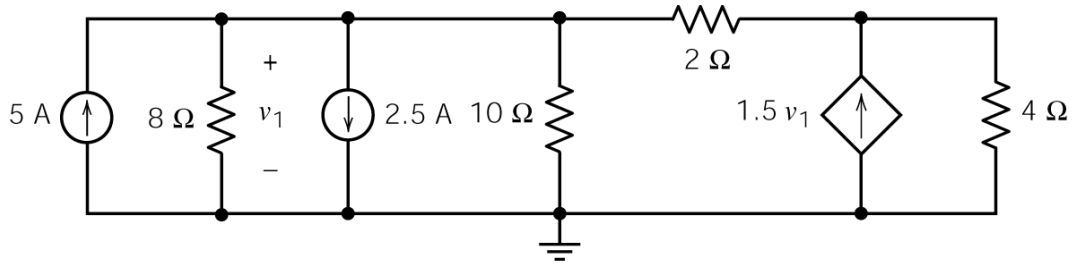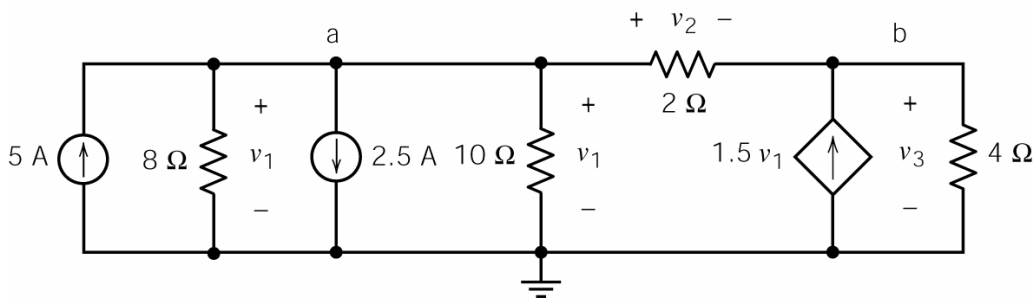**Example:** Determine the power supplied by each of the sources, independent and dependent, in this circuit:

Solution: We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element voltages:

Notice that the 8 Ω resistor, the 10 Ω resistor and the two independent current sources are all connected in parallel. Consequently, the element voltages of theses elements can be labeled so that they are equal. Similarly, the 4 Ω resistor and the dependent current source are connected in parallel so their voltages can be labeled so as to be equal.

Using Ohm's Law we see that the current directed downward in the 8 Ω resistor is $\dfrac{v_1}{8}$, current directed downward in the 10 Ω resistor is $\dfrac{v_1}{10}$, and the current directed from left to right in the 2 Ω resistor is $\dfrac{v_2}{2}$. Applying Kirchhoff's Current Law (KCL) at node a gives

$$5 = \frac{v_1}{8} + 2.5 + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225\,v_1 + 0.5\,v_2 = 2.5 \tag{1}$$

Using Ohm's Law we see that the current directed downward in the 4 Ω resistor is $\dfrac{v_3}{4}$. Applying Kirchhoff's Current Law (KCL) at node a gives

1

$$\frac{v_2}{2} + 1.5 v_1 = \frac{v_3}{4} \quad \Rightarrow \quad 1.5 v_1 + 0.5 v_2 - 0.25 v_3 = 0 \tag{2}$$

