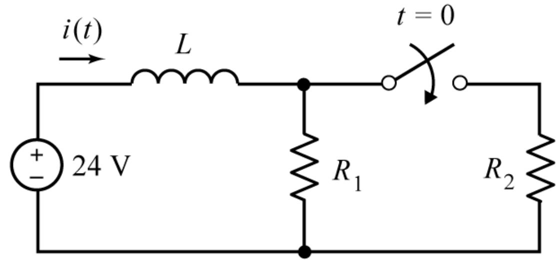
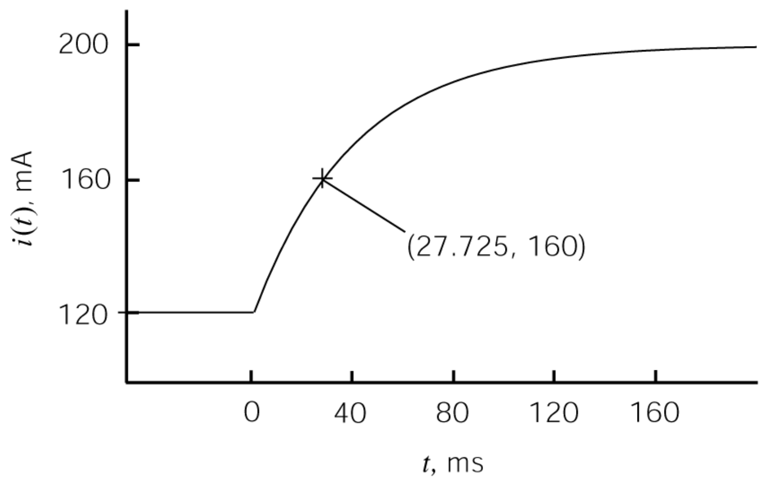
EE221 - Practice for the 1st Midterm Exam

1. Consider this circuit and corresponding plot of the inductor current:





Determine the values of L, R_1 and R_2: $L =$ __4.8__ H, $R_1 =$ __200__ Ω and $R_2 =$ __300__ Ω.

Hint: Use the plot to determine values of D, E, F and a such that the inductor current can be represented as

$$i(t) = \begin{cases} D & \text{for } t \le 0 \\ E + F\,e^{-at} & \text{for } t \ge 0 \end{cases}$$

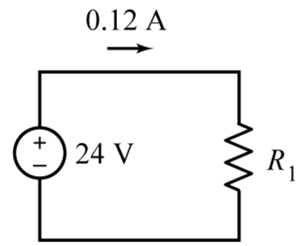
Solution:
From the plot $D =$ $i(t)$ for $t < 0$ =120 mA = 0.12 A, $E + F = i(0+)$ = 120 mA = 0.12 A and $E = \lim_{t\to\infty} i(t) = 200$ mA = 0.2 A . The point labeled on the plot indicates that $i(t)$ = 160 mA when t = 27.725 ms = 0.027725 s. Consequently

$$160 = 200 - 80\,e^{-a(0.027725)} \quad \Rightarrow \quad a = \frac{\ln\left(\frac{160-200}{80}\right)}{-0.027725} = 25\ \frac{1}{\text{s}}$$

Then

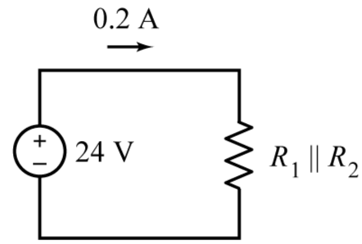
$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t \le 0 \\ 200 - 80\,e^{-25t} \text{ mA} & \text{for } t \ge 0 \end{cases}$$

When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.



$$R_1 = \frac{24}{0.12} = 200\ \Omega$$

As $t \to \infty$, the circuit is again at steady state so the inductor acts like a short circuit.



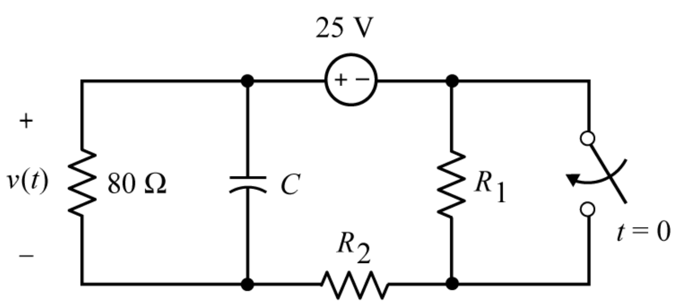
$$R_1 \parallel R_2 = \frac{24}{0.2} = 120\ \Omega$$

$$120 = 200 \parallel R_2 \quad \Rightarrow \quad R_2 = 300\ \Omega$$

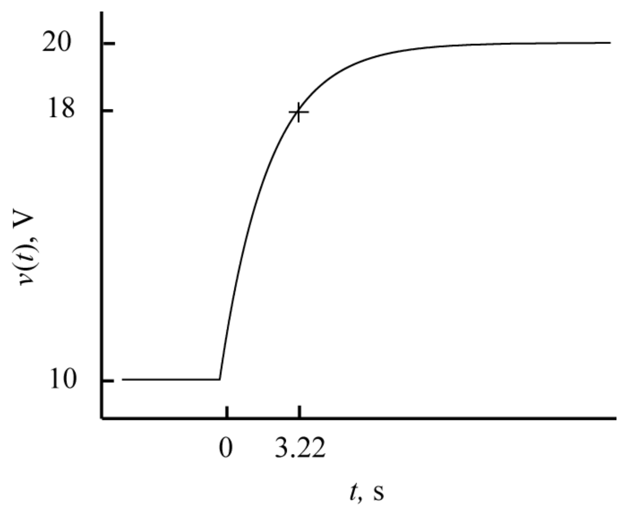
Next, the inductance can be determined using the time constant:

$$25 = a = \frac{1}{\tau} = \frac{R_1 \parallel R_2}{L} = \frac{120}{L} \quad \Rightarrow \quad L = \frac{120}{25} = 4.8\ \text{H}$$

2.



