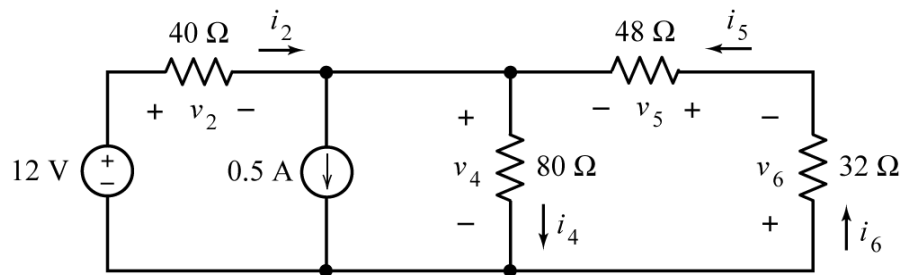


**Solution:**



Apply KCL to the node at which the current source and the 40 Ω, 48 Ω and 80 Ω resistors are connected together.

$$\text{EQN 1:} \quad i_2 + i_5 = 0.5 + i_4$$

Apply KCL to the node at which the 48 Ω and 32 Ω resistors are connected together.

$$\text{EQN 2:} \quad i_5 = i_6$$

Apply KVL to the loop consisting of the voltage source and the 40 Ω and 80 Ω resistors.

$$\text{EQN 3:} \quad 12 = v_2 + v_4$$

Apply KVL to the loop consisting of the 48 Ω, 32 Ω and 80 Ω resistors.

$$\text{EQN 4:} \quad v_4 + v_5 + v_6 = 0$$

Apply Ohm's law to the resistors.

$$\text{Ohm's law:} \quad v_2 = 40 i_2, \quad v_4 = 80 i_4, \quad v_5 = 48 i_5, \quad v_6 = 32 i_6$$

Use the Ohm's law equations to eliminate the variables representing resistor voltages from the KVL equations.

$$\text{EQN 5 (from EQN 3):} \quad 12 = 40 i_2 + 80 i_4$$

$$\text{EQN 6 (from EQN 4):} \quad 80 i_4 + 48 i_5 + 32 i_6 = 0$$

Use EQN 2 to eliminate  $i_6$  from EQN 6.

$$\text{EQN 7:} \quad 80 i_4 + 48 i_5 + 32 i_5 = 0 \Rightarrow 80 i_4 + 80 i_5 = 0 \Rightarrow i_4 = -i_5$$

Use EQN 7 to eliminate  $i_5$  from EQN 1.

$$\text{EQN 8:} \quad i_2 - i_4 = 0.5 + i_4 \Rightarrow i_2 = 0.5 + 2i_4$$

Use EQN 8 to eliminate  $i_4$  from EQN 5. Solve the resulting equation to determine the value of  $i_2$ .

$$\text{EQN 9:} \quad 12 = 40i_2 + 80\left(\frac{i_2 - 0.5}{2}\right) = 80i_2 - 20 \Rightarrow i_2 = \frac{12 + 20}{80} = 0.4 \text{ A}$$

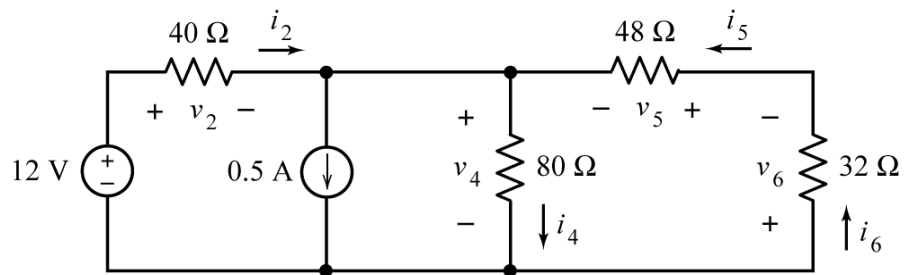
Determine the values of the rest of the resistor voltages and currents.

$$i_4 = \frac{i_2 - 0.5}{2} = \frac{0.4 - 0.5}{2} = -0.05 \text{ A}, \quad i_6 = i_5 = -i_4 = 0.05 \text{ A},$$

$$v_2 = 40i_2 = 40(0.4) = 1.6 \text{ V}, \quad v_4 = 80i_4 = 80(-0.05) = -4 \text{ V},$$

$$v_5 = 48i_5 = 48(0.05) = 2.4 \text{ V} \quad \text{and} \quad v_6 = 32i_6 = 32(0.05) = 1.6 \text{ V}$$

