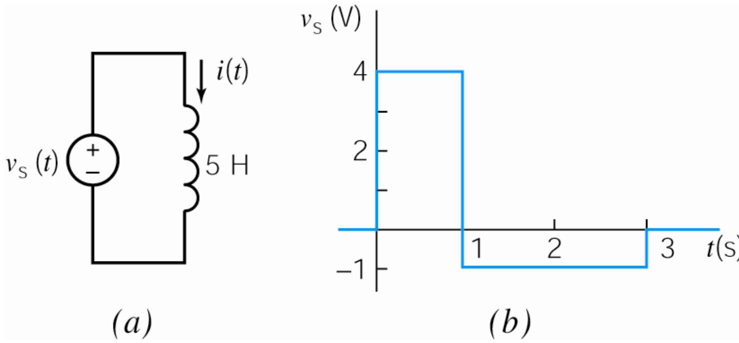
Example:



Determine $i(t)$ for $t \geq 0$ for the circuit in (a) when $i(0) = -2$ A and $v_s(t)$ is the voltage in (b).

Solution:

$$i(t) = i(0) + \frac{1}{L}\int_0^t v_s(\tau)\, d\tau = -2 + \frac{1}{5}\int_0^t v_s(\tau)\, d\tau$$

For $0 \leq t \leq 1$ s

$$i(t) = -2 + \frac{1}{5}\int_0^t 4\, d\tau = -2 + \frac{4}{5}\tau\int_0^t \;= -2 + \frac{4}{5}t$$

For example $i(0) = -2,\ i(1) = -\frac{6}{5}$ and $i\left(\frac{1}{2}\right) = -\frac{8}{5}$

For $1 \leq t \leq 3$ s

$$i(t) = -2 + \frac{1}{5}\int_0^1 4\, dt + \frac{1}{5}\int_1^t -1\, d\tau = -2 + \frac{4}{5} - \frac{1}{5}\tau\Big|_0^t = -\frac{6}{5} - \frac{1}{5}(t-1) = -1 - \frac{t}{5}$$

For example $i(1) = -\frac{6}{5},\ i(2) = -\frac{7}{5},\ i(3) = -\frac{8}{5}$

Notice that $$i(t) = i(1) + \frac{1}{5}\int_1^t -1\, d\tau = -\frac{6}{5} - \frac{1}{5}(t-1) = -1 - \frac{t}{5}$$

For $3 \leq t$

$$i(t) = i(0) + \frac{1}{5}\int_0^1 4\, d\tau + \frac{1}{5}\int_1^3 -1\, d\tau + \frac{1}{5}\int_3^t 0\, d\tau = -\frac{8}{5}$$

In summary

$$i(t) = \begin{cases} -2 + 0.8t & 0 \leq t \leq 1 \\ -1 - 0.2t & 1 \leq t \leq 3 \\ -1.6 & 3 \leq t \end{cases}$$

Example:
The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the capacitance when the voltage is $v(t) = 12\cos(500t - 45°)$ V and the current is $i(t) = 3\cos(500t + 45°)$ mA.

Solution:

$$\left(3\times10^{-3}\right)\cos\left(500t+45^{\circ}\right) = C\frac{d}{dt}\,12\cos\left(500t-45^{\circ}\right) = C\left(12\right)\left(-500\right)\sin\left(500t-45^{\circ}\right)$$

$$= C\left(6000\right)\cos\left(500t+45^{\circ}\right)$$

so
$$C = \frac{3\times10^{-3}}{6\times10^{3}} = \frac{1}{2}\times10^{-6} = \frac{1}{2}\ \mu\text{F}$$

Example:
The voltage across a 40-μF capacitor is 25 V at $t_0 = 0$. If the current through the capacitor as a function of time is given by $i(t) = 6e^{-6t}$ mA for $t < 0$, find $v(t)$ for $t > 0$.

Solution:

$$v\left(t\right) = v\left(0\right) + \frac{1}{C}\int_0^t i\left(\tau\right)d\tau = 25 + 2.5\times10^{4}\int_0^t\left(6\times10^{-3}\right)e^{-6\tau}\,d\tau$$

$$= 25 + 150\int_0^t e^{-6\tau}\,d\tau$$

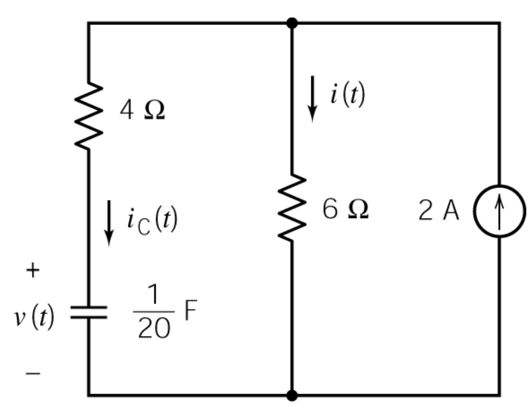
$$= 25 + 150\left[-\frac{1}{6}e^{-6\tau}\right]_0^t = \underline{50 - 25e^{-6t}\ \text{V}}$$

Example:
The capacitor voltage in this circuit is given by

$$v(t) = 12 - 10e^{-2t}\ \text{V} \quad \text{for} \quad t \ge 0$$

Determine $i(t)$ for $t > 0$.