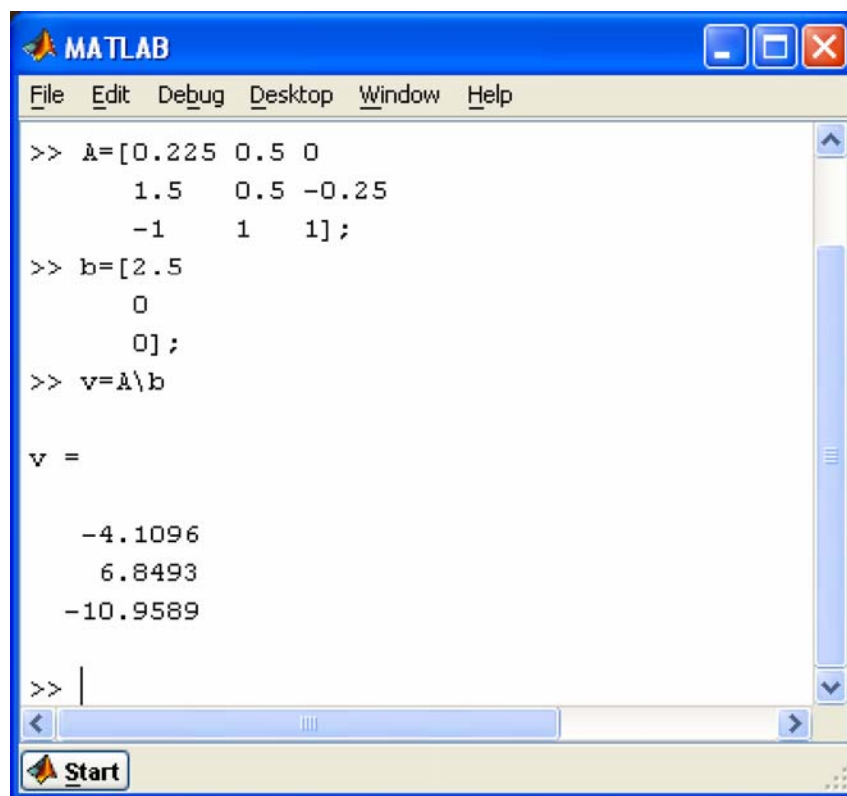Applying Kirchhoff's Voltage Law (KVL) to the mesh consisting of the 10 Ω resistor, the 2 Ω resistor and the dependent source to get

$$v_2 + v_3 - v_1 = 0 \tag{3}$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 0.225 & 0.5 & 0 \\ 1.5 & 0.5 & -0.25 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:


Hence

$$v_1 = -4.1096 \text{ V}, \quad v_2 = 6.8493 \text{ V} \quad \text{and} \quad v_3 = -10.9589 \text{ V}$$

2

The power supplied by the 5 A current source is $5v_1 = 5(-4.1096) = -20.548 \text{ W}$. The power supplied by the 2.5 A current source is $-2.5v_1 = -2.5(-4.1096) = 10.247 \text{ W}$. The power supplied by the dependent current source is $(1.5v_1)v_3 = 1.5(-4.1096)(-10.9589) = 67.555 \text{ W}$.

Observation: Changing the order of the 8 Ω resistor, the 10 Ω resistor and the two independent current sources only changes the order of the terms in the KCL equation at node a. We know that addition is commutative, so change the order of the terms will not affect the values of the voltages v_1, v_2 and v_3. For example, if the positions of the 2.5 A current source and 8 Ω resistor are swithched:
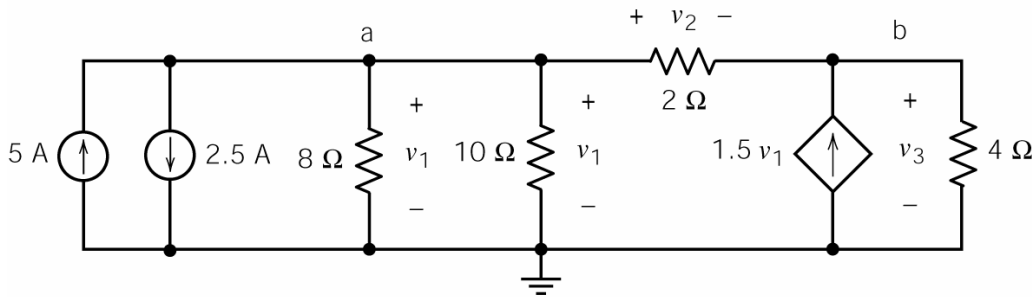
The KCL equation at node a is

