# 1. Kirchhoff's Laws

## Introduction

The circuits in this problem set are comprised of unspecified circuit elements. (We don't know if a particular circuit element is a resistor or a voltage source or something else.) The current and voltage of each circuit element is labeled, sometimes as a value and sometimes as a variable. Some of these problems ask that we determine the value of a particular voltage or current. Other problems ask for the values of the power supplied or received by a particular circuit element. Kirchhoff's laws are used to determine values of currents or voltages. The passive convention is used to decide if the product of a particular element current and voltage is the power supplied or received by the circuit element.

The passive convention is discussed in Section 1.5 of *Introduction to Electric Circuits* (7e) by R. C. Dorf and J. A. Svoboda and summarized in Table 1.5-1. Kirchhoff's laws are discussed in Section 3.2 of *Introduction to Electric Circuits*.

## Worked Examples

**Example 1:**

Consider the circuit shown in Figure 1. Determine the power supplied by element *D* and the power received by element *F*.

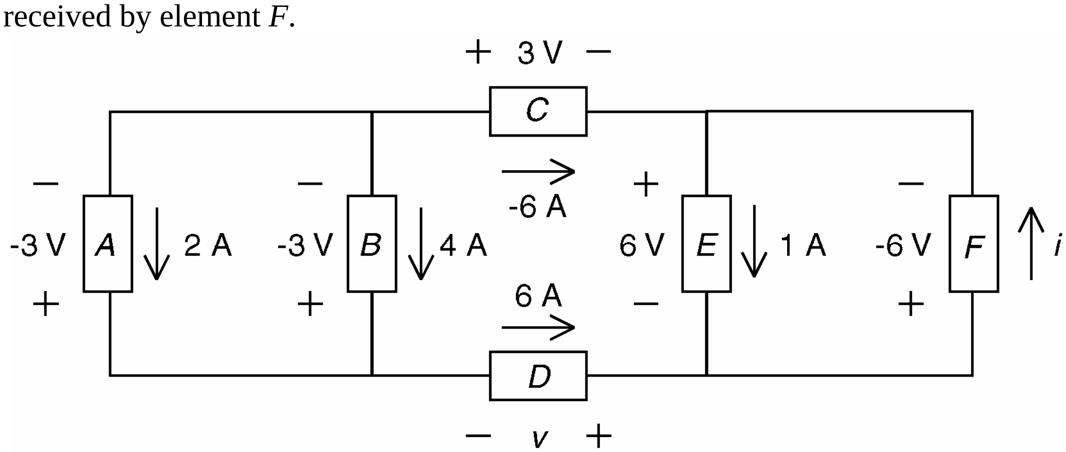


**Figure 1**. The circuit considered in Example 1

**Solution:** Figure 1 provides a value for the current in element *D* but not for the voltage, *v*, across element *D*. The voltage and current of element *D* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *D*. Similarly, Figure 1 provides a value for the voltage across element *F* but not for the current, *i*, in element *F*. The voltage and current of element *F* given in Figure 1 do adhere to the passive convention so the product of this voltage and current is the power *received* by element *F*.

We need to determine the voltage, *v*, across element *D* and the current, *i*, in element *F*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 2.