(*a*)



(*b*)

Design the circuit in (*a*) to have the response in (*b*) by specifying the values of *C*, R_1 and R_2.

$C =$ ___0.125__ F, $R_1 =$ ____100___ Ω and $R_2 =$ ____20____ Ω.

Solution:

The voltage $v(t)$ is represented by an equation of the form $v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\, e^{-at} & \text{for } t > 0 \end{cases}$

where *D*, *E*, *F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D = v(t) \ \text{ when } t < 0, \quad E = \lim_{t \to \infty} v(t), \quad E + F = \lim_{t \to 0+} v(t)$$

From the plot, we see that $D = 10,\ E = 20,\ \text{and}\ E + F = 10\ \text{V}$

Consequently, $$v(t) = \begin{cases} 10 & \text{for } t < 0 \\ 20 - 10\, e^{-at} & \text{for } t > 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot. We see $v(3.22) = 18\ \text{V}$, that is, the value of the voltage is 18 volts at time 3.22 seconds. Substituting these into the equation for $v(t)$ gives

$$18 = 20 - 10e^{-a(3.22)} \quad \Rightarrow \quad a = \frac{\ln(0.2)}{-3.22} = 0.5$$

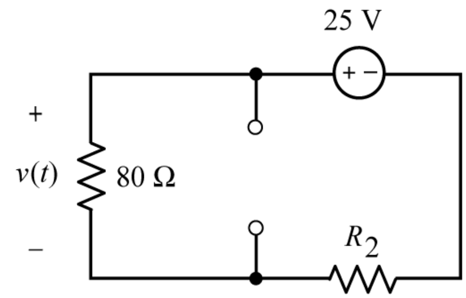
Consequently

$$v(t) = \begin{cases} 10 & \text{for } t < 0 \\ 20 - 10\, e^{-0.5t} & \text{for } t > 0 \end{cases}$$

Now let's turn our attention to the circuit. When the circuit is at steady state, the capacitor acts like an open circuit.

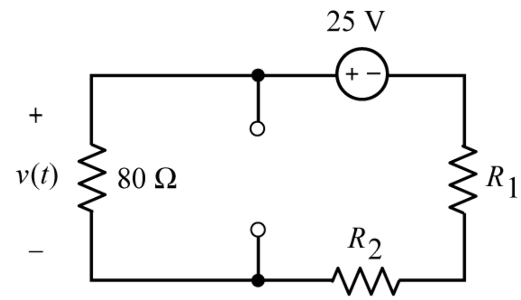
After $t = 0$, the switch is closed and the steady state voltage is determined from the plot to be $v(t) = E = 20$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by a short circuit. Using voltage division gives

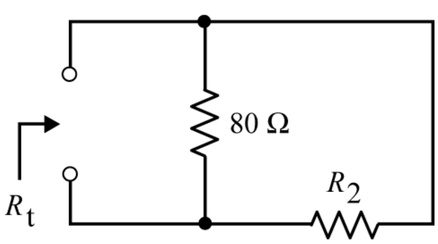
$$20 = \frac{80}{80 + R_2}(25) \quad \Rightarrow \quad R_2 = 20\ \Omega$$



Before $t = 0$, the switch is open and the steady state voltage is determined from the plot to be $v(t) = E + F = 10$ V. On the right we see the circuit that results from replacing the capacitor by an open circuit and the switch by an open circuit. Using voltage division gives

$$10 = \frac{80}{80 + R_1 + R_2}(25) = \frac{80}{100 + R_1}(25) \quad \Rightarrow \quad R_1 = 100\ \Omega$$

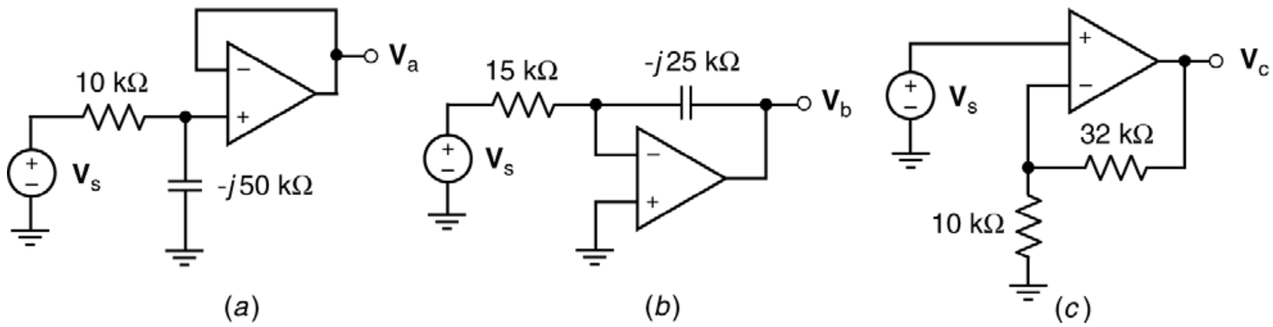




Recalling that $a = 0.5$ from the plot, consider the time constant $2 = \frac{1}{a} = \tau = C\, R_t$. After $t = 0$, the Thevenin resistance of the part of the circuit connected to the capacitor is $R_t = 80 \,||\, R_2 = 80 \,||\, 20 = 16\ \Omega$.

