

# Capacitors and Inductors with Current and Voltage Represented Analytically

## Introduction

Each of the circuits in this problem set contains of a single capacitor or inductor and a single independent source. When the independent source is a current source, the current source current is equal to the current in the capacitor or inductor. Similarly, when the independent source is a voltage source, the voltage source voltage is equal to the voltage across the capacitor or inductor.

These problems can be solved using the constitutive equations for the capacitor and inductor. (An “constitutive equation” is the equation that describes the relationship between the element voltage and element current.) Capacitors and inductors are described in Sections 7.2 and 7.5 of *Introduction to Electric Circuits* by R.C. Dorf and J.A Svoboda.

## Worked Examples

### Example 1:

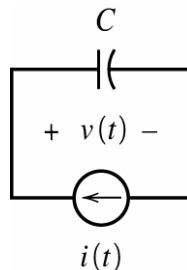
The input to the circuit shown in Figure 1 is the current

$$i(t) = 3.75e^{-1.2t} \text{ A for } t > 0$$

The output is the capacitor voltage

$$v(t) = 4 - 1.25e^{-1.2t} \text{ V for } t > 0$$

Find the value of the capacitance,  $C$ .



**Figure 1** The circuit considered in Example 1.

**Solution:** The capacitor voltage is related to the capacitor current by

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$$v(t) = \frac{1}{C} \int_0^t i(\tau) d\tau + v(0)$$

That is

$$4 - 1.25e^{-1.2t} = \frac{1}{C} \int_0^t 3.75e^{-1.2\tau} d\tau + v(0) = \frac{3.75}{C(-1.2)} e^{-1.2\tau} \Big|_0^t + v(0) = \frac{-3.125}{C} (e^{-1.2t} - 1) + v(0)$$

Equating the coefficients of  $e^{-1.2t}$  gives

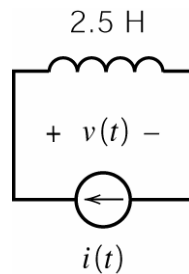
$$12.5 = \frac{3.125}{C} \Rightarrow C = \frac{3.125}{12.5} = 0.25 = 250 \text{ mF}$$

### Example 2:

The input to the circuit shown in Figure 2 is the current

$$i(t) = 3 - 4.5e^{-6t} \text{ A for } t > 0$$

Determine the inductor voltage,  $v(t)$ , for  $t > 0$ .



**Figure 2** The circuit considered in Example 2.

**Solution:** The inductor voltage is related to the inductor current by

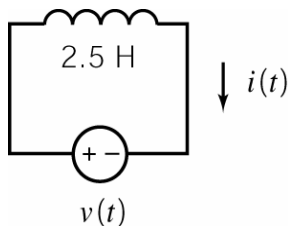
$$v(t) = L \frac{d}{dt} i(t) = 2.5 \frac{d}{dt} (3 - 4.5e^{-6t}) = 2.5 (-4.5(-6)e^{-6t}) = 67.5e^{-6t} \text{ V}$$

### Example 3:

The input to the circuit shown in Figure 3 is the voltage

$$v(t) = 15e^{-4t} \text{ V for } t > 0$$

The initial current in the inductor is  $i(0) = 2$  A. Determine the inductor current,  $i(t)$ , for  $t > 0$ .



**Figure 3** The circuit considered in Example 3.

**Solution:** The inductor current is related to the inductor voltage by

$$i(t) = \frac{1}{L} \int_0^t v(\tau) d\tau + i(0)$$

That is

$$i(t) = \frac{1}{2.5} \int_0^t 15e^{-4\tau} d\tau + 2 = \frac{15}{2.5(-4)} e^{-4\tau} \Big|_0^t + 2 = -1.5(e^{-4t} - 1) + 2 = 3.5 - 1.5e^{-4t} \text{ A for } t > 0$$

**Example 4:**

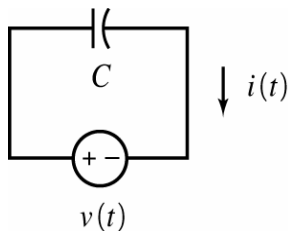
The input to the circuit shown in Figure 4 is the voltage

$$v(t) = -6 + 12e^{-5t} \text{ V for } t > 0$$

The output is the capacitor current

$$i(t) = -4e^{-5t} \text{ mA for } t > 0$$

Find the value of the capacitance,  $C$ .



**Figure 4** The circuit considered in Example 4.

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**Solution:** The capacitor current is related to the capacitor voltage by

$$i(t) = C \frac{d}{dt} v(t)$$

That is

$$-0.004 e^{-5t} = C \frac{d}{dt} (-6 + 12 e^{-5t}) = C (12(-5) e^{-5t}) = -60 C e^{-5t} \text{ V}$$

Equating the coefficients of  $e^{-5t}$  gives

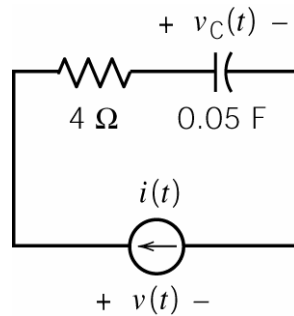
$$-0.004 = -60 C \Rightarrow C = \frac{0.004}{60} = 6.67 \times 10^{-5} \text{ F} = 66.7 \mu\text{F}$$

**Example 5:**

The input to the circuit shown in Figure 5 is the current

$$i(t) = 3e^{-25t} \text{ A for } t > 0$$

The initial capacitor voltage is  $v_C(0) = -2 \text{ V}$ . Determine the current source voltage,  $v(t)$ , for  $t > 0$ .