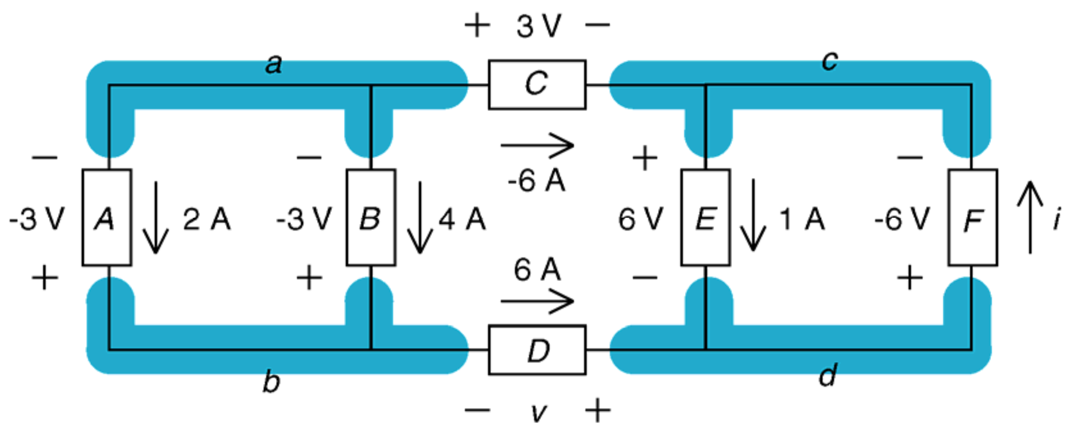


**Figure 2.** Labeling the nodes of the circuit from Figure 1.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *E*, *D* and *B* to get

$$3 + 6 + v + (-3) = 0 \quad \Rightarrow \quad v = -6 \text{ V}$$

The value of the current in element *D* in Figure 2 is 6 A. The voltage and current of element *D* given in Figure 2 do not adhere to the passive convention so

$$p_D = v\,(6) = (-6)\,(6) = -36 \text{ W}$$

is the power *supplied* by element *D*. (Equivalently, we could say that element *D receives* 36 W.)

Next, apply Kirchhoff's current law (KCL) at node *c* to get

$$-6 + i = 1 \quad \Rightarrow \quad i = 7 \text{ A}$$

The value of the voltage across element *F* in Figure 2 is -6 V. The voltage and current of element *F* given in Figure 2 adhere to the passive convention so

$$p_F = (-6)\,i = (-6)\,(7) = -42 \text{ W}$$

is the power *received* by element *F*. (Equivalently, we could say that element *F supplies* 42 W.)

**Example 2:**
Consider the circuit shown in Figure 3. Determine the power supplied by element *B* and the power supplied by element *C*.

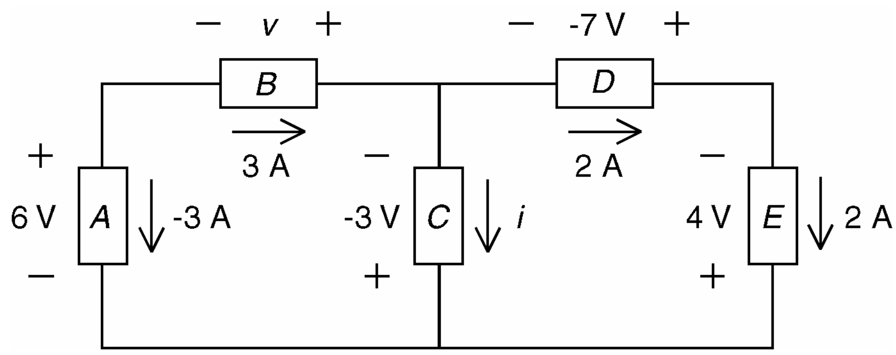


**Figure 3**. The circuit considered in Example 2

**Solution:** Figure 3 provides a value for the current in element *B* but not for the voltage, *v*, across element *B*. The voltage and current of element *B* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *B*. Similarly, Figure 3 provides a value for the voltage across element *C* but not for the current, *i*, in element *C*. The voltage and current of element *C* given in Figure 1 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *C*.

We need to determine the voltage, *v*, across element *B* and the current, *i*, in element *C*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 4.

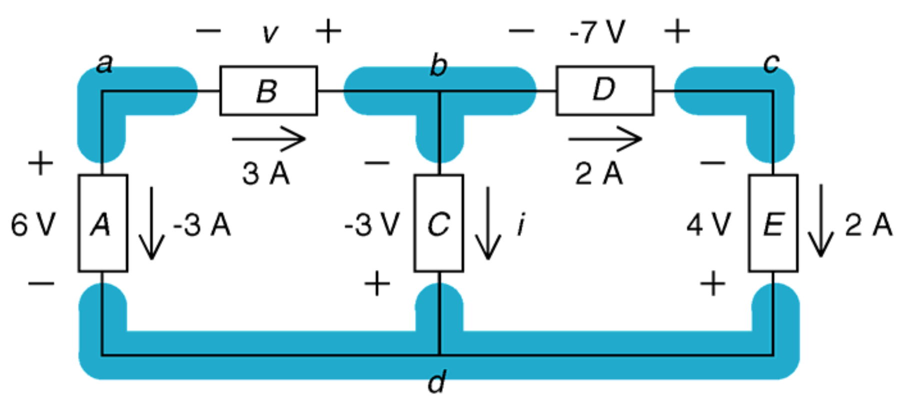


**Figure 4.** Labeling the nodes of the circuit from Figure 3.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *B*, *C* and *A* to get