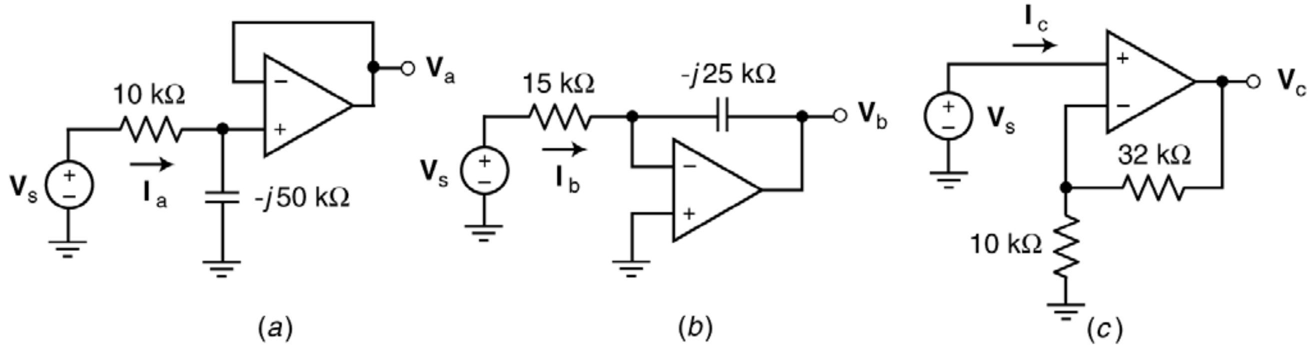
Then $C = \frac{2}{R_t} = \frac{2}{16} = 0.125\ \text{F}$.

3.



Here are three ac circuits, each represented in the frequency domain. The input to each of these circuits is the phasor voltage $\mathbf{V}_s = 2.5\angle -75°$ V . Let P_a , P_b and P_c denote the average power supplied by the source in circuit (*a*), (*b*) and (*c*) respectively. Determine the values of P_a , P_b and P_c:

P_a = ___0.0120__ mW, P_b = __0.2084__ mW and P_c = ___0____ mW



,

$$\mathbf{I}_a = \frac{2.5\angle -75°}{10 - j50} = \frac{2.5\angle -75°}{51\angle -78.7°} = 0.0490\angle 3.7° \text{ mA} \Rightarrow \mathbf{P}_a = \frac{2.5(0.0490)}{2}\cos(-75 - 3.7) = 0.0120 \text{ mW}$$

$$\mathbf{I}_b = \frac{2.5\angle -75}{15} = 0.1667\angle -75° \text{ mA} \Rightarrow \mathbf{P}_b = \frac{2.5(0.1667)}{2}\cos(-75 - (-75)) = 0.208375 \text{ mW}$$

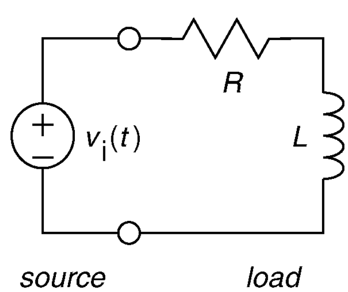
and

$$\mathbf{I}_c = 0 \Rightarrow \mathbf{P}_c = 0$$

4. Given that

$$v_i(t) = 24\cos(3t + 75°) \quad \text{V}$$

answer the following questions:



a) Suppose $R = 9\ \Omega$ and $L = 5$ H . What are the average, complex and reactive powers delivered by the source to the load?

P = __8.47__ W, $\mathbf{S}$ = __ 8.47 + j14.1__ VA and Q = __14.1__ VAR

b) Suppose the source delivers 8.47 + *j* 14.12 VA to the load. What are the values of the resistance, *R*, and the inductance, *L*?

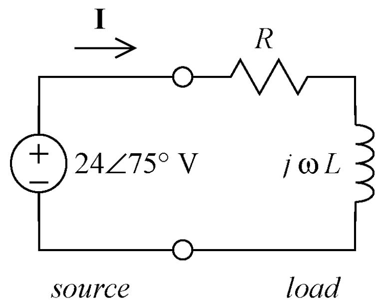
R= __9__ Ω and *L* = __5__ H

c) Suppose the source delivers 14.12 W to the load at a power factor of 0.857 lagging. What are the values of the resistance, *R*, and the inductance, *L*?

R= __15__ Ω and *L* = __3__ H

Solution:

Represent the circuit in the frequency domain as



a)

$$\mathbf{I} = \frac{24\angle 75^\circ}{9 + j15} = \frac{24\angle 75^\circ}{17.5\angle 59^\circ} = 1.37\angle 16^\circ \text{ A}$$

$$\mathbf{S} = \frac{1}{2}(24\angle 75^\circ)(1.37\angle 16^\circ)^* = \frac{24(1.37)}{2}\angle(75-16)^\circ = 16.44\angle 59^\circ = 8.47 + j14.1 \text{ VA}$$

b)

$$\mathbf{I} = \left(\frac{2(8.47 + j14.12)}{24\angle 75^\circ}\right)^* = \left(\frac{2(16.44\angle 59^\circ)}{24\angle 75^\circ}\right)^* = (1.37\angle -16^\circ)^* = 1.37\angle 16^\circ \text{ A}$$

$$R + j3L = \frac{24\angle 75^\circ}{1.37\angle 16^\circ} = 17.5\angle 59^\circ = 9 + j15\ \Omega \quad \text{so} \quad R = 9\ \Omega \quad \text{and} \quad L = \frac{15}{3} = 5 \text{ H}$$

c)

$$pf = 0.857 \text{ lagging} \quad \Rightarrow \quad \begin{cases} 0.857 = \cos(\theta) \\ \text{and} \\ \theta > 0 \end{cases} \quad \text{so} \quad \theta = 31^\circ.$$

Next

$$14.12 = P = |\mathbf{S}|\cos\theta = |\mathbf{S}|(0.857) \quad \text{so} \quad |\mathbf{S}| = \frac{14.12}{0.857} = 16.48 \text{ VA}$$

