Ohm's and Kirchhoff's Laws

1. Consider this circuit.

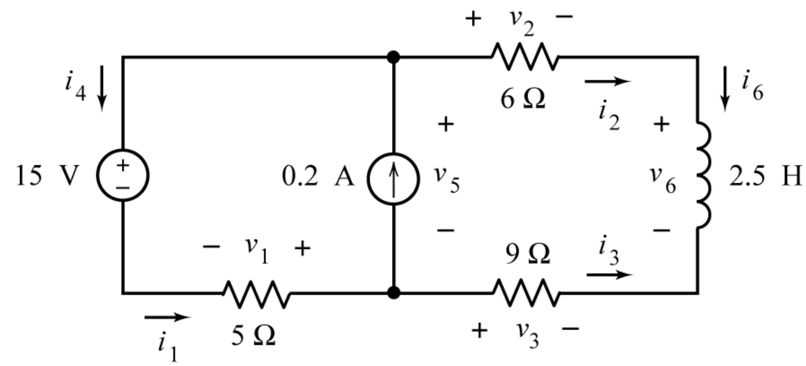


Figure 1

For $t > 0$, the inductor current and voltage are given by

$$i_6(t) = 0.8 - 0.6e^{-8t} \text{ A} \quad \text{and} \quad v_6(t) = 12e^{-8t} \text{ V} \tag{1}$$

Determine the voltage source current $i_4(t)$, the current source voltage, $v_5(t)$ and the resistor voltage $v_1(t)$.

Solution
From **KCL**

$$i_2(t) = i_6(t) = 0.8 - 0.6e^{-8t} \text{ A} \quad \text{and} \quad i_3(t) + i_6(t) = 0 \quad \Rightarrow \quad i_3(t) = -0.8 + 0.6e^{-8t} \text{ A}$$

From **Ohm's law**

$$v_2(t) = 6i_2(t) = 6\left(0.8 - 0.6e^{-8t}\right) = 4.8 - 3.6e^{-8t} \text{ V} \tag{2}$$

and

$$v_3(t) = 9i_3(t) = 9\left(-0.8 + 0.6e^{-8t}\right) = -7.2 + 5.4e^{-8t} \text{ V} \tag{3}$$

(Notice that the current and voltage of the 6 Ω resistor, $i_2(t)$ and $v_2(t)$, adhere to the passive convention. Similarly, $i_3(t)$ and $v_3(t)$, adhere to the passive convention.)

From **KVL**

$$v_2(t) + v_6(t) - v_3(t) - v_5(t) = 0 \quad \Rightarrow \quad v_5(t) = v_2(t) + v_6(t) - v_3(t) \tag{4}$$

so

$$v_5(t) = \left(4.8 - 3.6e^{-8t}\right) + 12e^{-8t} - \left(-7.2 + 5.4e^{-8t}\right) = 12 + 3e^{-8t} \text{ V} \tag{5}$$

From **KCL**

$$0.2 = i_2(t) + i_4(t) \quad \Rightarrow \quad i_4(t) = 0.2 - i_2(t) = 0.2 - \left(0.8 - 0.6e^{-8t}\right) = -0.6 + 0.6e^{-8t} \text{ A}$$

and

$$i_1(t) = i_4(t) = -0.6 + 0.6e^{-8t} \text{ A}$$

From **Ohm's law**

$$v_1(t) = -5i_1(t) = -5\left(-0.6 + 0.6e^{-8t}\right) = 3 - 3e^{-8t} \text{ V}$$

(Notice that the current and voltage of the 5 Ω resistor, $i_1(t)$ and $v_1(t)$, **do not** adhere to the passive convention. Consequently $v_1(t) = -5i_1(t)$ rather than $v_1(t) = 5i_1(t)$.)

As a check, apply KVL to the left mesh to get

$$v_5(t) + v_1(t) - 15 = 0 \quad \Rightarrow \quad v_1(t) = -v_5(t) + 15 = -\left(12 + 3e^{-8t}\right) + 15 = 3 - 3e^{-8t} \text{ V}$$

as before.