$$-v - (-3) - 6 = 0 \qquad \Rightarrow \qquad v = -3 \text{ V}$$

The value of the current in element *B* in Figure 4 is 3 A. The voltage and current of element *B* given in Figure 4 do not adhere to the passive convention so

$$p_B = v\,(3) = (-3)\,(3) = -9 \text{ W}$$

is the power *supplied* by element *B*. (Equivalently, we could say that element *B receives* 9 W.)

Next, apply Kirchhoff's current law (KCL) at node *b* to get

$$2 + i = 3 \qquad \Rightarrow \qquad i = 1 \text{ A}$$

The value of the voltage across element *C* in Figure 4 is -3 V. The voltage and current of element *C* given in Figure 4 do not adhere to the passive convention so

$$p_C = (-3)\,i = (-3)\,(1) = -3 \text{ W}$$

is the power *supplied* by element *C*. (Equivalently, we could say that element *C receives* 3 W.)

**Example 3:**

Consider the circuit shown in Figure 5. Determine the power supplied by element *C* and the power received by element *D*.

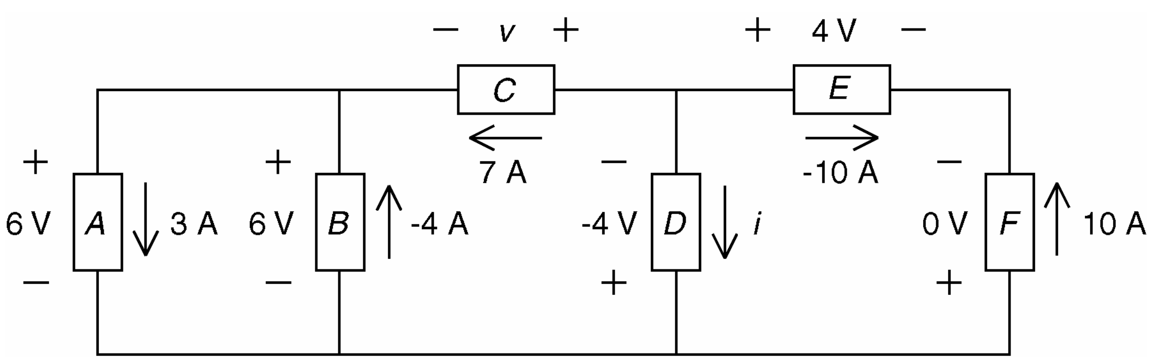


**Figure 5**. The circuit considered in Example 3

**Solution:** Figure 5 provides a value for the current in element *C* but not for the voltage, *v*, across element *C*. The voltage and current of element *C* given in Figure 5 adhere to the passive convention so the product of this voltage and current is the power *received* by element *C*. Similarly, Figure 5 provides a value for the voltage across element *D* but not for the current, *i*, in element *D*. The voltage and current of element *D* given in Figure 5 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage, *v*, across element *C* and the current, *i*, in element *D*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 6.