Then

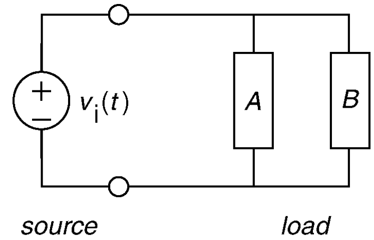
$$\mathbf{S} = 16.48\angle 31^\circ = 14.12 + j8.49 \quad \text{and} \quad \mathbf{I} = \left(\frac{2\mathbf{S}}{\mathbf{V}}\right)^* = \left[\frac{2(16.48\angle 31^\circ)}{24\angle 75^\circ}\right]^* = 1.37\angle 44^\circ$$

$$R + j3L = \frac{24\angle 75^\circ}{1.37\angle 44^\circ} = 17.5\angle 31^\circ = 15 + j9\ \Omega \quad \text{so} \quad R = 15\ \Omega \quad \text{and} \quad L = 3 \text{ H}$$

5. Given that

$$v_i(t) = 24\cos(3t + 75°) \quad \text{V}$$

Determine the impedance of the load and the complex power delivered by the source to the load under each of the following conditions:



a) The source delivers 14.12 + *j* 8.47 VA to load *A* and 8.47 + *j* 14.12 VA to load *B*.

$$\mathbf{Z} = \_\_\ 9.016\angle 45°\ \_\_\ \Omega,\ \ \mathbf{S} = \_\_\ 22.59 + j22.59\ \_\_\ \text{VA}$$

b) The source delivers 8.47 + *j* 14.12 VA to load *A* and the impedance of load *B* is $15 + j9\,\Omega$.

$$\mathbf{Z} = \_\_\ 9.016\angle 45°\ \_\_\ \Omega,\ \ \mathbf{S} = \_\_\ 22.59 + j22.59\ \_\_\ \text{VA}$$

c) The source delivers 14.12 W to load *A* at a power factor of 0.857 lagging and the impedance of load *B* is $9 + j15\,\Omega$.

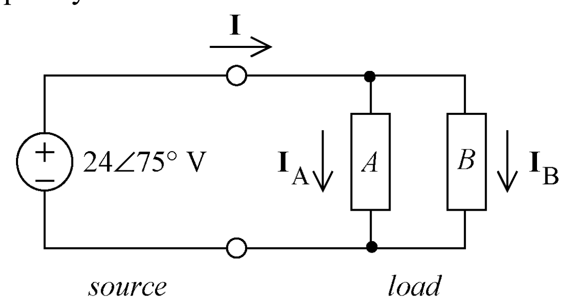
$$\mathbf{Z} = \_\_\ 9.016\angle 45°\ \_\_\ \Omega,\ \ \mathbf{S} = \_\_\ 22.59 + j22.59\ \_\_\ \text{VA}$$

d) The impedance of load *A* is $15 + j9\,\Omega$ and the impedance of load *B* is $9 + j15\,\Omega$.

$$\mathbf{Z} = \_\_\ 9.016\angle 45°\ \_\_\ \Omega,\ \ \mathbf{S} = \_\_\ 22.59 + j22.59\ \_\_\ \text{VA}$$

Solution:

Represent the circuit in the frequency domain as



(a) $$\mathbf{I}_A = \left(\frac{2(14.12 + j8.47)}{24\angle 75°}\right)^* = 1.37\angle 44°\ \text{A} \quad \text{and} \quad \mathbf{I}_B = \left(\frac{2(8.47 + j14.12)}{24\angle 75°}\right)^* = 1.37\angle 16°\ \text{A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = (1.37\angle 44°) + (1.37\angle 16°) = (0.986 + j0.954) + (1.319 + j0.377)$$
$$= 2.305 + j1.331 = 2.662\angle 30°\ \text{A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45°$$

$$\mathbf{S} = \frac{1}{2}(24\angle 75°)(2.662\angle 30°)^* = 31.9\angle 45° = 22.59 + j22.59\ \text{VA}$$

(b) $$\mathbf{I}_A = \left(\frac{2(8.47 + j14.12)}{24\angle 75°}\right)^* = 1.37\angle 16°\ \text{A} \quad \text{and} \quad \mathbf{I}_B = \frac{24\angle 75°}{15 + j9} = 1.37\angle 44°\ \text{A}$$

$$\mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30°\ \text{A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45°\ \Omega \quad \text{and} \quad \mathbf{S} = 22.59 + j22.59 \text{ VA}$$

(c)

$$\mathbf{P} = 14.12 \text{ W} = \frac{24|\mathbf{I}_A|}{2}\cos(75 - \theta_A)$$

$$\left.\begin{array}{r} 0.857 = \cos(75 - \theta_A) \\ 75 - \theta_A > 0 \end{array}\right\} \quad \Rightarrow \quad \theta_A = 75° - 31° = 44°$$

Then

$$|\mathbf{I}_A| = \frac{2(14.12)}{24\cos(31°)} = 1.37 \quad \text{so} \quad \mathbf{I}_A = 1.37\angle 44° \text{ A}$$

Also

$$\mathbf{I}_B = \frac{24\angle 75°}{9 + j15} = 137\angle 16° \text{ A} \quad \text{so} \quad \mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30° \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45°\ \Omega \quad \text{and} \quad \mathbf{S} = 22.59 + j22.59 \text{ VA}$$

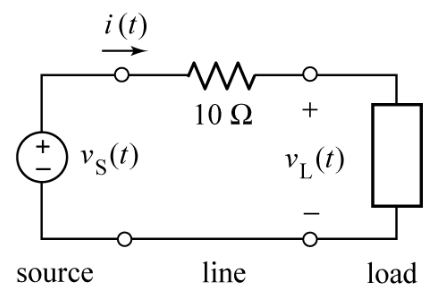
(d)

$$\mathbf{I}_A = \frac{24\angle 75°}{15 + j9} = 1.37\angle 44° \text{ and } \mathbf{I}_B = \frac{24\angle 75°}{9 + j15} = 1.37\angle 16° \text{ then } \mathbf{I} = \mathbf{I}_A + \mathbf{I}_B = 2.662\angle 30° \text{ A}$$

$$\mathbf{Z} = \frac{24\angle 75°}{2.662\angle 30°} = 9.016\angle 45°\ \Omega \quad \text{and} \quad \mathbf{S} = 22.59 + j22.59 \text{ VA}$$

6. In this circuit an ac source is connected to a load by the line. The load voltage is $\mathbf{V}_L$ = 120∠0° Vrms and the load receives 50 W at a power factor of 0.8 lagging. The line current is

$$\mathbf{I} = 0.5208\angle -36.87° \text{ Arms}$$



Determine the RMS value of required source voltage, $v_s(t)$, and the average power supplied by the source, P_s.