**Figure 5** The circuit considered in Example 5.

**Solution:** Apply KVL to the mesh to get

$$v(t) = 4i(t) + v_C(t) = 4i(t) + \left[ \frac{1}{0.05} \int_0^t i(\tau) d\tau + v(0) \right]$$

That is,

$$\begin{aligned} v(t) &= 4(3e^{-25t}) + \frac{1}{0.05} \int_0^t 3e^{-25\tau} d\tau - 2 = 12e^{-25t} + \frac{3}{0.05(-25)} (e^{-25t} - 1) - 2 \\ &= 12e^{-25t} - 2.4(e^{-25t} - 1) - 2 = 9.6e^{-25t} + 0.4 \text{ V for } t > 0 \end{aligned}$$

**Example 6:**

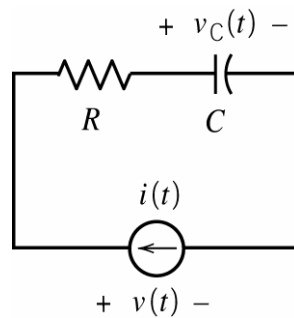
The input to the circuit shown in Figure 6 is the current

$$i(t) = 3e^{-25t} \text{ A for } t > 0$$

The output is the voltage

$$v(t) = 9.6e^{-25t} + 0.4 \text{ V for } t > 0$$

The initial capacitor voltage is  $v_C(0) = -2 \text{ V}$ . Determine the values of the capacitance,  $C$ , and resistance,  $R$ .



**Figure 6** The circuit considered in Example 6.

**Solution:** Apply KVL to the mesh to get

$$v(t) = Ri(t) + v_C(t) = Ri(t) + \left[ \frac{1}{C} \int_0^t i(\tau) d\tau + v(0) \right]$$

That is

$$\begin{aligned} 9.6e^{-25t} + 0.4 &= R(3e^{-25t}) + \left[ \frac{1}{C} \int_0^t 3e^{-25\tau} d\tau - 2 \right] \\ &= 3Re^{-25t} + \frac{3}{C(-25)}(e^{-25t} - 1) - 2 = 3\left(R - \frac{1}{25C}\right)e^{-25t} + \frac{3}{25C} - 2 \end{aligned}$$

Equating coefficients gives

$$0.4 = \frac{3}{25C} - 2 \Rightarrow C = 0.05 = 50 \text{ mF}$$

and

$$9.6 = 3\left(R - \frac{1}{25C}\right) = 3\left(R - \frac{1}{25(0.05)}\right) = 3(R - 0.8) \Rightarrow R = 4 \Omega$$

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**Example 7:**

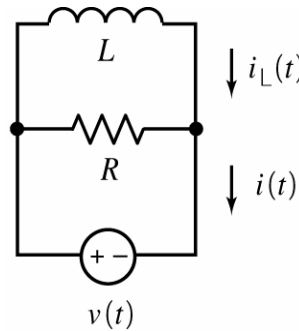
The input to the circuit shown in Figure 7 is the voltage

$$v(t) = 4e^{-20t} \text{ V for } t > 0$$

The output is the current

$$i(t) = -1.2e^{-20t} - 1.5 \text{ A for } t > 0$$

The initial inductor current is  $i_L(0) = -3.5 \text{ A}$ . Determine the values of the inductance,  $L$ , and resistance,  $R$ .



**Figure 7** The circuit considered in Example 7.

**Solution:** Apply KCL at either node to get

$$i(t) = \frac{v(t)}{R} + i_L(t) = \frac{v(t)}{R} + \left[ \frac{1}{L} \int_0^t v(\tau) d\tau + i(0) \right]$$

That is

$$\begin{aligned} -1.2e^{-20t} - 1.5 &= \frac{4e^{-20t}}{R} + \frac{1}{L} \int_0^t 4e^{-20\tau} d\tau - 3.5 = \frac{4e^{-20t}}{R} + \frac{4}{L(-20)}(e^{-20t} - 1) - 3.5 \\ &= \left( \frac{4}{R} - \frac{1}{5L} \right) e^{-20t} + \frac{1}{5L} - 3.5 \end{aligned}$$

Equating coefficients gives

$$-1.5 = \frac{1}{5L} - 3.5 \Rightarrow L = 0.1 \text{ H}$$

and

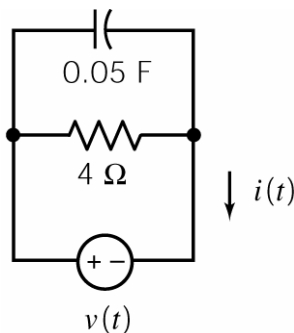
$$-1.2 = \frac{4}{R} - \frac{1}{5L} = \frac{4}{R} - \frac{1}{5(0.1)} = \frac{4}{R} - 2 \Rightarrow R = 5 \Omega$$

**Example 8:**

The input to the circuit shown in Figure 8 is the voltage

$$v(t) = 8 + 5e^{-10t} \text{ V for } t > 0$$

Determine the current,  $i(t)$  for  $t > 0$ .



**Figure 8** The circuit considered in Example 8.

**Solution:** Apply KCL at either node to get

$$\begin{aligned} i(t) &= \frac{v(t)}{4} + 0.05 \frac{d}{dt} v(t) = \frac{8 + 5e^{-10t}}{4} + 0.05 \frac{d}{dt} (8 + 5e^{-10t}) \\ &= 2 + 1.25e^{-10t} + 0.05(5)(-10)e^{-10t} = 2 - 1.25e^{-10t} \text{ A for } t > 0 \end{aligned}$$