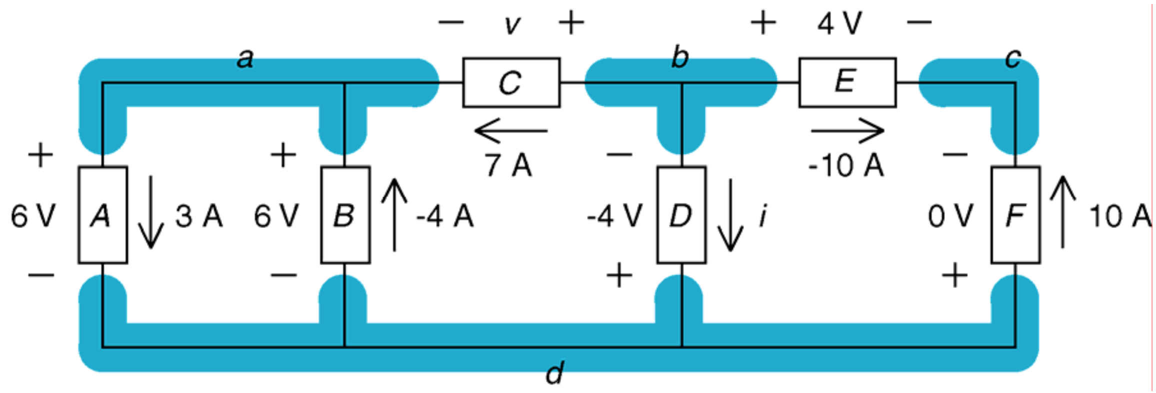


**Figure 6.** Labeling the nodes of the circuit from Figure 5.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C*, *D* and *B* to get

$$-v-(-4)-6=0 \quad \Rightarrow \quad v=-2\text{ V}$$

The value of the current in element *C* in Figure 6 is 7 A. The voltage and current of element *C* given in Figure 6 adhere to the passive convention so

$$p_D = v\,(7) = (-2)\,(7) = -14\text{ W}$$

is the power *received* by element *C*. Therefore element *C supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node *b* to get

$$7+(-10)+i=0 \quad \Rightarrow \quad i=3\text{ A}$$

The value of the voltage across element *D* in Figure 6 is -4 V. The voltage and current of element *D* given in Figure 6 do not adhere to the passive convention so the power *supplied* by element *F* is given by

$$p_F = (-4)\,i = (-4)\,(3) = -12\text{ W}$$

Therefore, element *D receives* 12 W.

**Example 4:**

Consider the circuit shown in Figure 7. Determine the power supplied by element *B* and the power received by element *F*.

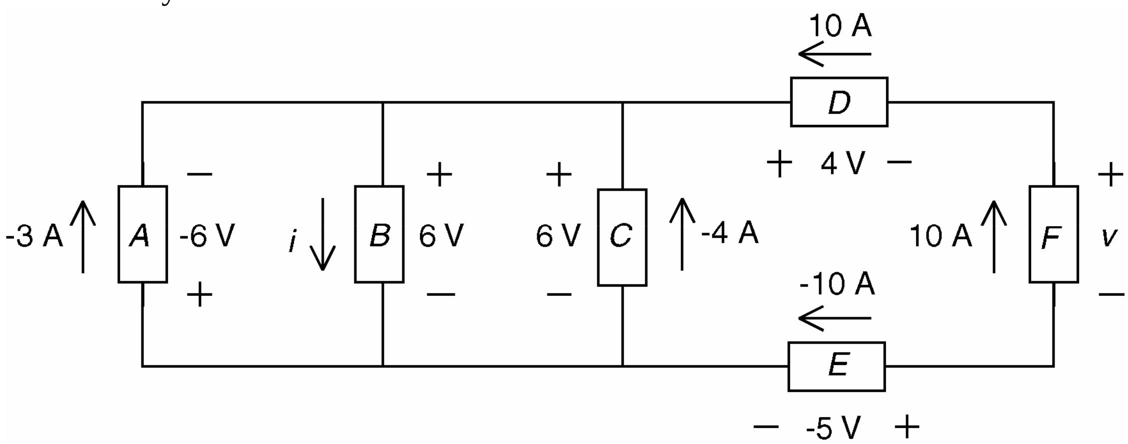


**Figure 7**. The circuit considered in Example 4

**Solution:** Figure 7 provides a value for the voltage across element *B* but not for the current, *i*, in element *B*. The voltage and current of element *B* given in Figure 7 adhere to the passive convention so the product of this voltage and current is the power *received* by element *B*. Similarly, Figure 7 provides a value for the current in element *F* but not for the voltage, *v*, across element *F*. The voltage and current of element *F* given in Figure 7 do not adhere to the passive convention so the product of this voltage and current is the power *supplied* by element *F*.