$|\mathbf{V}s|$ = ___124.2__ Vrms and P_s = ___52.71__ W

Using KVL

$$\mathbf{V}_S = 10(0.5208\angle -36.87°) + 120\angle 0° = 124.2 - j3.125 = 124.2\angle -1.45° \text{ Vrms}$$

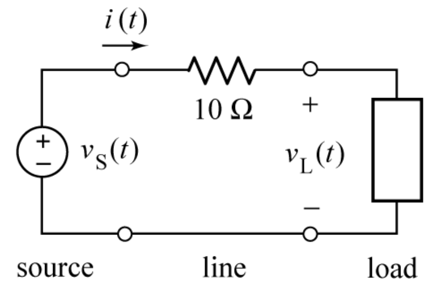
The complex power delivered by the source is

$$\mathbf{S} = (124.2\angle -1.45°)(0.5208\angle -36.87°)^* = 64.68\angle 35.42° = 52.71 + j37.49 \text{ VA}$$

7. In this circuit an ac source is connected to a load by the line. The load voltage is $\mathbf{V}_L = 120\angle 0°$ Vrms and the load receives 50 W at a power factor of 0.8 lagging. The line current is

$$\mathbf{I} = B\angle\phi \text{ Arms}$$

Determine the values of B and ϕ.



B = __0.5208__ Arms and ϕ = __–36.87__ °

The complex power delivered to the load is

$$\mathbf{S}_L = 50 + j\frac{50}{0.8}\sin\left(\cos^{-1}(0.8)\right) = 50 + j37.5 = 62.5\angle 36.87° \text{ VA}$$

The line current is $$\mathbf{I} = \left(\frac{62.5\angle 36.87°}{120\angle 0°}\right)^* = 0.5208\angle -36.87° \text{ Arms}$$

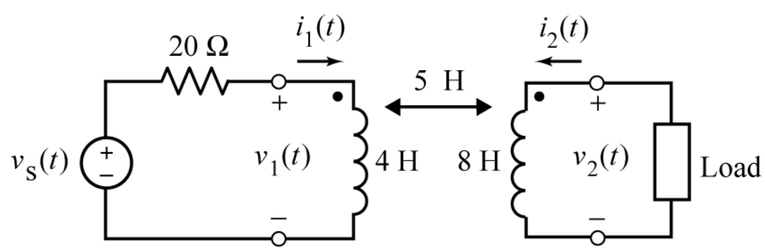
.

8. The input to this circuit shown is

$$v_s(t) = 12\cos(5t) \text{ V}$$

The impedance of the load is 20 + j 15 Ω.

Noticing that $i_1(t)$ and $i_2(t)$ are mesh currents, we can represent this circuit by the mesh equations

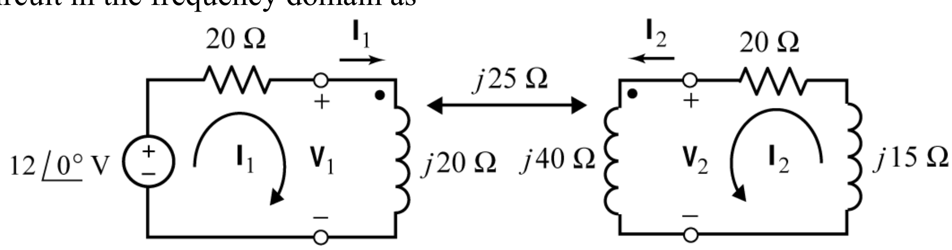


$$\begin{bmatrix} 20 + ja & jb \\ jc & 20 + jd \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 12\angle 0° \\ 0 \end{bmatrix}$$

where a, b, c, and d are real constants. Determine the values of a, b, c, and d.

a = ____20____ Ω, b = ___25____ Ω, c = ___25____ Ω, and d = ____55___ Ω

Represent the circuit in the frequency domain as



The coil voltages are given by

$$\mathbf{V}_1 = j20\mathbf{I}_1 + j25\mathbf{I}_2 \quad \text{and} \quad \mathbf{V}_2 = j40\mathbf{I}_2 + j25\mathbf{I}_1$$

Using KVL

$$20\mathbf{I}_1 + \mathbf{V}_1 - 12\angle 45° = 0 \quad \text{and} \quad 20\mathbf{I}_2 + j15\mathbf{I}_2 + \mathbf{V}_2 = 0$$

Substituting the coil voltages:

$$20\mathbf{I}_1 + j20\mathbf{I}_1 + j25\mathbf{I}_2 = 12\angle 0°$$

$$20\mathbf{I}_2 + j15\mathbf{I}_2 + j40\mathbf{I}_2 + j25\mathbf{I}_1 = 0$$

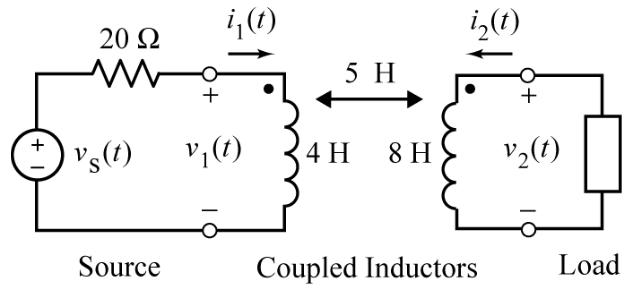
Solving gives $$\mathbf{I}_1 = 0.4676\angle -22.8° \text{ A} \quad \text{and} \quad \mathbf{I}_2 = 0.1998\angle 177.1° \text{ A}$$

9. This circuit consists of a source connected to a load by coupled coils. The input is

$$v_s(t) = 12\cos(5t)\ \text{V}$$

The impedance of the load is 20 + *j* 15 Ω.