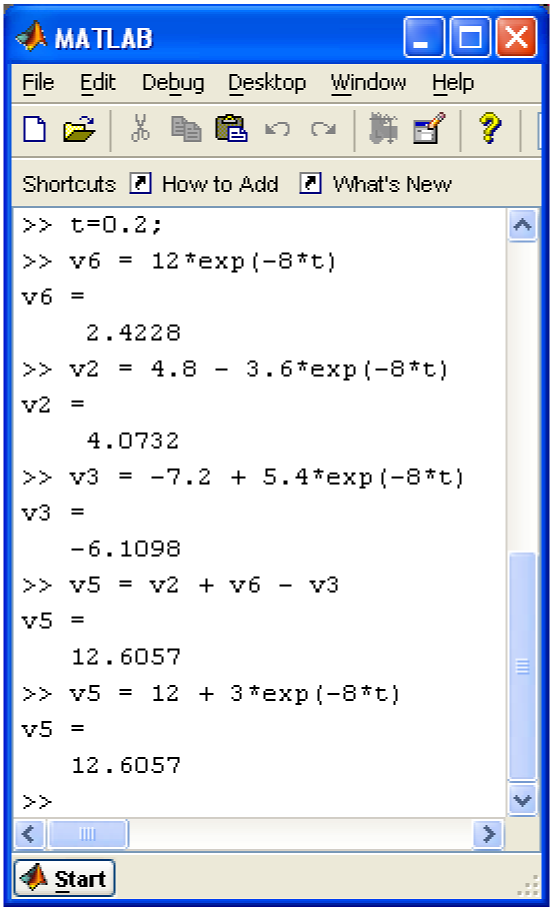


Figure 2 Using MATLAB to check a KVL equation.

The computer program MATLAB (Hanselman and Littlefield, 2005) provides additional ways to check our results. For example, Equation 4 is valid at all times $t > 0$. In Figure 2, Equation 4 is checked at time t = 0.2 seconds using MATLAB. Voltages v_6, v_2, and v_3 are evaluated at time $t = 0.2$ seconds using equations 1, 2 and 3. Next, The voltage v_5 is calculated twice, using equations 4 and 5. The two values of v_5 are identical, indicating that our results are correct.

The computer program MATLAB also helps us to visualize the voltages and currents in our circuit. For example, Figure 3 shows a plot of the voltage v_5 as a function of time. We see that v_5 makes an exponential transition from 15 V at time zero to an eventual value of 12 V. These are the same values that are obtained from equation 5 by taking the limit of v_5 as time goes to zero and to infinity.

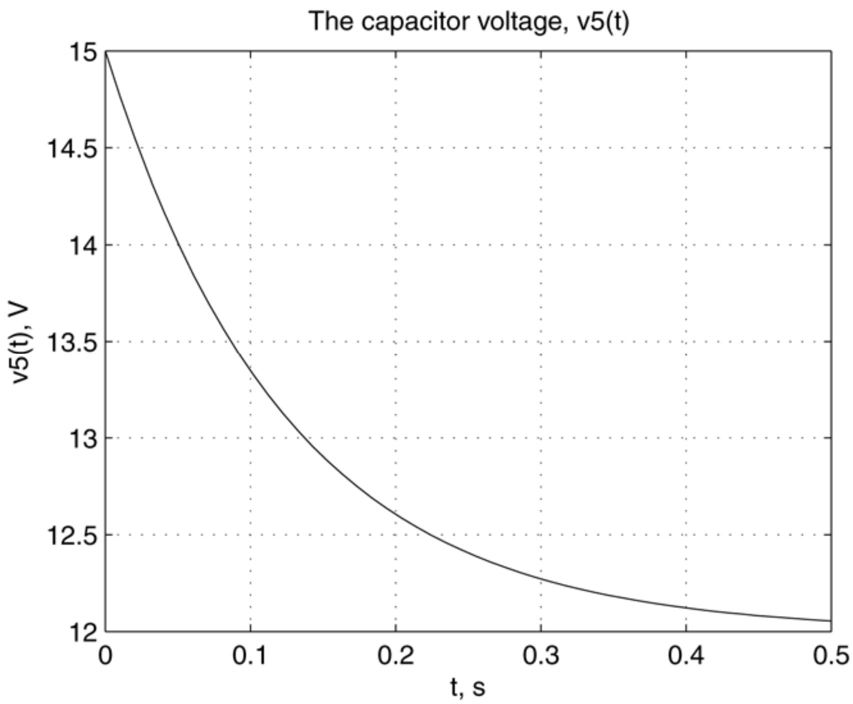
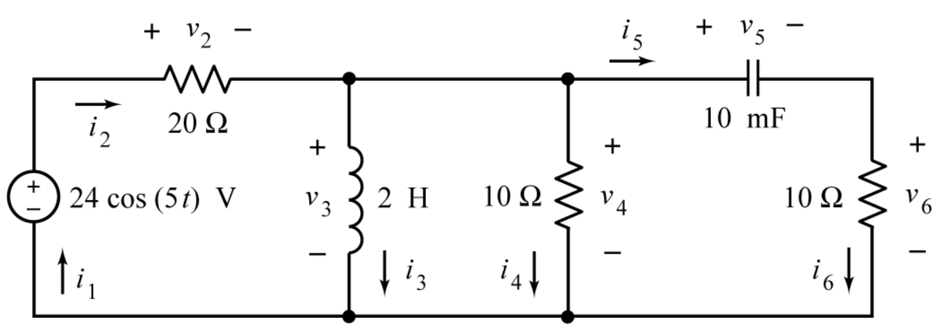


