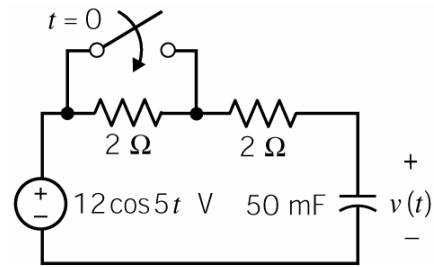


Example: Before time $t = 0$, this circuit is at steady state so all its voltages and currents are sinusoidal with $\omega = 5$ rad/s. At time $t = 0$, the switch closes disturbing the circuit. Immediately after $t = 0$, the currents and voltages are not sinusoidal. Eventually, the disturbance dies out and the circuit is again at steady state (most likely a different steady state). Once again the currents and voltages are all sinusoidal with $\omega = 5$ rad/s.

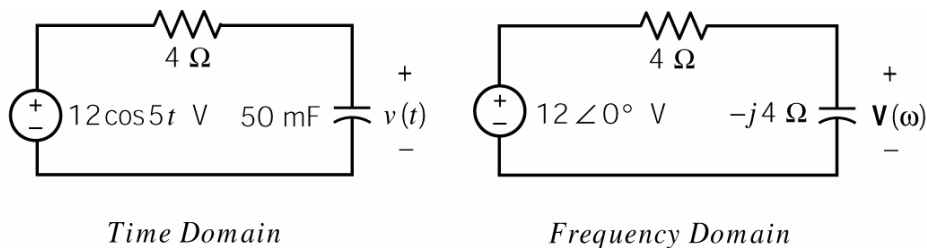


Determine the capacitor voltage, $v(t)$, both before and after the switch closes.

Solution:

Step 1: For $t < 0$, the switch is open and the circuit is at steady state.

The steady state response of the circuit can be obtained by representing the circuit in the frequency domain using impedances and phasors.



Using voltage division in the frequency domain gives

$$\mathbf{V}(\omega) = \left(\frac{-j4}{4 - j4} \right) (12 \angle 0^\circ) = \frac{48 \angle -90^\circ}{5.66 \angle -45^\circ} = 8.485 \angle -45^\circ \text{ V}$$

In the time domain

$$v(t) = 8.485 \cos(5t - 45^\circ) \text{ V}$$

Immediately before the switch closes, the capacitor voltage is

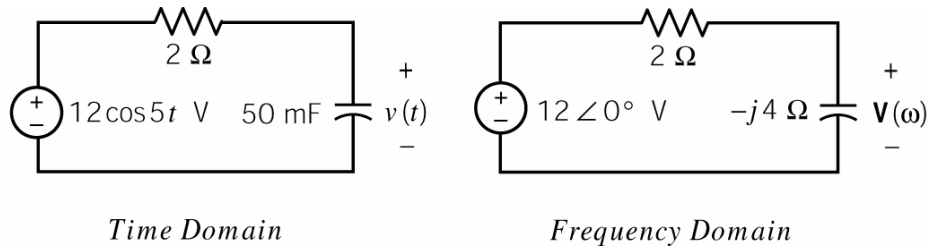
$$v(0^-) = \lim_{t \rightarrow 0^-} v(t) = 8.485 \cos(0 - 45^\circ) = 6 \text{ V}$$

The capacitor voltage is continuous, so the capacitor voltage immediately after the switch closes is the same as immediately before the switch closes. That is

$$v(0^+) = v(0^-) = 6 \text{ V}$$

Step 2: For $t > 0$, the switch is closed. Eventually, the circuit will reach a new steady state.

The steady state response of the circuit can be obtained by representing the circuit in the frequency domain using impedances and phasors.



Using voltage division in the frequency domain gives

$$\mathbf{V}(\omega) = \left(\frac{-j4}{2 - j4} \right) (12 \angle 0^\circ) = \frac{48 \angle -90^\circ}{4.47 \angle -63.4^\circ} = 10.74 \angle -26.6^\circ \text{ V}$$

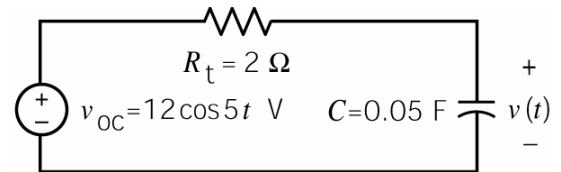
In the time domain

$$v(t) = 10.74 \cos(5t - 26.6^\circ) \text{ V}$$

Step 3: Immediately after $t = 0$, the switch is closed but the circuit is not at steady state. We must find the complete response of a first order circuit.

The time constant of the circuit is

$$\tau = R_t C = 2 \times 0.05 = 0.01 \text{ 1/s}$$



The natural response of the circuit is

$$v_n(t) = K e^{-10t}$$

The steady state response for $t > 0$ can be used as the forced response so

$$v_f(t) = 10.74 \cos(5t - 26.6^\circ) \text{ V}$$

The complete response is

$$v(t) = v_n(t) + v_f(t) = K e^{-10t} + 10.74 \cos(5t - 26.6^\circ)$$

The constant, K , is evaluated using the initial capacitor voltage, $v(0+)$,

$$6 = v(0+) = K e^{-0} + 10.74 \cos(0 - 26.6^\circ) = K + 9.6$$

Thus $K = -3.6$ and

$$v(t) = -3.6 e^{-10t} + 10.74 \cos(5t - 26.6^\circ) \text{ V}$$

Step 4: Summary the results:

The capacitor voltage is

$$v(t) = \begin{cases} 8.485 \cos(5t - 45^\circ) \text{ V} & \text{for } t \leq 0 \\ -3.6 e^{-10t} + 10.74 \cos(5t - 26.6^\circ) \text{ V} & \text{for } t \geq 0 \end{cases}$$

Here is a plot showing the capacitor voltage as a function of time:

Complete Response of a Switched Circuit with Sinusoidal Input