The mesh currents $i_1(t)$ and $i_2(t)$ are



$$i_1(t) = 0.4676\cos(5t - 22.8°)\ \text{A} \quad \text{and} \quad i_2(t) = 0.1998\cos(5t + 177.1°)\ \text{A}$$

Determine the values of **S**, the complex power supplied by the source, $\mathbf{S}_c$, the complex power received by the coupled inductors and $\mathbf{S}_L$, the complex power received by the load.

S = ________ + *j* _______ VA, $\mathbf{S}_c$ = ________ + *j* ________ VA and $\mathbf{S}_L$ = ________ + *j* ________ VA

The complex power delivered by the source is

$$\mathbf{S} = \frac{(12\angle 0°)\mathbf{I}_1{}^*}{2} = \frac{(12\angle 0°)(0.4676\angle -22.8°)^*}{2} = 2.5855 + j1.0893\ \text{VA}$$

The complex power received by the 20 Ω resistor is

$$\mathbf{S} = \frac{|\mathbf{I}_1|^2}{2}(20) = \frac{(0.4676)^2}{2}(20) = 2.1865 + j0\ \text{VA}$$

The complex power received by the coupled inductors is

$$\mathbf{S} = \frac{\mathbf{V}_1\mathbf{I}_1{}^*}{2} + \frac{\mathbf{V}_2\mathbf{I}_2{}^*}{2} = 0 + j0.79\ \text{VA}$$

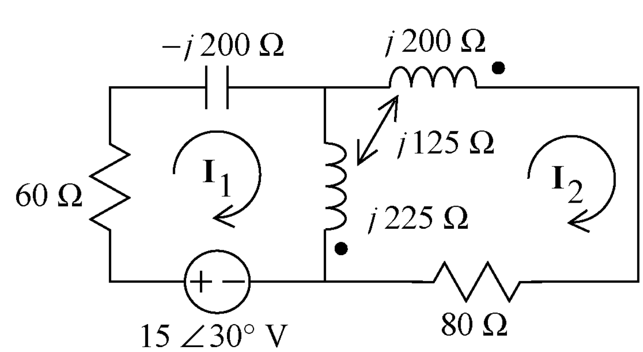
The complex power received by the load is

$$\mathbf{S} = \frac{|\mathbf{I}_2|^2}{2}(20 + j15) = \frac{(0.1998)^2}{2}(20 + j15) = 0.399 + j0.299\ \text{VA}$$

10. Here is a circuit containing coupled coils, represented in the frequency domain. The currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are mesh currents. The mesh equations representing this circuit can be expressed as

$$(a + jb)\mathbf{I}_1 + (c + jd)\mathbf{I}_2 = 15\angle 30°$$

$$(c + jd)\mathbf{I}_1 + (80 + jf)\mathbf{I}_2 = 0$$



where $a + jb$, $c + jd$, and $40 + jf$ represent complex numbers in rectangular form. Determine the following:

a =___60_____, b =___25___, c =____0_____, d =____-100___, f =___175____

Apply KVL to the left mesh to get

$$(-j200)\mathbf{I}_1 + \left[(j225)(\mathbf{I}_1 - \mathbf{I}_2) + (j125)\mathbf{I}_2\right] - 15\angle 30° + 60\mathbf{I}_1 = 0$$

$$(60 + j25)\mathbf{I}_1 - (j100)\mathbf{I}_2 = 15\angle 30°$$

Apply KVL to the right mesh to get

$$\left[(j200)\mathbf{I}_2 + (j125)(\mathbf{I}_1 - \mathbf{I}_2)\right] + 80\mathbf{I}_2 - \left[(j225)(\mathbf{I}_1 - \mathbf{I}_2) + (j125)\mathbf{I}_2\right] = 0$$

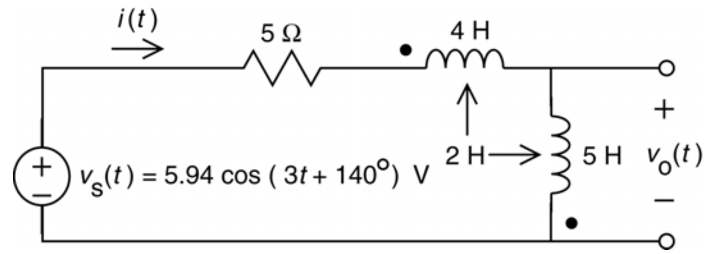
$$(-j100)\mathbf{I}_1 + (80 + j175)\mathbf{I}_2 = 0$$

11. The current $i(t)$ and voltage $v(t)$ labeled on the circuit drawing are

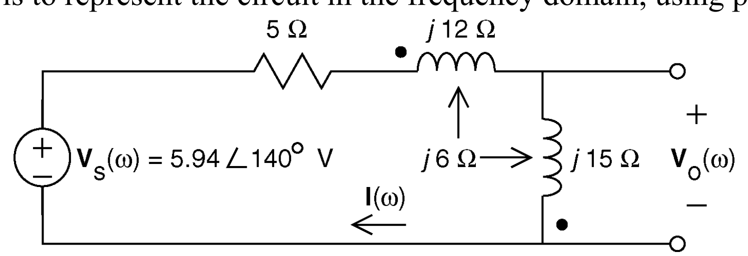
$i(t) =$ __0.376__ $\cos(3t + 68.4°)$ A

and

$v(t) =$ __3.38___ $\cos(3t +$ __158.4__ $°)$ V



Solution: The first step is to represent the circuit in the frequency domain, using phasors and impedances.



