Capacitors

$$\begin{array}{cccc}
+ & v(t) & - \\
 & \downarrow & \\
\hline
i(t) & C
\end{array}$$

$$i(t) = C \frac{d}{dt} v(t)$$
 and $v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau) d\tau = v(0) + \frac{1}{C} \int_{0}^{t} i(\tau) d\tau$

These equations describe an voltage and current that adhere to the passive convention.

All of the currents and voltages are constant in a dc circuit. When the capacitor voltage is constant, the capacitor current is zero.

Capacitors act like open circuits in dc circuits.

Suppose the capacitor voltage is discontinuous, for example

$$v(t) = \begin{cases} 4.3 & t < 2.5 \\ 4.4 & t > 2.5 \end{cases}$$

That is, the voltage changes from 4.3 V to 4.4 V abruptly at time t = 2.5 s . At t = 2.5 s , the derivative of the capacitor voltage is

$$\frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t - \Delta t)}{(t + \Delta t) - (t - \Delta t)} = \lim_{\Delta t \to 0} \frac{4.4 - 4.3}{2 \Delta t} = \infty$$

Consequently, discontinuous capacitor voltages require infinite capacitor currents. Infinite currents are physically impossible, so discontinuous capacitor voltages are physically impossible.

In the absence of infinite currents, capacitor voltages must be continuous.