We need to determine the current, *i*, in element *B* and the voltage, *v*, across element *F*. We will use Kirchhoff's laws to determine values of *i* and *v*. First, we identify and label the nodes of the circuit as shown in Figure 8.

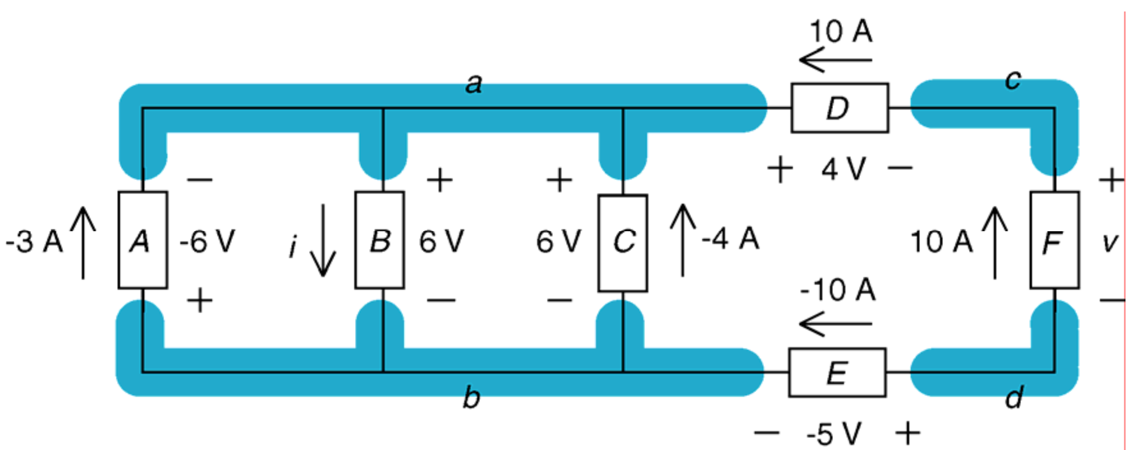


**Figure 8.** Labeling the nodes of the circuit from Figure 7.

Apply Kirchhoff's current law (KCL) at node *a* to get

$$i = -3 + (-4) + 10 \quad \Rightarrow \quad i = 3 \text{ A}$$

The value of the voltage across element *B* in Figure 8 is 6 V. The voltage and current of element *B* given in Figure 8 adhere to the passive convention so

$$p_B = (6)\, i = (6)\,(3) = 18 \text{ W}$$

is the power *received* by element *B*. Therefore element *B supplies* -18 W.

Next, apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *D*, *F*, *E* and *C* to get

$$4 + v + (-5) - (6) = 0 \quad \Rightarrow \quad v = 7 \text{ V}$$

The value of the current in element *F* in Figure 8 is 10 A. The voltage and current of element *F* given in Figure 8 do not adhere to the passive convention so

$$p_D = v\,(10) = (7)\,(10) = 70 \text{ W}$$

is the power *supplied* by element *F*. Therefore element *F receives* –70 W.

**Example 5:**
Consider the circuit shown in Figure 9. Determine the values of the currents in and voltages across the various circuit elements.

