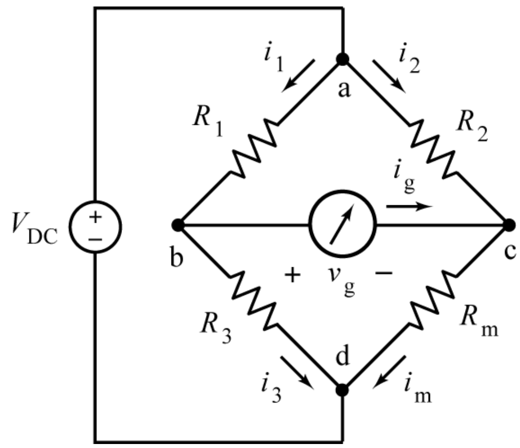
The Wheatstone Bridge

The Wheatstone Bridge consists of a dc voltage source, four resistors and a detector. The detector is a type of ammeter called a galvanometer.



The galvanometer is used to detect the condition $i_g = 0$. When the circuit satisfies the condition $i_g = 0$ we say that "the bridge is balanced".

Because the galvanometer is a type of ammeter, $v_g = 0$. (It's always true that $v_g = 0$, whether the bridge is balanced or not. When the bridge is balanced it is also true that $i_g = 0$.) Apply KVL to the top mesh of the bridge to get

$$R_2 i_2 - v_g - R_1 i_1 = 0 \quad \Rightarrow \quad R_1 i_1 = R_2 i_2 \tag{1}$$

Apply KVL to the bottom mesh of the bridge to get

$$v_g + R_m i_m - R_3 i_3 = 0 \quad \Rightarrow \quad R_3 i_3 = R_m i_m \tag{2}$$

When the bridge is balanced $i_g = 0$. Apply KCL to node b of the balanced bridge to get

$$i_1 = i_g + i_3 = 0 \quad \Rightarrow \quad i_1 = i_3 \tag{3}$$

Apply KCL to node c of the balanced bridge to get

$$i_2 + i_g = i_m \quad \Rightarrow \quad i_2 = i_m \tag{4}$$

Using equations 3 and 4 to substitute for the currents in equation 2 gives

$$R_3 i_1 = R_m i_2 \tag{5}$$

Dividing equation 5 by equation 1 gives

$$\frac{R_3}{R_1} = \frac{R_m}{R_2} \tag{6}$$

Now and solving for R_m we get

$$R_m = \frac{R_2}{R_1} R_3 \tag{6}$$

Typically, R_1 and R_2 are fixed resistors and R_3 is a variable resistor. R_m is the resistance that is being measured. R_3 is adjusted until the detector indicates that the bridge is balanced. Then the value of R_m is determined using equation 6.

Example
Consider using a Wheatstone bridge having $R_1 = 200\ \Omega$ and $R_2 = 2000\ \Omega$ to measure a resistance R_m. The bridge is balanced by adjusting R_3 until $R_3 = 250\ \Omega$. What is the value of R_m?

Solution
From equation 6

$$R_m = \frac{R_2}{R_1} R_3 = \frac{2000}{200} 250 = 2500\ \Omega$$

Example
Consider using a Wheatstone bridge having $R_1 = 200\ \Omega$ and $R_2 = 2000\ \Omega$ to measure a resistance, R_m, of a temperature sensor. Suppose the resistance of the temperature sensor, R_m, in Ω, is related to the temperature *T*, in °C, by the equation

$$R_m = 1500 + 25T$$

The bridge is balanced by adjusting R_3 until $R_3 = 250\ \Omega$. What is the value of the temperature?

Solution
From equation 6

$$R_m = \frac{R_2}{R_1} R_3 = \frac{2000}{200} 250 = 2500\ \Omega$$

Next, the temperature in °C is given by

$$T = \frac{R_m - 1500}{25} = \frac{2500 - 1500}{25} = \frac{1000}{25} = 40\ ^\circ\mathrm{C}$$

Example
Consider using a Wheatstone bridge having $R_1 = 200\ \Omega$ and $R_2 = 2000\ \Omega$ to measure a resistance, R_{m}, of a temperature sensor. Suppose the resistance of the temperature sensor, R_{m}, in Ω, is related to the temperature *T*, in °C, by the equation

$$R_{\mathrm{m}} = 1500 + 25T$$

The temperature is expected to vary over the range 0 to 100 °C. Over what range must R_3 vary in order for the bridge to measure temperature over the range 0 to 100 °C?

Solution:
Solve equation 6 for R_3:

$$R_3 = \frac{R_1}{R_2} R_{\mathrm{m}} \tag{7}$$

When $T = 0$ °C, $R_{\mathrm{m}} = 1500\ \Omega$ and

$$R_3 = \frac{R_1}{R_2} R_{\mathrm{m}} = \frac{200}{2000} 1500 = 150\ \Omega$$

When $T = 100$ °C, $R_{\mathrm{m}} = 1500 + 25(100) = 4000\ \Omega$ and

$$R_3 = \frac{R_1}{R_2} R_{\mathrm{m}} = \frac{200}{2000} 4000 = 400\ \Omega$$

R_3 could be implemented as a 150 Ω resistor in series with a 250 Ω potentiometer: