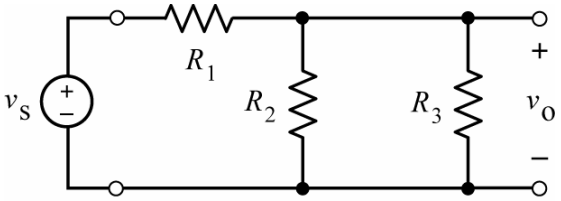
DC Circuit with Specifications

Example:
Consider this circuit:



The input to this circuit is the voltage of the voltage source, v_s. The output is the voltage v_o. We want to choose values of the resistances R_1, R_2 and R_3 to satisfy the following specifications:

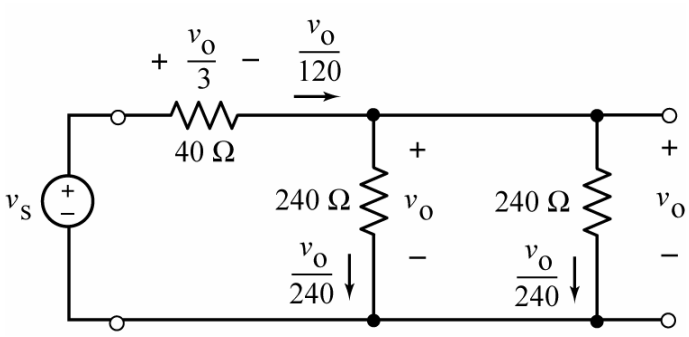
Specifications:

1. $v_o = A v_s$ where $A = \dfrac{v_o}{v_s}$ is the gain of the circuit.
2. The value of the gain is $A = \dfrac{3}{4}$ V/V.
3. All three resistors are quarter watt resistors.
4. The value of the input is restricted by $|v_s| \le 10$ V.

Problem 1
Are the specifications satisfied when $R_1 = 40\ \Omega$, $R_2 = 240\ \Omega$ and $R_3 = 240\ \Omega$?

Problem 2
Are the specifications satisfied when $R_1 = 25\ \Omega$, $R_2 = 150\ \Omega$ and $R_3 = 150\ \Omega$?

Solution 1:
We can label the element voltages and currents as



(Apply KCL at the left node of the 40 Ω resistor to label the current in that resistor.) Apply KVL to the mesh at the left side of the circuit to write

$$\frac{v_o}{3} + v_o - v_s = 0 \quad \Rightarrow \quad v_o = \frac{3}{4} v_s$$

Therefore specifications 1 and 2 are satisfied. It remains to check the values of the power received by the resistors:

$$p_1 = \frac{\left(\frac{v_o}{3}\right)^2}{40} = \frac{{v_o}^2}{360} \quad \text{and} \quad p_2 = p_3 = \frac{{v_o}^2}{360}$$

These power are largest when v_o is largest and v_o is largest when v_s. Consequently

$$p_1 \le \frac{7.5^2}{360} = 0.15625 \text{ W} \quad \text{and} \quad p_2 = p_3 \le \frac{7.5^2}{240} = 0.234375 \text{ W}$$

Therefore, all the resistors can be quarter watt resistors and specification 3 is satisfied.

We conclude that the specifications are satisfied when $R_1 = 40\ \Omega$, $R_2 = 240\ \Omega$ and $R_3 = 240\ \Omega$.

Solution 2
Similar calculations show that when $R_1 = 25\ \Omega$, $R_2 = 150\ \Omega$ and $R_3 = 150\ \Omega$, the gain is correct but resistors R_2 and R_3 each receive more than a quarter watt of power. The specifications are not satisfied.