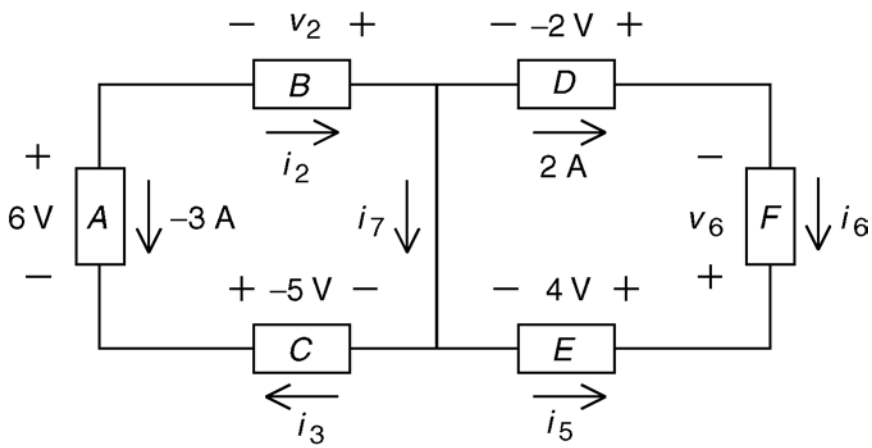


**Figure 9**. The circuit considered in Example 5

**Solution:** First, we identify and label the nodes of the circuit as shown in Figure 10.

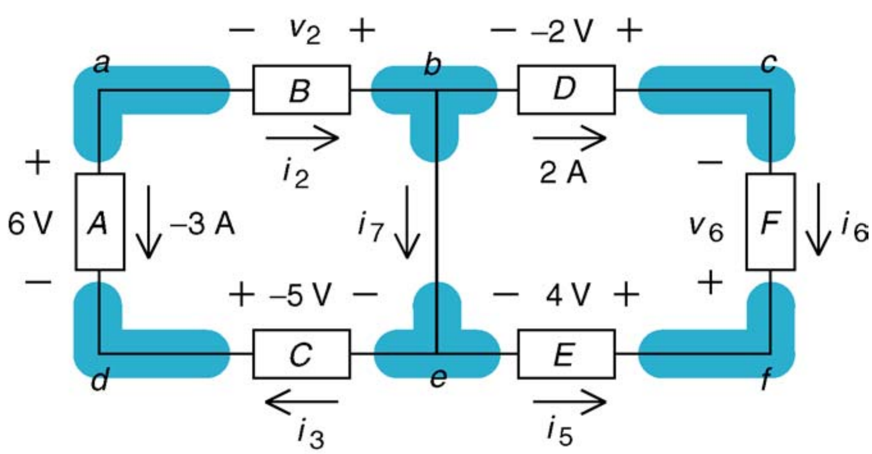


**Figure 10.** Labeling the nodes of the circuit from Figure 9.

Apply Kirchhoff's current law (KCL) at node *a* to get

$$i_2 + (-3) = 0 \quad \Rightarrow \quad i_2 = 3 \text{ A}$$

Apply KCL at node *d* to get

$$i_3 + (-3) = 0 \quad \Rightarrow \quad i_3 = 3 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *C* to get

$$-(v_2) - (-5) - 6 = 0 \quad \Rightarrow \quad v_2 = 1 \text{ V}$$

Apply KCL at node *c* to get

$$i_6 = 2 \text{ A}$$

Apply KCL at node *f* to get

$$i_5 + i_6 = 0 \quad \Rightarrow \quad i_5 = -i_6 = -2 \text{ A}$$

Apply KVL to the loop consisting of elements *D*, *E* and *F* to get

$$-(-2) - (v_6) + 4 = 0 \quad \Rightarrow \quad v_6 = 6 \text{ V}$$

Finally, apply KCL at node *b* to get

$$i_2 = i_7 + 2 \quad \Rightarrow \quad i_7 = 2 - i_2 = 2 - 3 = -1 \text{ A}$$

**Example 6:**
Consider the circuit shown in Figure 12. Determine the values of the currents in and voltages across the various circuit elements.

