What is a Thevenin Equivalent Circuit?

Figure 1 shows a circuit that has been separated into two parts, Circuit A and Circuit B. These parts are connected at a single pair of terminals.

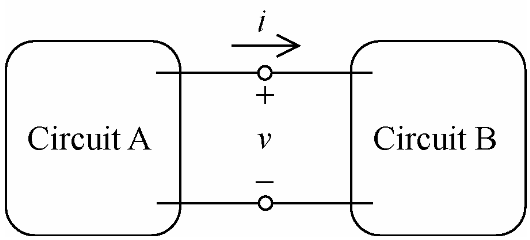

Figure 1. Two circuits connected at a pair of terminals.

We'd like to understand the interaction between these two circuits. Circuit A and Circuit B share two things: the current, *i*, and the voltage, *v*. It seems reasonable to ask

1. Are *i* and *v* related to each other? If so, how?
2. Does Circuit B affect *i* and *v*? How?
3. Suppose we knew *i* and *v*. What would they tell us about Circuit A?

To start our investigation, let's simply the problem by considering the case where Circuit B is a single resistor:

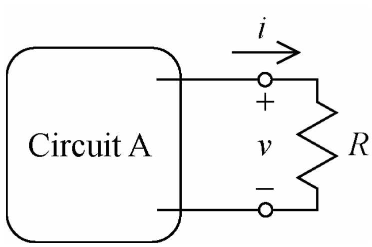

Figure 2. Circuit B is a resistor.

Let's go to the laboratory and take some data:

R, Ω	1	2	3	4	5	6	8	10	15
i, A	0.476	0.455	0.435	0.417	0.400	0.385	0.357	0.333	0.286
v, V	0.476	0.909	1.304	1.667	2.000	2.308	2.857	3.333	4.286

R, Ω	20	25	30	40	50	100			
i, A	0.250	0.222	0.200	0.167	0.143	0.083			
v, V	5.000	5.556	6.000	6.667	7.143	8.333			

(This data was obtained by computer simulation using PSpice [] rather than laboratory measurements. The circuit labeled "Circuit A2" in Figure 10 was used as Circuit A.)

Maybe a picture will help us to see a pattern in this data. Figure 3 shows a graph of v as a function of i.

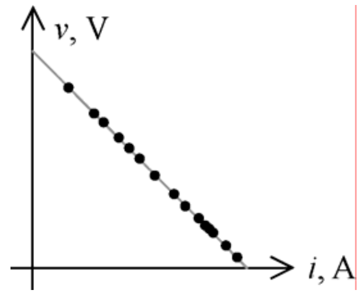

Figure 3. A graph of the data.

The data points all lie on a segment of a straight line. That line segment connects two data points that we did not measure, the points on the axes. The point on the i axis corresponds to $v = 0$. To cause $v = 0$, we set $R = 0$ in Figure 2. Similarly, the point on the v axis corresponds to $i = 0$ and $R = \infty$.

$$i_{sc} = \text{the current when } R = 0 \quad \text{(a short circuit)}$$

and

$$v_{oc} = \text{the voltage when } R = \infty \quad \text{(an open circuit)}$$

We go back into the lab and measure i_{sc} and v_{oc}. Figure 4 illustrates the procedure for making those measurements. For this particular choice of Circuit A, we find $v_{oc} = 10 \text{ V}$ and $i_{sc} = 0.5 \text{ A}$. Figure 5 shows the graph of v versus i, updated to show the new data.

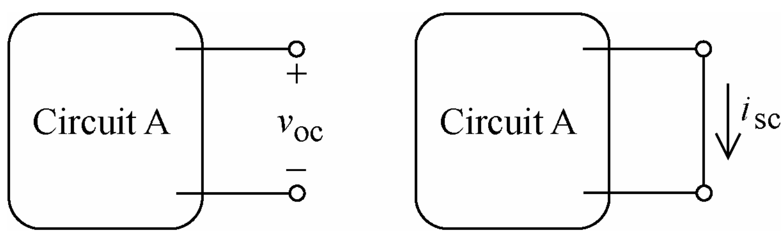

Figure 4. The laboratory procedure for measuring i_{sc} and v_{oc}.

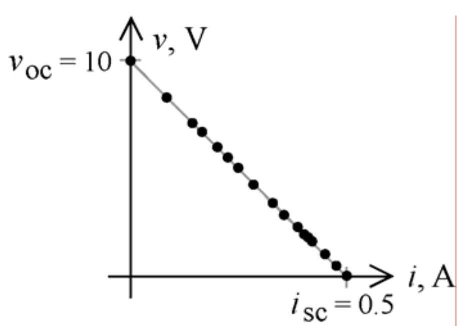

Figure 5. The graph of the data after measuring i_{sc} and v_{oc}.