Figure 3 MATLAB plot of the voltage v_5 as a function of time.

2. Consider this circuit.



The voltages across the 10 Ω resistors are given to be

$$v_4(t) = 6.656 \cos(5t + 19.4°) \text{ V} \quad \text{and} \quad v_6(t) = 2.977 \cos(5t + 82.9°) \text{ V}$$

Determine the voltages $v_2(t)$ and $v_5(t)$ and the current $i_3(t)$.

Solution
From **KVL**

$$v_2(t)+v_4(t)-24\cos(5t)=0 \quad\Rightarrow\quad \begin{aligned} v_2(t) &= -6.656\cos(5t+19.4°)+24\cos(5t) \\ &= -\left(6.278\cos(5t)+2.211\sin(5t)\right)+24\cos(5t) \\ &= 17.722\cos(5t)-2.211\sin(5t) \\ &= 17.859\cos(5t-7.2°)\text{ V} \end{aligned}$$

Alternately, we can solve this problem using complex arithmetic instead of trigonometry. We use phasors to associate a complex number with each cosine:

$$A\cos(\omega t+\theta) \quad\leftrightarrow\quad A\angle\theta$$

Representing the cosines as complex numbers gives

$$-6.656\cos(5t+19.4°)+24\cos(5t)\leftrightarrow -6.656\angle 19.4°+24\angle 0°$$

Adding the complex numbers gives

$$-6.656\angle 19.4°+24\angle 0° = -(6.278+j\,2.211)+(24+j\,0)=17.722-j\,2.211=17.859\angle -7.2°$$

Next, $v_2(t)$ is the sinusoidal voltage corresponding to $17.859\angle -7.2°$, that is

$$v_2(t)=17.859\cos(5t-7.2°)\text{ V}$$

From **KVL**

$$v_5(t)+v_6(t)-v_4(t)=0 \quad\Rightarrow\quad \begin{aligned} v_5(t) &= -2.977\cos(5t+82.9°)+6.656\cos(5t+19.4°) \\ &= -\left(0.3680\cos(5t)+2.954\sin(5t)\right)+\left(6.278\cos(5t)+2.211\sin(5t)\right) \\ &= 5.910\cos(5t)-0.743\sin(5t) \\ &= 5.957\cos(5t-7.2°)\text{ V} \end{aligned}$$

Using **Ohm's law** we can determine the resistor currents:

$$i_4(t)=\frac{v_4(t)}{10}=\frac{6.656\cos(5t+19.4°)}{10}=0.6656\cos(5t+19.4°)\text{A}$$

$$i_6(t)=\frac{v_6(t)}{10}=\frac{2.977\cos(5t+82.9°)}{10}=0.2977\cos(5t+82.9°)\text{ A}$$

$$i_2(t) = \frac{v_2(t)}{20} = \frac{17.859\cos(5t - 7.2°)}{20} = 0.8930\cos(5t - 7.2°) \text{ A}$$

(Notice that the current and voltage of each resistor adhere to the passive convention.)

