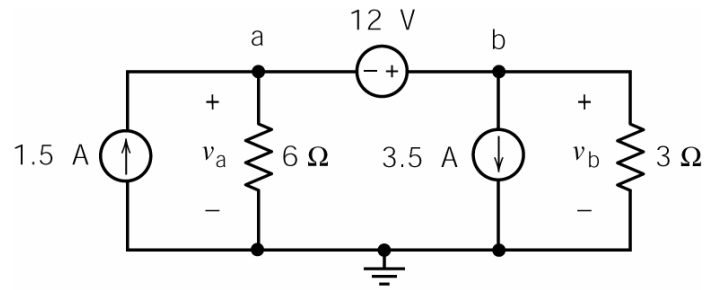


### Example



Determine the values of the node voltages,  $v_a$  and  $v_b$ , for this circuit.

### Solution

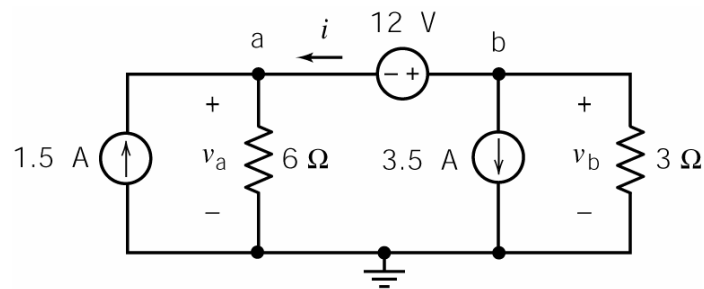
We can write one node equation by considering the voltage source. The voltage source voltage is related to the node voltages at by

$$v_b - v_a = 12 \Rightarrow v_b = v_a + 12$$

In order to write the second node equation, we must decide what to do about the voltage source current. (Notice that there is no easy way to express the voltage source current in terms of the node voltages.) In this example, illustrate two methods of writing the second node equation.

**Method 1:** Assign a name to the voltage source current. Apply KCL at both of the voltage source nodes. Eliminate the voltage source current from the KCL equations.

Here's the circuit after labeling the voltage source current.



The KCL equation at node a is

$$1.5 + i = \frac{v_a}{6}$$

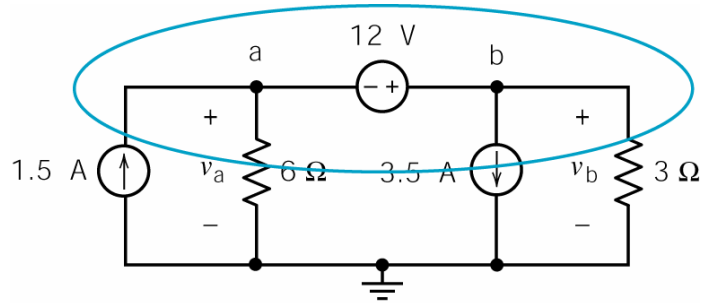
The KCL equation at node b is

$$i + 3.5 + \frac{v_b}{3} = 0$$

Combining these two equations gives

$$1.5 - \left( 3.5 + \frac{v_b}{3} \right) = \frac{v_a}{6} \Rightarrow -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

**Method 2:** Apply KCL to the supernode corresponding to the voltage source. Shown below in blue, this super node separates the voltage source and its nodes from the rest of the circuit. (In this small circuit, the rest of the circuit is just the reference node.)



Apply KCL to the supernode to get

$$1.5 = \frac{v_a}{6} + 3.5 + \frac{v_b}{3} \Rightarrow -2.0 = \frac{v_a}{6} + \frac{v_b}{3}$$

This is the same equation that was obtained using method 1. Applying KCL to the supernode is a shortcut for doing three things:

1. labeling the voltage source current as  $i$
2. applying KCL at both nodes of the voltage source
3. eliminating  $i$  from the KCL equations

In summary, the node equations are

$$v_b - v_a = 12$$

and

$$\frac{v_a}{6} + \frac{v_b}{3} = -2.0$$

Solving the node equations gives

$$v_a = -12 \text{ V} \quad \text{and} \quad v_b = 0 \text{ V}$$

(We might be surprised that  $v_b$  is 0 V but it is easy to check that these values are correct by substituting them into the node equations.)