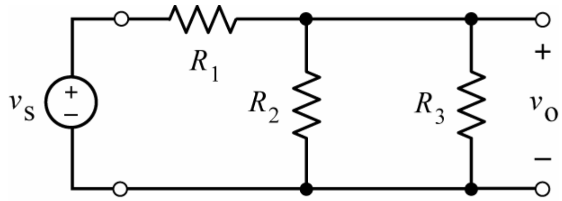
DC Circuit with Specifications

Example:
Consider this circuit:

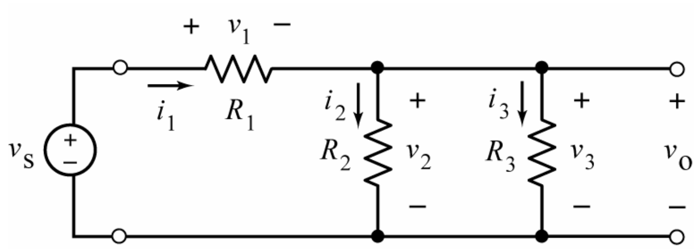


The input to this circuit is the voltage of the voltage source, v_s. The output is the voltage v_o. We want to choose values of the resistances R_1, R_2 and R_3 to satisfy the following specifications:

Specifications:

1. $v_o = A v_s$ where $A = \frac{v_o}{v_s}$ is the gain of the circuit.
2. The value of the gain is $A = \frac{3}{4}$ V/V.
3. All three resistors are quarter watt resistors.
4. The value of the input is restricted by $|v_s| \le 10$ V.

Analysis:
Let's label the element voltage and current of each of the resistors. (In anticipation of using Ohm's law, we will choose the element current and voltage of each resistor to adhere to the passive convention.)



Notice that

$$v_2 = v_3 = v_o$$

It's reasonable to choose $R_2 = R_3$. (Otherwise the resistor with the smaller resistance will receive more power than the resistor having the larger resistance. Consequently, specification 3 will be harder to satisfy.) Then

$$i_2 = \frac{v_2}{R_2} = \frac{v_o}{R_2} = \frac{v_o}{R_3} = \frac{v_3}{R_3} = i_3$$

Next, apply KCL at the node where resistors R_1, R_2 and R_3 are connected together to get

$$i_1 = i_2 + i_3 = i_2 + i_2 = 2\, i_2 \quad \Rightarrow \quad i_2 = i_3 = \frac{i_1}{2}$$

Now apply KVL to the loop consisting of the voltage source and resistors R_1 and R_2 to get

$$v_1 + v_2 - v_s = 0 \quad \Rightarrow \quad R_1\, i_1 + R_2\, i_2 - v_s = 0 \quad \Rightarrow \quad v_s = R_1\, i_1 + R_2\, i_2 = i_1\left(R_1 + \frac{R_2}{2} \right)$$

So

$$i_1 = \frac{v_s}{R_1 + \frac{R_2}{2}} = \frac{2\, v_s}{2\, R_1 + R_2}$$

And

$$i_2 = i_3 = \frac{i_1}{2} = \frac{v_s}{2\, R_1 + R_2}$$

Finally, use Ohm's law to write

$$v_o = R_3\, i_3 = R_2\, i_2 = \frac{R_2\, v_s}{2\, R_1 + R_2} = \frac{R_2}{2\, R_1 + R_2} v_s$$

Thus, specification 1 is satisfied with

$$A = \frac{R_2}{2\, R_1 + R_2}$$

From specification 2, we require

$$\frac{3}{4} = A = \frac{R_2}{2\, R_1 + R_2} \quad \Rightarrow \quad 3\left(2\, R_1 + R_2\right) = 4\, R_2 \quad \Rightarrow \quad 6\, R_1 = R_2$$

Partial Result:

Specifications 1 and 2 are satisfied by choosing resistances R_1, R_2 and R_3 to satisfy

$$R_1 = R \quad \text{and} \quad R_2 = R_3 = 6\, R$$

This observation reduces the problem form one of finding three values; R_1, R_2 and R_3; to one of finding a single value, *R*.