$$5 = 2.5 + \frac{v_1}{8} + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

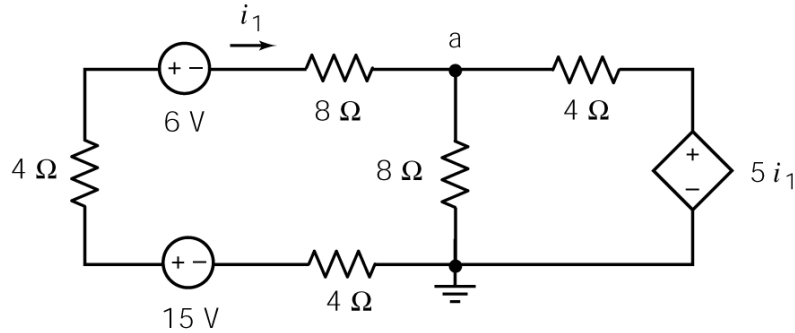Similarly, when the circuit is drawn as
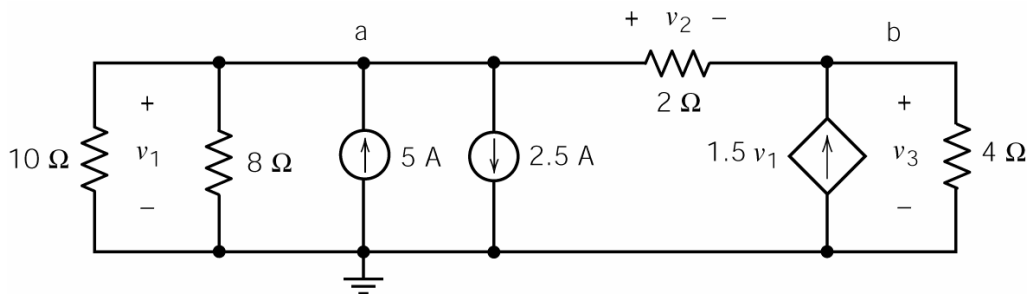
The KCL equation at node a is

$$5 = \frac{v_1}{10} + \frac{v_1}{8} + 2.5 + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

The changes do not affect the values of the voltages v_1, v_2 and v_3.

3

Example: Determine the power supplied by each of the sources, independent and dependent, in this circuit:

Solution: We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element currents:
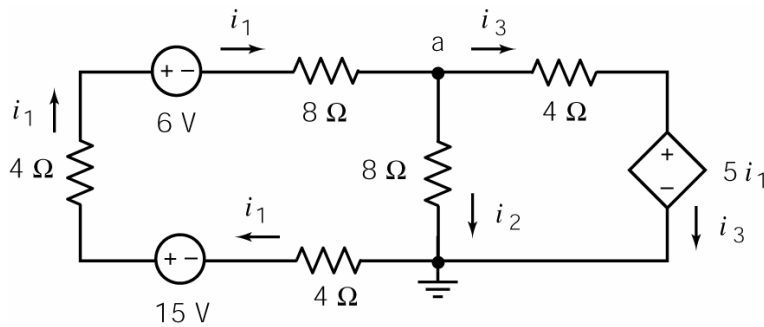
Notice that two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources are all connected in series. Consequently, the element currents of theses elements can be labeled so that they are equal. Similarly, a 4 Ω resistor and the dependent voltage source are connected in series so their currents can be labeled so as to be equal.

The current in each resistor has been labeled so we can use Ohm's Law to calculate resistor voltages from the resistor currents and the resistances. Apply Kirchhoff's Voltage Law (KVL) to the left mesh to get

$$6 + 8i_1 + 8i_2 + 4i_1 - 15 + 4i_1 = 0 \quad \Rightarrow \quad 16i_1 + 8i_2 = 9 \tag{1}$$

Apply Kirchhoff's Voltage Law (KVL) to the right mesh to get
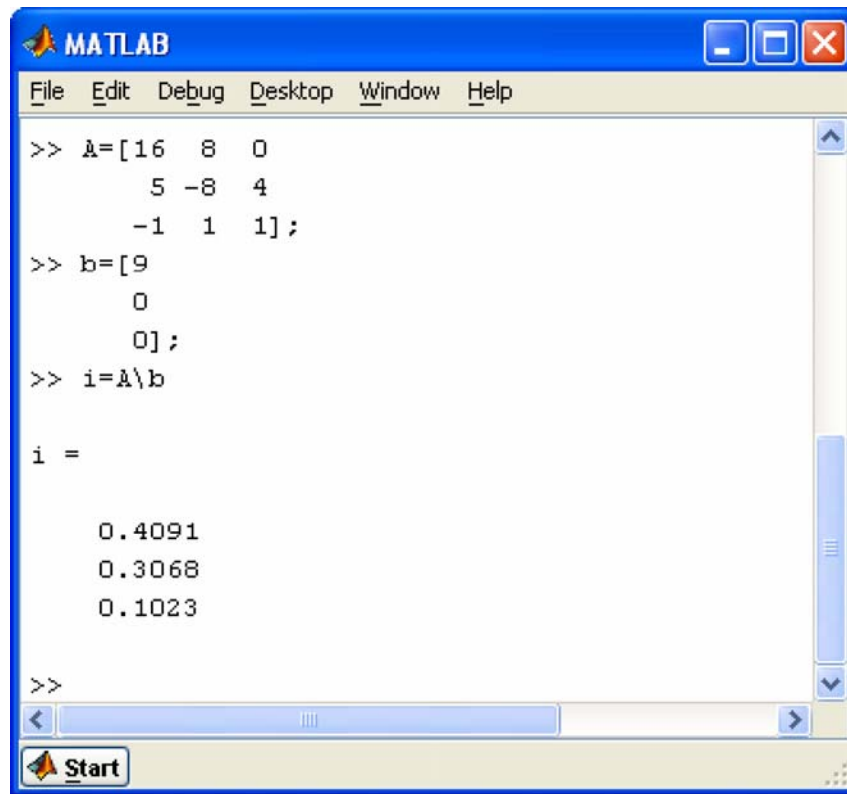
$$4i_3 + 5i_1 - 8i_2 = 0 \tag{2}$$

