevenin or Norton equivalent circuits:





When the open circuit voltage, V_{oc}, is constant after t = 0, the capacitor voltage is given by

$$v(t) = V_{oc} + \left(v(0) - V_{oc}\right)e^{-at} \quad \text{for } t \geq 0$$

where

$$a = \frac{1}{R_t C}$$

When the open circuit voltage, I_{sc}, is constant after t = 0, the inductor current is given by

$$i(t) = I_{sc} + \left(i(0) - I_{sc}\right)e^{-at} \quad \text{for } t \geq 0$$

where

$$a = \frac{R_t}{L}$$

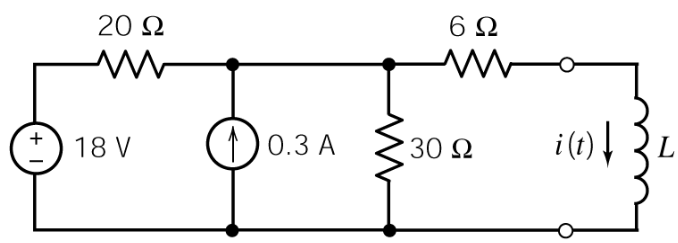
Example 1:

This diagram represents a circuit for time $t \geq 0$. Given the initial condition

$$i(0) = 2.4 \text{ A}$$

and the inductance

$$L = 6 \text{ H}$$



Represent the inductor current $i(t)$ as a function of t for $t \geq 0$.

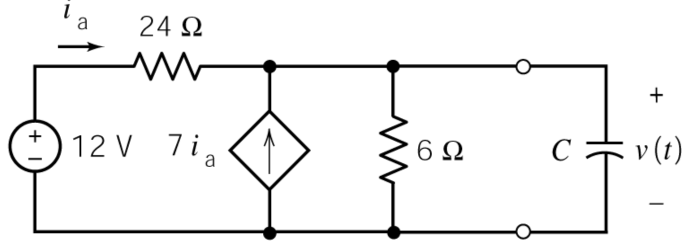
Example 2:

This diagram represents a circuit for time $t \geq 0$. Given the initial condition

$$v(0) = 2 \text{ V}$$

and the capacitance

$$C = 0.1 \text{ F}$$

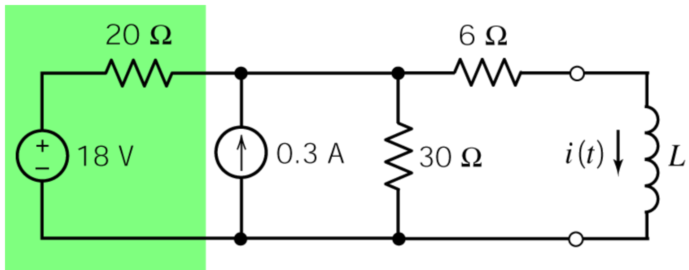


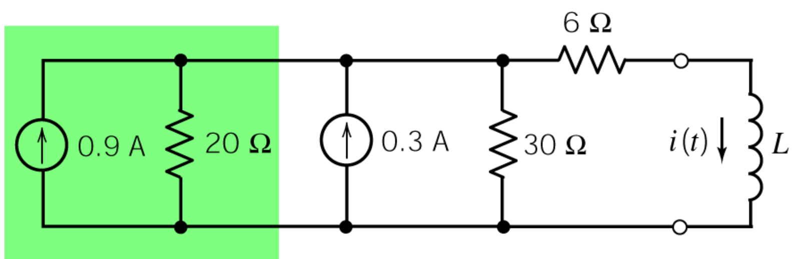
Represent the capacitor voltage $v(t)$ as a function of t for $t \geq 0$.

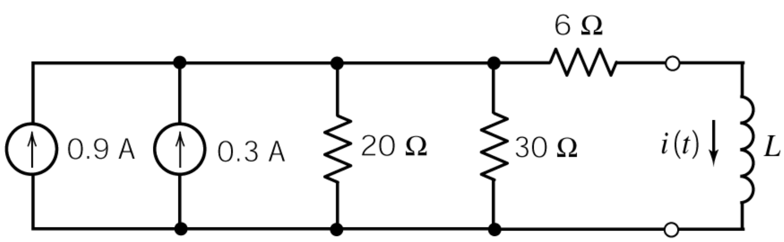
Solutions

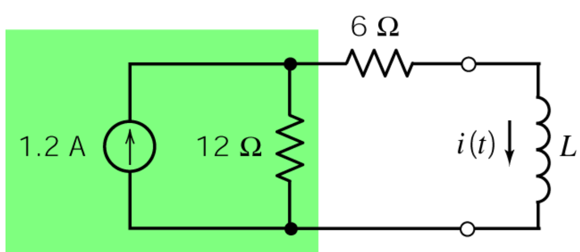
Example 1.