From **KCL**

$$i_2(t) = i_3(t) + i_4(t) + i_6(t) \quad \Rightarrow \quad i_3(t) = i_2(t) - (i_4(t) + i_6(t))$$

so

$$i_3(t) = 0.8930\cos(5t + 172.8°) - (0.6656\cos(5t + 19.4°) + 0.2977\cos(5t + 82.9°))$$

Adding the complex numbers gives

$$\begin{aligned} &0.8930\angle 7.2° - (0.6656\angle 19.4° + 0.2977\angle 82.9°) \\ &\quad = 0.8861 + j0.1105 - (0.6278 + j0.2211 + 0.0368 + j0.2954) \\ &\quad = 0.2215 - j0.6270 = 0.6650\angle -70.5° \end{aligned}$$

Finally, $i_3(t)$ is the sinusoidal voltage corresponding to $0.6650\angle -70.5°$, that is

$$i_3(t) = 0.6650\cos(5t - 70.5°) \text{ V}$$

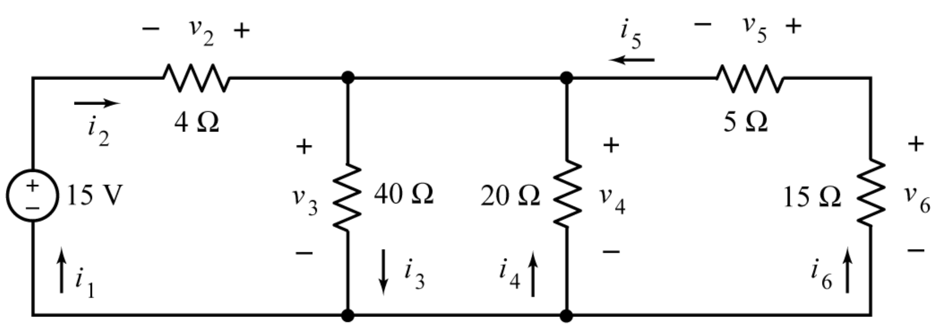
In summary, the required voltages and current are

$$v_2(t) = 17.859\cos(5t - 7.2°) \text{ V}, \quad v_5(t) = 5.957\cos(5t - 7.2°) \text{ V}$$

and

$$i_3(t) = 0.6650\cos(5t - 70.5°) \text{ V}$$

3. Consider this circuit.



The voltages across the 40 Ω and 15 Ω resistors are given to be

$$v_3(t) = 10 \text{ V} \quad \text{and} \quad v_6(t) = 7.5 \text{ V}$$

Determine the voltages $v_2(t)$ and $v_5(t)$ and the current $i_4(t)$.

Solution:
From **KVL**

$$-v_5(t)+v_6(t)-v_3(t)=0 \quad \Rightarrow \quad v_5(t)=v_6(t)-v_3(t)=7.5-10=-2.5 \text{ V}$$

and

$$-v_2(t)+v_3(t)-15=0 \quad \Rightarrow \quad v_2(t)=v_3(t)-15=10-15=-5 \text{ V}$$

From **Ohm's law**

$$i_3(t)=\frac{v_3(t)}{40}=\frac{10}{40}=0.25 \text{ A} \quad \text{and} \quad i_5(t)=\frac{v_5(t)}{5}=\frac{-2.5}{5}=-0.5 \text{ A}$$

(Notice that the current and voltage of the 40 Ω and 5 Ω resistors adhere to the passive convention.) Also,

$$i_2(t)=-\frac{v_2(t)}{4}=-\frac{-5}{4}=1.25 \text{ A}$$

(Notice that the current and voltage of the 4 Ω resistor **does not** adhere to the passive convention.)

Finally, from **KCL**

$$i_2(t)+i_4(t)+i_5(t)=i_3(t) \quad \Rightarrow \quad i_4(t)=i_3(t)-\left(i_2(t)+i_5(t)\right)=0.25-\left(1.25+(-0.5)\right)=-0.5 \text{ A}$$