Applying Kirchhoff's Current Law (KCL) at node a to get

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad -i_1 + i_2 + i_3 = 0 \tag{3}$$

4

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 16 & 8 & 0 \\ 5 & -8 & 4 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:

```
MATLAB
File  Edit  Debug  Desktop  Window  Help

>> A=[16   8   0
       5  -8   4
      -1   1   1];
>> b=[9
      0
      0];
>> i=A\b

i =

    0.4091
    0.3068
    0.1023

>>
Start
```

Hence
$$i_1 = 0.4091 \text{ A}, \quad i_2 = 0.3068 \text{ A} \quad \text{and} \quad i_3 = 0.1023 \text{ A}$$

The power supplied by the 15 V voltage source is $15 i_1 = 15(0.4091) = 6.1365 \text{ W}$. The power supplied by the 6 V voltage source is $-6 i_1 = -6(0.4091) = -2.4546 \text{ W}$. The power supplied by the dependent voltage source is $-(5 i_2) i_3 = -5(0.3068)(0.1023) = 0.1569 \text{ W}$.

Observation: Changing the order of the two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources in the left mesh changes the order of the terms in the KVL equation for that mesh. We know that addition is commutative, so change the order of the terms will not affect the values of the currents i_1, i_2 and i_3. For example, when the circuit is drawn as
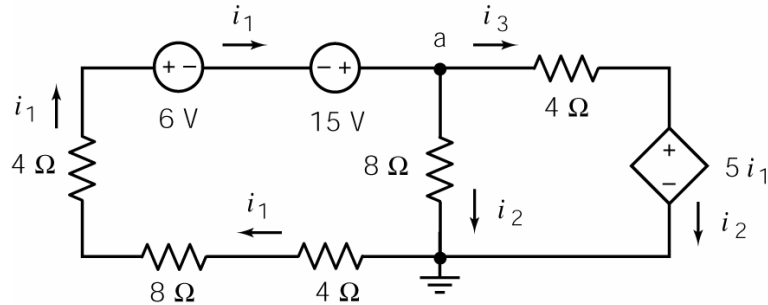
The KVL equation for the left mesh is

$$6 - 15 + 8i_2 + 4i_1 + 8i_1 + 4i_1 = 0 \quad \Rightarrow \quad 16i_1 + 8i_2 = 9$$

When the circuit is drawn as
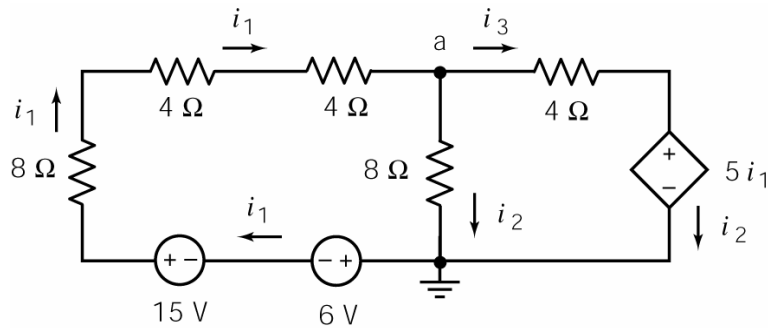
The KVL equation for the left mesh is

$$4i_1 + 4i_1 + 8i_2 + 6 - 15 + 8i_1 = 0 \quad \Rightarrow \quad 16i_1 + 8i_2 = 9$$

These changes do not affect the values of the currents i_1, i_2 and i_3.