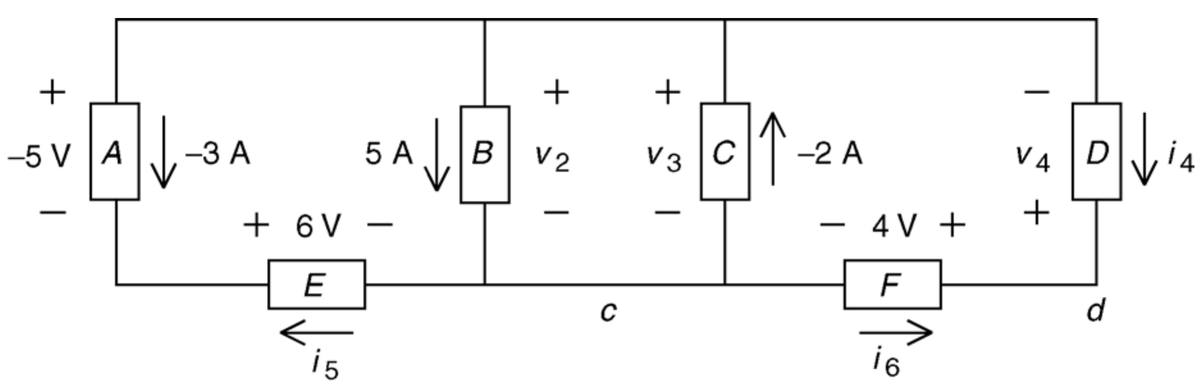


**Figure 12**. The circuit considered in Example 6

**Solution:** First, we identify and label the nodes of the circuit as shown in Figure 13.

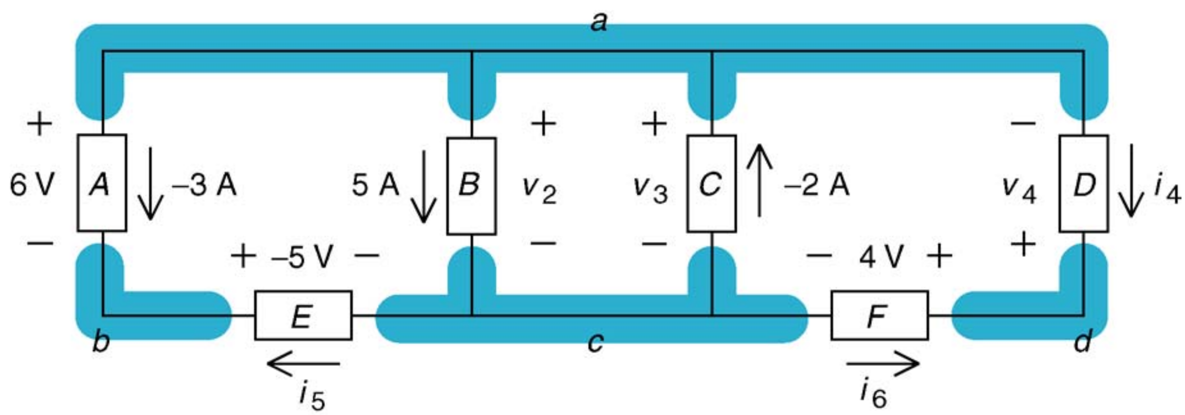


**Figure 13.** Labeling the nodes of the circuit from Figure 12.

Apply Kirchhoff's current law (KCL) at node *b* to get

$$i_5 + (-3) = 0 \quad \Rightarrow \quad i_5 = 3 \text{ A}$$

Apply KCL at node *a* to get

$$-2 = -3 + 5 + i_4 \quad \Rightarrow \quad i_4 = -4 \text{ A}$$

Apply KCL at node *d* to get

$$i_4 + i_6 = 0 \quad \Rightarrow \quad i_6 = -i_4 = -(-4) = 4 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *E* to get

$$v_2 - (-5) - 6 = 0 \quad \Rightarrow \quad v_2 = 1 \text{ V}$$

Apply KVL to the loop consisting of elements *B* and *C* to get

$$v_3 - v_2 = 0 \quad \Rightarrow \quad v_3 = v_2 = 1 \text{ V}$$

Finally, apply KVL to the loop consisting of elements *C*, *D* and *F* to get

$$-v_4 + 4 - v_3 = 0 \quad \Rightarrow \quad v_4 = 4 - v_2 = 4 - 1 = 3 \text{ V}$$

**Example 7:**
Consider the circuit shown in Figure 14. Determine the values of the currents in and voltages across the various circuit elements.