### MATLAB Solution:



### Consecutive equations:

The above algebra shows that this circuit can be represented by these equations:

$$12 = 80i_2 - 20, \quad i_4 = \frac{i_2 - 0.5}{2},$$

$$i_6 = i_5 = -i_4,$$

$$v_2 = 40i_2, \quad v_4 = 80i_4,$$

$$v_5 = 48i_5 \quad \text{and} \quad v_6 = 32i_6$$

These equations can be solved consecutively using MATLAB.

```
>> i2=(12+20)/80
i2 =
    0.4000
>> i4=(i2-0.5)/2
i4 =
   -0.0500
>> i5=-i4;
>> i6=i5;
>> v2=40*i2
v2 =
    16
>> v4=80*i4
v4 =
   -4.0000
>> v5=48*i5
v5 =
    2.4000
>> v6=32*i6
v6 =
    1.6000
>>
```

## Simultaneous Equations

We can avoid some algebra if we are willing to solve simultaneous equations.

After applying Kirchhoff's laws and then using the Ohm's law equations to eliminate the variables representing resistor voltages we have

$$i_2 + i_5 = 0.5 + i_4, \quad i_5 = i_6, \quad 12 = 40i_2 + 80i_4$$

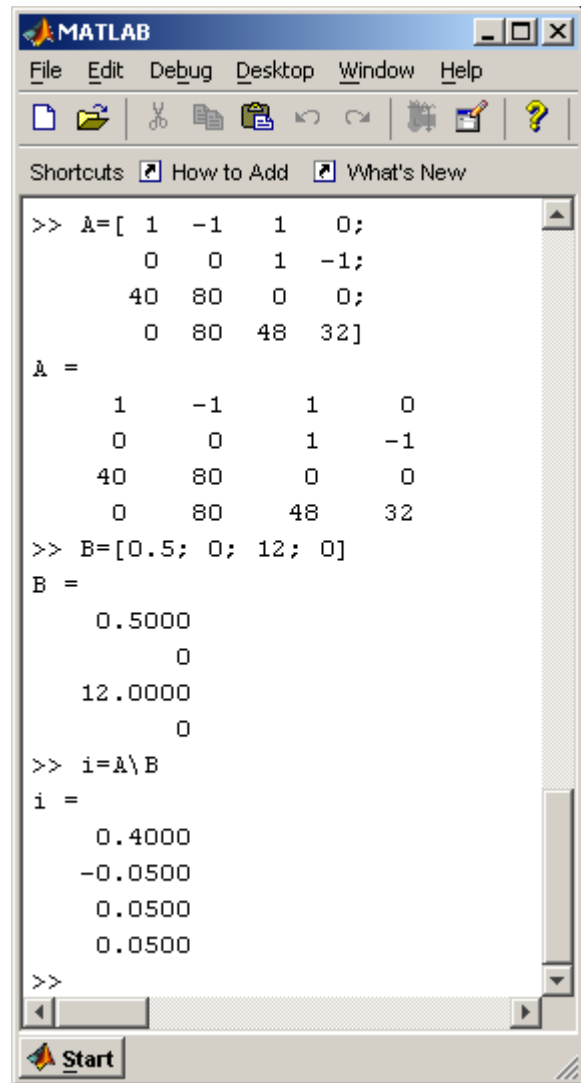
and

$$80i_4 + 48i_5 + 32i_6 = 0$$

This set of 4 simultaneous equations in  $i_2$ ,  $i_4$ ,  $i_5$  and  $i_6$  can be written as a single matrix equation.

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 40 & 80 & 0 & 0 \\ 0 & 80 & 48 & 32 \end{bmatrix} \begin{bmatrix} i_2 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 12 \\ 0 \end{bmatrix}$$

This matrix equation can be solved using MATLAB.



```
MATLAB
File Edit Debug Desktop Window Help
[Icons]
Shortcuts [How to Add] [What's New]
>> A=[ 1 -1 1 0;
      0 0 1 -1;
      40 80 0 0;
      0 80 48 32]
A =
     1    -1     1     0
     0     0     1    -1
    40    80     0     0
     0    80    48    32
>> B=[0.5; 0; 12; 0]
B =
    0.5000
         0
   12.0000
         0
>> i=A\B
i =
    0.4000
   -0.0500
    0.0500
    0.0500
>>
```