This circuit consists of a single mesh. Notice that the mesh current, **I**(ω), enters the dotted end of the left-hand coil and the undotted end of the right-hand coil. Apply KVL to the mesh to get

$$5\,\mathbf{I}(\omega) + \left(j12\,\mathbf{I}(\omega) - j6\,\mathbf{I}(\omega)\right) + \left(-j6\,\mathbf{I}(\omega) + j15\,\mathbf{I}(\omega)\right) - 5.94\angle 140° = 0$$

$$5\,\mathbf{I}(\omega) + (j12 - j6 - j6 + j15)\,\mathbf{I}(\omega) - 5.94\angle 140° = 0$$

$$\mathbf{I}(\omega) = \frac{5.94\angle 140°}{5 + j(12 - 6 - 6 + 15)} = \frac{5.94\angle 140°}{5 + j15} = \frac{5.94\angle 140°}{15.8\angle 71.6} = 0.376\angle 68.4° \text{ A}$$

Notice that the voltage, $\mathbf{V_o}(\omega)$, across the right-hand coil and the mesh current, **I**(ω), adhere to the passive convention. The voltage across the right-hand coil is given by is given by

$$\mathbf{V_o}(\omega) = j\,15\,\mathbf{I}(\omega) - j\,6\,\mathbf{I}(\omega) = j\,9\,\mathbf{I}(\omega) = j\,9\,(0.376\angle 68.4°)$$

$$= (9\angle 90°)(0.376\angle 68.4°) = 3.38\angle 158.4° \text{ V}$$

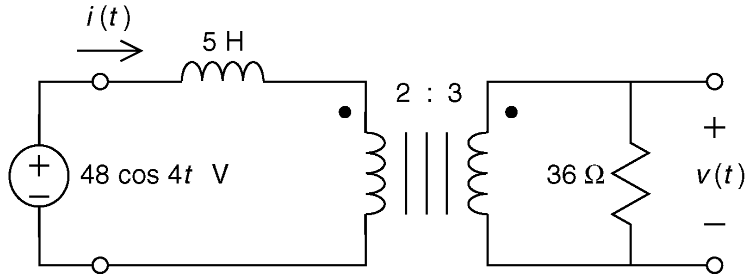
In the time domain, the output voltage is given by $v_o(t) = 3.38\cos(3t + 158.4°)$ V

12. The current $i(t)$ and voltage $v(t)$ labeled on the circuit drawing are

$i(t) =$ __1.87__ $\cos(4t - 51.3°)$ A

and

$v(t) =$ ___45___ $\cos(4t -$ __51.3__ $°)$ V



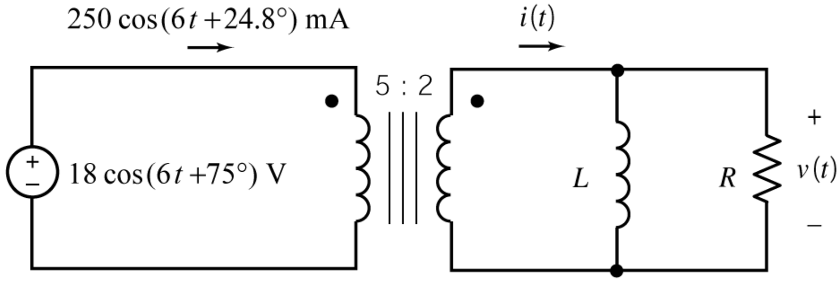
Represent the circuit in the frequency domain. Then

$$\mathbf{I} = \frac{48\angle 0°}{j20 + \left(\frac{2}{3}\right)^2 (36)} = \frac{48\angle 0°}{j20 + 16} = 1.874\angle -51.3° \text{ A}$$

and

$$\mathbf{V} = -\left(\frac{2}{3}\right)(1.874\angle -51.3°)(36) = 44.978\angle -51.3° \text{ V}$$

13.



Determine the values of R and L: $R =$ ____18____ Ω and $L =$ ____2.5____ H

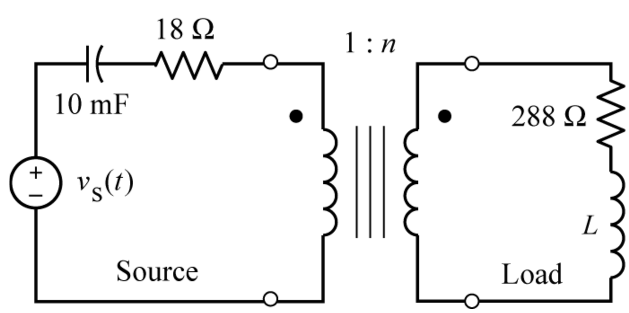
$$\frac{18\angle 75°}{0.250\angle 24.8°} = \left(\frac{5}{2}\right)^2 (R \,\|\, j6L) \;\Rightarrow\; \frac{1}{R} + \frac{1}{j6L} = \frac{1}{\frac{18\angle 75°}{0.250\angle 24.8°}\left(\frac{2}{5}\right)^2} = \left(\frac{5}{2}\right)^2 \left(\frac{0.250\angle 24.8°}{18\angle 75°}\right)$$

$$\frac{1}{R} - \frac{1}{j6L} = \left(\frac{5}{2}\right)^2 \left(\frac{0.250\angle 24.8°}{18\angle 75°}\right) = 0.0868\angle 50.2° = 0.05556 - j0.066687$$

$$R = \frac{1}{0.05556} = 18\ \Omega \quad \text{and} \quad 6L = \frac{1}{0.066687} = 15$$

14. This circuit consists of a load connected to a source through an ideal transformer. The input to the circuit is

$$v_s(t) = 12\cos(20t) \text{ V}$$



Determine the values of the turns ration, n, and load inductance, L, required for maximum power transfer to the load.