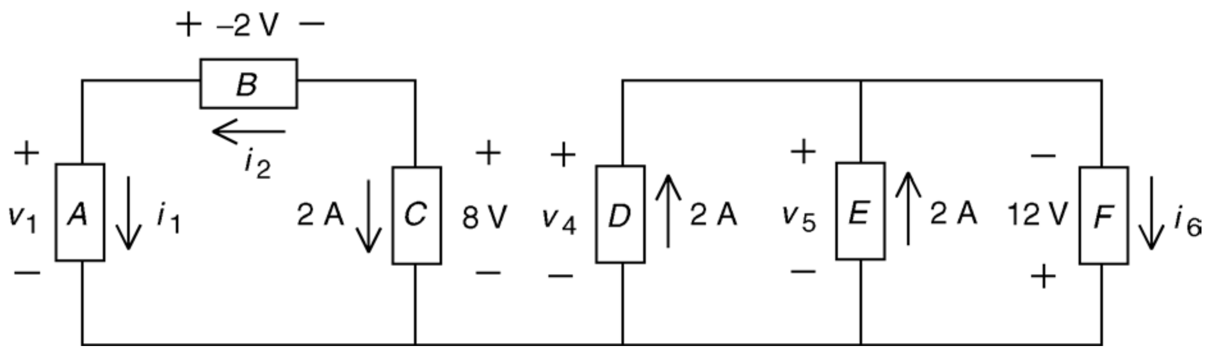


**Figure 14**. The circuit considered in Example 7

**Solution:** First, we identify and label the nodes of the circuit as shown in Figure 15.

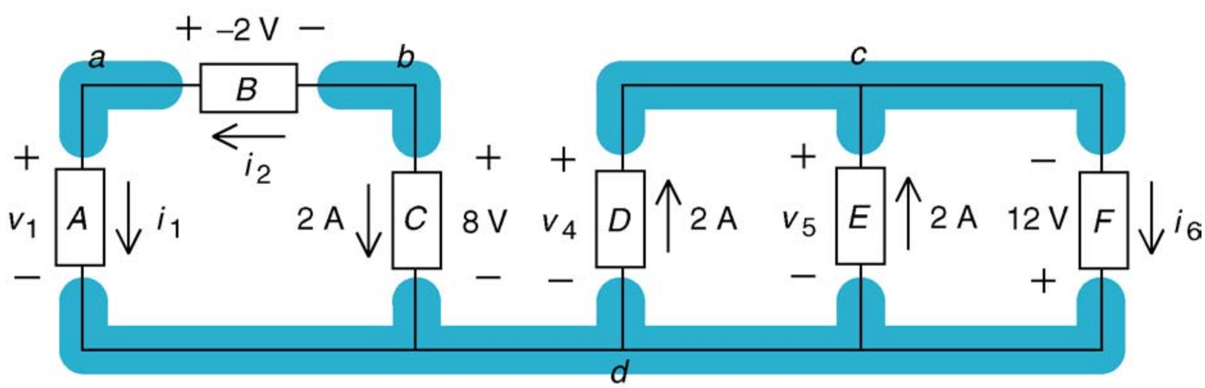


**Figure 15.** Labeling the nodes of the circuit from Figure 14.

Apply Kirchhoff's current law (KCL) at node *b* to get

$$i_2 + 2 = 0 \quad \Rightarrow \quad i_2 = -2 \text{ A}$$

Apply KCL at node *a* to get

$$i_1 = i_2 = -2 \text{ A}$$

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *A*, *B* and *C* to get

$$-2+8-v_1=0 \quad \Rightarrow \quad v_1=6 \text{ V}$$

Apply KCL at node *c* to get

$$i_6=2+2=4 \text{ A}$$

Apply KVL to the loop consisting of elements *E* and *F* to get

$$-12-v_5=0 \quad \Rightarrow \quad v_5=-12 \text{ V}$$

Finally, apply KVL to the loop consisting of elements *D* and *E* to get

$$v_5-v_4=0 \quad \Rightarrow \quad v_4=v_5=-12 \text{ V}$$

**Example 8:**
Verify that power is conserved in the circuit shown in Figure 14.

**Solution:** The values of the currents in and voltages across the various circuit elements were determined in Example 7. Let's summarize what we know in the following table.

| Element | Current, A | Voltage, V | Adhere to passive convention? | Power received, W |
|---|---|---|---|---|
| A | -2 | 6 | Yes | -12 |
| B | -2 | -2 | No | -4 |
| C | 2 | 8 | Yes | 16 |
| D | 2 | -12 | No | 24 |
| E | 2 | -12 | No | 24 |
| F | 4 | 12 | No | -48 |

The sum of the power received by all of the elements in the circuit is zero so power is conserved.