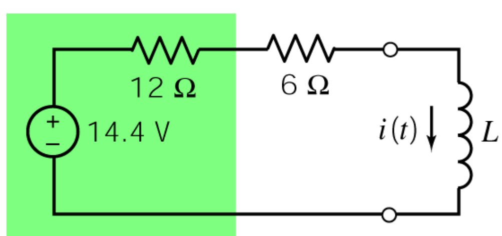
Use source transformations and equivalent resistances to simplify the part of the circuit connected to inductor until it is a Norton equivalent circuit.

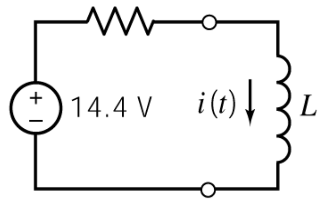


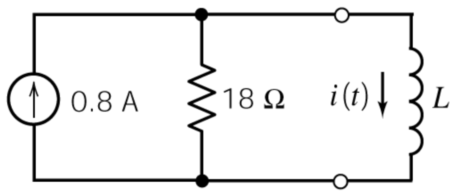












Now recognize that

$$I_{sc} = 0.8 \text{ A} \quad \text{and} \quad R_t = 18\ \Omega$$

Then

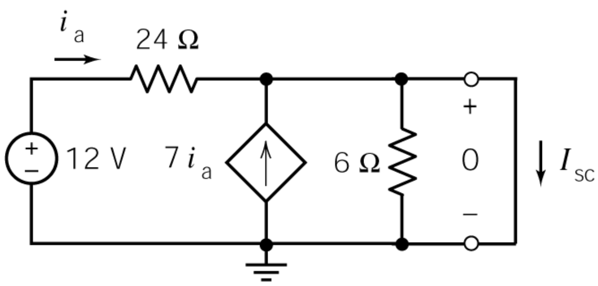
$$a = \frac{R_t}{L} = \frac{18}{6} = 3\ \frac{1}{\text{s}}$$

Finally

$$i(t) = I_{sc} + \left(i(0) - I_{sc}\right)e^{-at} = 0.8 + \left(2.4 - 0.8\right)e^{-3t} = 0.8 + 1.6e^{-3t} \quad \text{for } t \geq 0$$

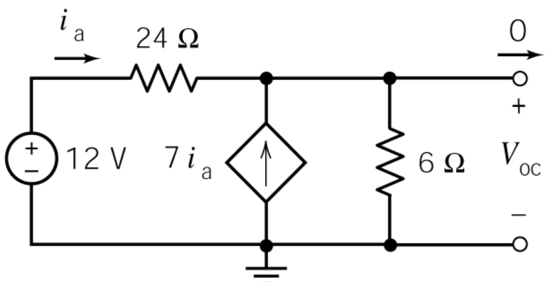
Example 2

We need to determine the values of I_{sc}, V_{oc} and R_{t}:



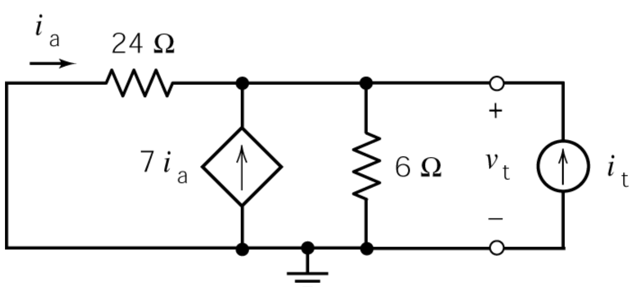
$$i_{\text{a}} = \frac{12-0}{24} = 0.5 \text{ A} \text{. Then } (7+1)\,0.5 = \frac{0}{6} + I_{\text{sc}}$$

$$I_{\text{sc}} = 4 \text{ A}$$



$$i_{\text{a}} = \frac{12-V_{\text{oc}}}{24} \text{. Then } (7+1)\left(\frac{12-V_{\text{oc}}}{24}\right) = \frac{V_{\text{oc}}}{6}$$

$$V_{\text{oc}} = \left(\frac{7+1}{7+5}\right)12 = 8 \text{ V}$$



$$i_{\text{a}} = -\frac{v_{\text{t}}}{24} \text{. Then } i_{\text{t}} + 7\left(-\frac{v_{\text{t}}}{24}\right) = \frac{v_{\text{t}}}{24} + \frac{v_{\text{t}}}{6}$$

$$R_{\text{t}} = \frac{v_{\text{t}}}{i_{\text{t}}} = \frac{24}{7+5} = 2\ \Omega$$

Using $v_{\text{oc}} = 8$ V and $R_{\text{t}} = 2\ \Omega$ we have

$$a = \frac{1}{R_{\text{t}}\, C} = \frac{1}{2 \cdot 0.1} = 5\ \frac{1}{\text{s}}$$

and

$$v(t) = V_{\text{oc}} + \left(v(0) - V_{\text{oc}}\right)e^{-at} = 8 + (2-8)e^{-5t} = 8 - 6e^{-5t} \quad \text{for } t \ge 0$$