Solution:



$$i_C\left(t\right) = \frac{1}{20}\frac{d}{dt}v\left(t\right)$$

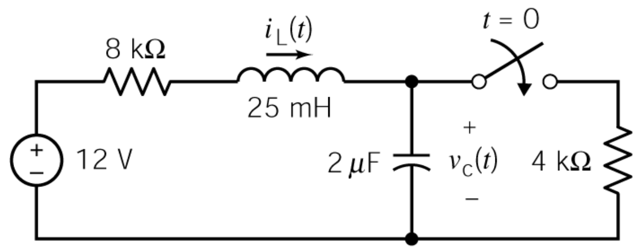
$$= \frac{1}{20}\left(+20e^{-2t}\right)$$

$$= e^{-2t}\ \text{A} \quad \text{for} \quad t > 0$$

Apply KCL to get

$$i\left(t\right) = 2 - i_C\left(t\right) = 2 - e^{-2t}\ \text{A} \quad \text{for} \quad t > 0$$

Example:



The switch in this circuit has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Solution:
The input is constant. When this circuit is at steady state, it is a dc circuit. Capacitors act like open circuits in dc circuits. Inductors act like short circuits in dc circuits.

In the absence of unbounded voltages and currents the capacitor voltage and inductor current are continuous functions of time . Consequently, we expect that

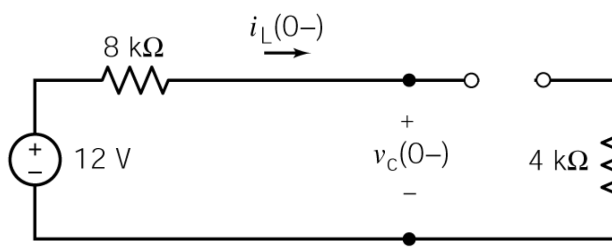
$$i_L\left(0^+\right) = i_L\left(0^-\right) \quad \text{and} \quad v_C\left(0^+\right) = v_C\left(0^-\right)$$

The switch is open before time $t = 0$. It is at steady state at time $t = 0–$ so

$$i_L\left(0^+\right) = i_L\left(0^-\right) = 0$$

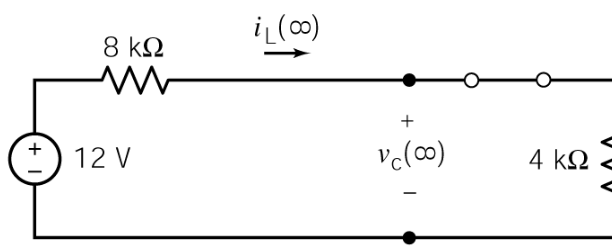
and

$$v_C\left(0^+\right) = v_C\left(0^-\right) = 12 \text{ V}$$

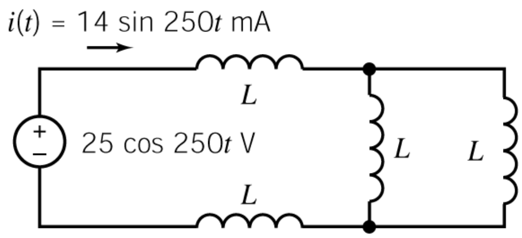


The switch is closed after time $t = 0$. The circuit will have reached steady state by time $t = \infty$ so

$v_c(\infty) = 4$ V, and $i_L(\infty) = 1$ mA



Example:



This circuit contains four identical inductors. Find the value of the inductance *L*.

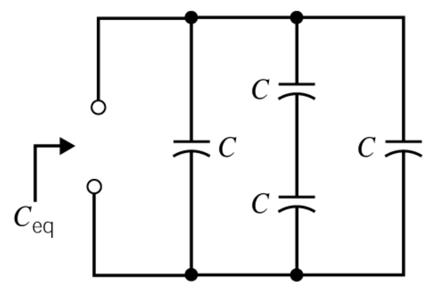
Solution:

$$L \parallel L = \frac{L \cdot L}{L+L} = \frac{L}{2} \quad \text{and} \quad L + L + \frac{L}{2} = \frac{5}{2} L$$

$$25\cos 250t = \left(\frac{5}{2} L\right) \frac{d}{dt}\left(\left(14\times10^{-3}\right)\sin 250t\right) = \left(\frac{5}{2} L\right)(14\times10^{-3})(250)\cos 250t$$

so $L = \dfrac{25}{\frac{5}{2}(14\times10^{-3})(250)} = 2.86 \text{ H}$

Example:

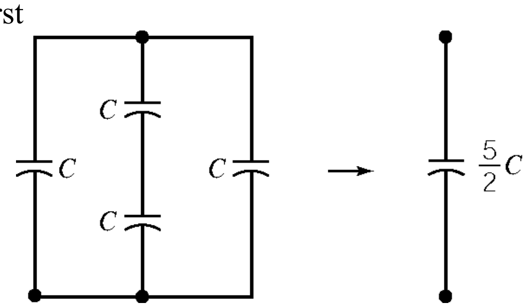


This circuit consists of four capacitors having equal capacitance, *C*.

a. Determine the value of the capacitance *C*, given that C_{eq} = 50 mF.
b. Determine the value of the equivalent capacitance C_{eq}, given that *C* = 50 mF.

Solution:

First



Then

$$50 = C_{\text{eq}} = \frac{5}{2}C \quad \Rightarrow \quad C = 20 \text{ mF}$$

$$C_{\text{eq}} = \frac{5}{2}C = \frac{5}{2}\left(50\ \mu\text{F}\right) = 125\ \mu\text{F}$$