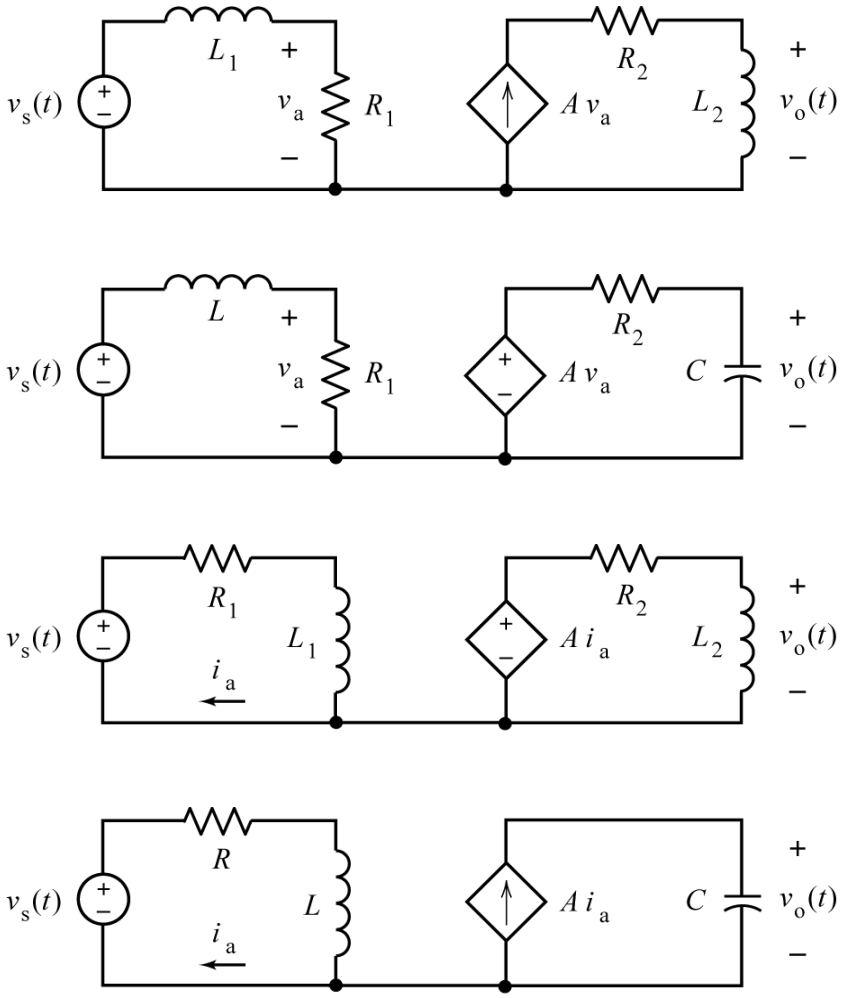
Design Exercise

The input to each of these four circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_o(t)$.



Design these four circuits to have these four network functions:

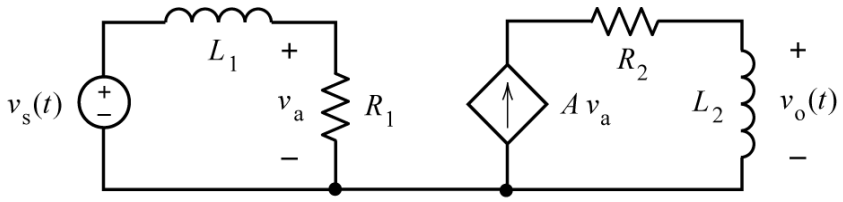
$$\mathbf{H}(\omega)=\frac{10\,(j\,\omega)}{\left(1+j\,\frac{\omega}{2}\right)\left(1+j\,\frac{\omega}{20}\right)},\quad \mathbf{H}(\omega)=\frac{10\,(j\,\omega)}{1+j\,\frac{\omega}{20}},\quad \mathbf{H}(\omega)=\frac{5}{j\,\omega\left(1+j\,\frac{\omega}{10}\right)}$$

and

$$\mathbf{H}(\omega)=\frac{10}{\left(1+j\,\frac{\omega}{2}\right)\left(1+j\,\frac{\omega}{50}\right)}$$

Solution

Consider:



The transfer function of this circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=L_2 A\frac{j\omega}{1+j\omega\dfrac{L_1}{R_1}}$$