The equation of the straight line in Figure 5is

$$v = m\,i + b$$

where

$$b = v_{oc} \quad \text{and} \quad m = -\frac{v_{oc}}{i_{sc}}$$

That is

$$v = \left(-\frac{v_{oc}}{i_{sc}}\right) i + v_{oc} = (-20)\,i + 10$$

Of course we can calculate the slope and intercept, hence i_{sc} and v_{oc}, from the coordinates of any two points on the line. For example, v = 2 V when i = 0.4 A and v = 5 V when i = 0.25 A, so

$$m = \frac{2-5}{0.4-0.25} = \frac{-3}{0.15} = -20$$

Thus

$$v = -20\,i + b$$

Next, using v = 2 V when i = 0.4 A

$$2 = (-20)\,0.4 + b \quad \Rightarrow \quad b = 10$$

As before,

$$v = -20\,i + 10$$

To summarize:

1. The plot of v versus i in Figure 1 is a straight line. Of course, the plot of v versus i in Ohm's law is also a straight line. Figure 6 shows these two straight lines when R = 6 Ω.

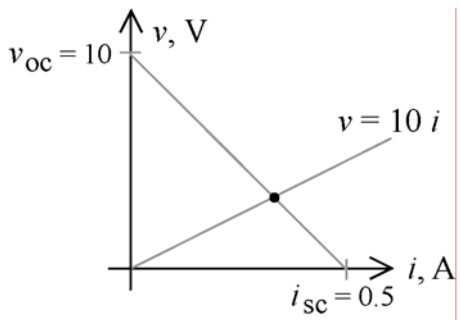

Figure 6. Another plot.

2. The intercepts of that straight line are i_{sc} and v_{oc}, the short circuit current and the open circuit voltage.
3. We can calculate i_{sc} and v_{oc} in either of two ways

a. Replace Circuit B with a resistor. Measure *v* and *i*. Change the resistance of the resistor. Measure *v* and *i* again. Do some arithmetic.
b. Measure i_{sc} and v_{oc} as shown in Figure 4.

Replacing Circuit B by a single resistor worked quite well. Let's see try something different. Consider replacing Circuit A in Figure 1 with a simple circuit as shown in Figure 7.

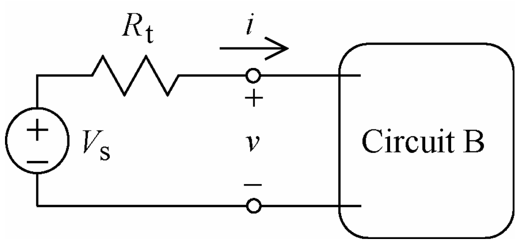

Figure 7.

Kirchhoff's voltage law gives

$$R_t\, i + v - V_s = 0 \quad \Rightarrow \quad v = -R_t\, i + V_s$$

This is the equation of the straight line shown in Figure 8.

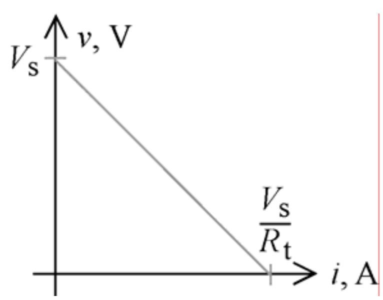

Figure 8. KVL for the circuit in Figure 7.

The plot in Figure 7 looks like the plot shown in Figure 3 with $v_{oc} = V_s$ and $i_{sc} = \frac{V_s}{R_t}$. Figure 9 shows …

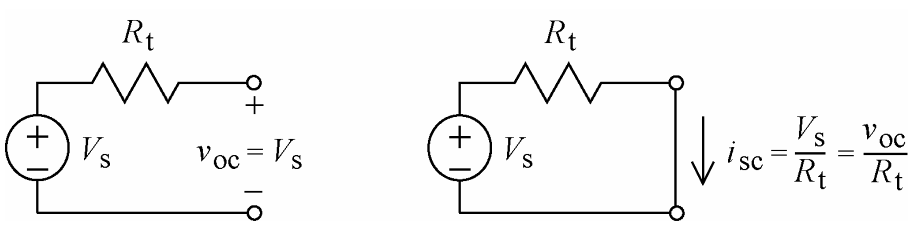

Figure 9.

The only characteristics of Circuit A that are important to Circuit B are i_{sc} and v_{oc}. For example, Circuits A1 and A2 shown in Figure 10 both have $i_{sc} = 0.6$ A and $v_{oc} = 12$ V so Circuit B would not be able to distinguish between these circuits.

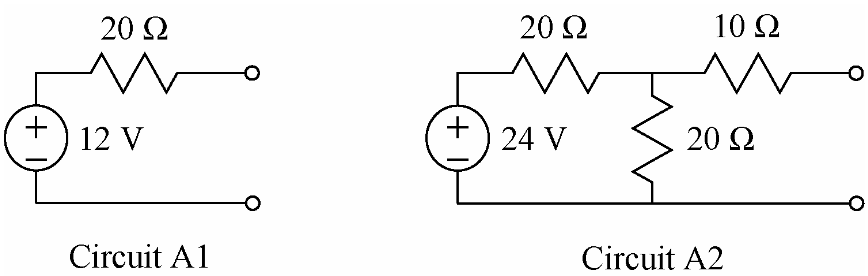

Figure 10. Two circuit with the same values of i_{sc} and v_{oc}.