More Analysis:

Now let's consider the power supplied by each resistor. The power supplied by resistor R_1 is given by

$$p_1 = R_1 i_1^{\,2} = R_1\left(\frac{2v_s}{2R_1+R_2}\right)^2 = R\left(\frac{2v_s}{2R+6R}\right)^2 = \left(\frac{1}{16}\right)\frac{v_s^{\,2}}{R}$$

Observe that p_1 will be maximum when v_s is maximum. In view of specifications 3 and 4, we require $p_1 \le 0.25$ when $v_s = 10$. That is

$$0.25 \ge \left(\frac{1}{16}\right)\frac{v_s^{\,2}}{R} = \frac{100}{16R} = \frac{25}{4R} \quad \Rightarrow \quad R \ge 25$$

The power supplied by resistor R_2 is given by

$$p_2 = R_2 i_2^{\,2} = R_2\left(\frac{v_s}{2R_1+R_2}\right)^2 = 6R\left(\frac{v_s}{2R+6R}\right)^2 = \left(\frac{3}{32}\right)\frac{v_s^{\,2}}{R}$$

Again, p_2 will be maximum when v_s is maximum. In view of specifications 3 and 4, we require $p_2 \le 0.25$ when $v_s = 10$. That is

$$0.25 \ge \left(\frac{3}{32}\right)\frac{v_s^{\,2}}{R} = \frac{300}{32R} = \frac{75}{8R} \quad \Rightarrow \quad R \ge 37.5$$

The power supplied by resistor R_3 is given by

$$p_3 = R_3 i_3^{\,2} = R_3\left(\frac{v_s}{2R_1+R_2}\right)^2 = 6R\left(\frac{v_s}{2R+6R}\right)^2 = \left(\frac{3}{32}\right)\frac{v_s^{\,2}}{R}$$

Once again, p_3 will be maximum when v_s is maximum. In view of specifications 3 and 4, we require $p_3 \le 0.25$ when $v_s = 10$. That is

$$0.25 \ge \left(\frac{3}{32}\right)\frac{v_s^{\,2}}{R} = \frac{300}{32R} = \frac{75}{8R} \quad \Rightarrow \quad R \ge 37.5$$

Result:

All the specifications can be satisfied by choosing resistances R_1, R_2 and R_3 to satisfy

$$R_1 = R \quad \text{and} \quad R_2 = R_3 = 6R$$

and

$$R \ge 37.5\ \Omega$$

Checking the Result:

Let R = 40 Ω. Then $R_1 = 40\ \Omega$ and $R_2 = R_3 = 240\ \Omega$. Let $v_s = 10$ V. In this case, we expect that the specifications will be satisfied. Analysis of the circuit gives

$$i_1 = 62.5 \text{ mA} \quad \text{and} \quad i_2 = i_3 = 31.25 \text{ mA}$$
$$v_1 = 2.5 \text{ V} \quad \text{and} \quad v_2 = v_3 = 7.5 \text{ V}$$

So

$$A = \frac{v_o}{v_s} = \frac{v_3}{v_s} = \frac{7.5}{10} = 0.75 \text{ V/V}$$
$$p_1 = 0.0625 \times 2.5 = 0.15625 \text{ W} \quad \text{and} \quad p_2 = p_3 = 0.03125 \times 7.5 = 0.234375 \text{ W}$$

The gain is correct and all of the resistor receive less than a quarter watt so the specifications are indeed satisfied.

As a second check, let R = 25 Ω. Then $R_1 = 25\ \Omega$ and $R_2 = R_3 = 150\ \Omega$. Let $v_s = 10$ V. In this case, we expect the gain to be 0.75 V/V and resistor R_1 to receive exactly one quarter watt of power. We also expect that resistors R_2 and R_3 to receive more than a quarter watt of power, so the specifications will not be satisfied. Analysis of the circuit gives

$$i_1 = 100 \text{ mA} \quad \text{and} \quad i_2 = i_3 = 50 \text{ mA}$$
$$v_1 = 2.5 \text{ V} \quad \text{and} \quad v_2 = v_3 = 7.5 \text{ V}$$

So

$$A = \frac{v_o}{v_s} = \frac{v_3}{v_s} = \frac{7.5}{10} = 0.75 \text{ V/V}$$
$$p_1 = 0.1 \times 2.5 = 0.25 \text{ W} \quad \text{and} \quad p_2 = p_3 = 0.05 \times 7.5 = 0.375 \text{ W}$$

The gain is correct but resistors R_2 and R_3 each receive more than a quarter watt of power, so the specifications are not satisfied.