Compare to

$$\mathbf{H}(\omega)=\frac{10(j\omega)}{1+j\dfrac{\omega}{20}}$$

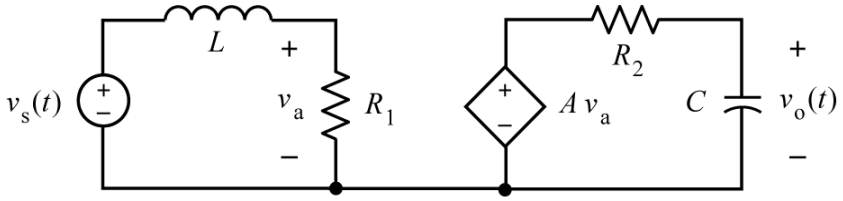
To get

$$\frac{L_1}{R_1}=\frac{1}{20}\quad\text{and}\quad L_2 A=10$$

For example

$$L_1=L_2=2\text{ H},\quad R_1=40\ \Omega,\quad R_2=10\ \Omega\quad\text{and}\quad A=5\text{ A/V}$$

Next, consider:



The transfer function of this circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{A}{\left(1+j\omega\dfrac{L}{R_1}\right)\left(1+j\omega C R_2\right)}$$

Compare to

$$\mathbf{H}(\omega)=\frac{10}{\left(1+j\dfrac{\omega}{2}\right)\left(1+j\dfrac{\omega}{50}\right)}$$

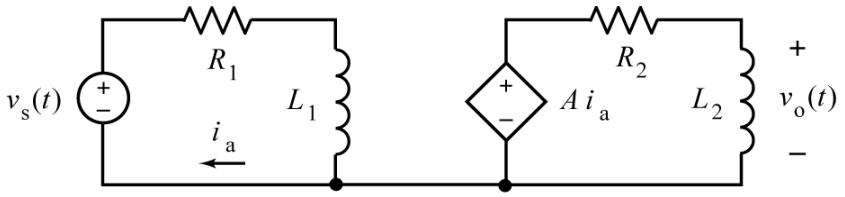
To get

$$A = 10, \quad \frac{L}{R_1} = \frac{1}{2} \quad \text{and} \quad C R_2 = \frac{1}{50}$$

For example

$$L = 2 \text{ H}, \quad R_1 = R_2 = 4\ \Omega, \quad C = 5 \text{ mF} \quad \text{and} \quad A = 10 \text{ V/V}$$

Next, consider:



The transfer function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{L_2 A}{R_1 R_2} \frac{j\omega}{\left(1 + j\omega \frac{L_1}{R_1}\right)\left(1 + j\omega \frac{L_2}{R_2}\right)}$$

Compare to

$$\mathbf{H}(\omega) = \frac{10\,(j\omega)}{\left(1 + j\frac{\omega}{2}\right)\left(1 + j\frac{\omega}{20}\right)}$$

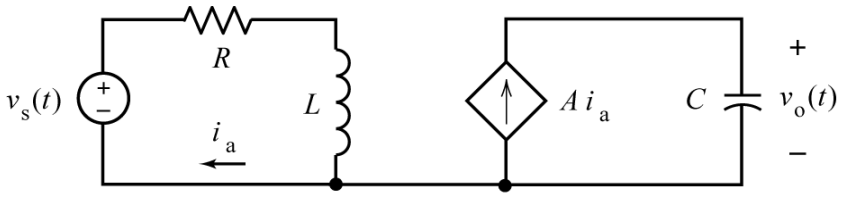
To get

$$\frac{L_1}{R_1} = \frac{1}{2}, \quad \frac{L_2}{R_2} = \frac{1}{20} \quad \text{and} \quad \frac{L_2 A}{R_1 R_2} = 10$$

For example

$$L_1 = L_2 = 2 \text{ H}, \quad R_1 = 4\ \Omega, \quad R_2 = 40\ \Omega \quad \text{and} \quad A = 800 \text{ V/A}$$

Finally, consider:



The transfer function of this circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)} = \frac{\frac{A}{CR}}{j\omega\left(1 + j\omega \frac{L}{R}\right)}$$

Compare to

$$\mathbf{H}(\omega) = \frac{5}{j\,\omega\left(1 + j\,\frac{\omega}{10}\right)}$$

To get

$$\frac{L}{R} = \frac{1}{10} \quad \text{and} \quad \frac{A}{C\,R} = 5$$

For example

$$L = 2\ \text{H}, \quad R = 20\ \Omega, \quad C = 100\ \text{mF} \quad \text{and} \quad A = 10\ \text{A/A}$$