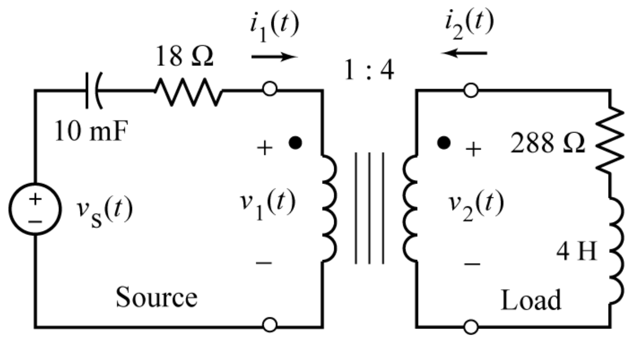
$n =$ ______4_____ and $L =$ _____4_____ H

For maximum power transfer: $\frac{1}{n^2}(288 + j20L) = \left(18 - j\frac{1}{20 \times 0.01}\right)^* = 18 + j5$

Equating real parts gives $n = \sqrt{\frac{288}{18}} = 4$. Equating imaginary parts gives $L = \frac{5(4^2)}{20} = 4$ H

15. This circuit consists of a load connected to a source through an ideal transformer. The input to the circuit is

$$v_s(t) = 12\cos(20t) \text{ V}$$



The coil voltages and currents are

$$v_1(t) = A\cos(20t + 15.5°) \text{ V},$$

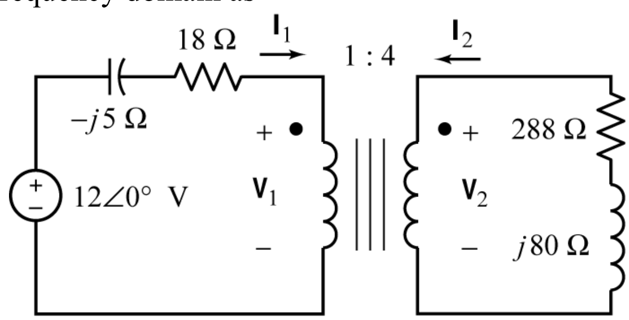
$$v_2(t) = B\cos(20t + 15.5°) \text{ V}$$

$$i_1(t) = C\cos(20t) \text{ A and } i_2(t) = D\cos(20t + 180) \text{ A}$$

Determine the values of *A*, *B*, *C* and *D*.

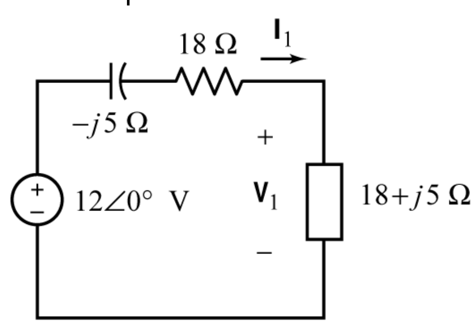
$A =$ ___6.227___ V, $B =$ ___24.91___ V, $C =$ ____0.33___ A and $D =$ __0.0833__ A

Represent the circuit in the frequency domain as



Replace the transformer and load by an equivalent impedance

$$\mathbf{Z}_{\text{equiv}} = \frac{1}{4^2}(288 + j80) = 18 + j5 \ \Omega$$



$$\mathbf{I}_1 = \frac{12\angle 0°}{(18 - j5) + (18 + j5)} = \frac{12\angle 0°}{36} = \frac{1}{3}\angle 0° \text{ A}$$

and
$$\mathbf{V}_1 = (18 + j5)\mathbf{I}_1 = (18 + j5)\left(\frac{1}{3}\angle 0°\right) = 6.227\angle 15.5° \text{ V}$$

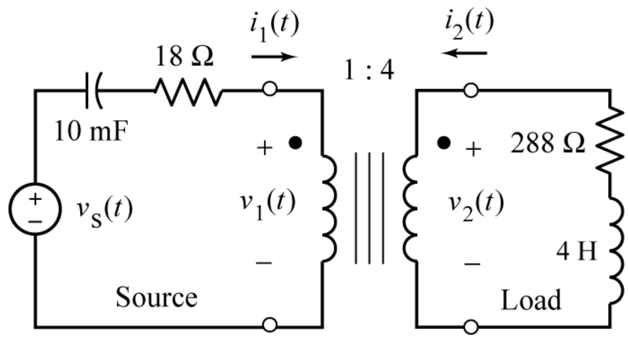
The secondary coil current and voltages

$$\mathbf{I}_2 = -\frac{1}{4}\mathbf{I}_1 = -\frac{1}{4}\left(\frac{1}{3}\angle 0°\right) = -\frac{1}{12}\angle 0° = -0.0833\angle 0° \text{ A}$$

and
$$\mathbf{V}_2 = \frac{4}{1}\mathbf{V}_1 = 24.91\angle 15.5° \text{ V}$$

16. This circuit consists of a load connected to a source through an ideal transformer. The input to the circuit is

$$v_s(t) = 12\cos(20t) \text{ V}$$



The coil voltages and currents are

$$v_1(t) = 6.227\cos(20t + 15.5°) \text{ V},$$

$$v_2(t) = 24.91\cos(20t + 15.5°) \text{ V}$$

$$i_1(t) = 0.333\cos(20t) \text{ A} \quad \text{and} \quad i_2(t) = 0.0833\cos(20t + 180) \text{ A}$$

Determine the values of $\mathbf{S}_p$, the complex power received by the primary (left) coil of the transformer and $\mathbf{S}_L$, the complex power received by the load.

$\mathbf{S}_p$ = ___1____ + j __0.277__ VA and $\mathbf{S}_L$= ____1____ + j __0.277__ VA

The complex power received by the primary (left) coil of the transformer is

$$\frac{\mathbf{V}_1\mathbf{I}_1{}^*}{2} = 1 + j0.277 \text{ VA} = \frac{|\mathbf{I}_1|^2}{2}(18 + j5)$$

The complex power received by the load is

$$-\frac{\mathbf{V}_2\mathbf{I}_2{}^*}{2} = 1 + j0.277 \text{ VA} = \frac{|\mathbf{I}_2|^2}{2}(288 + j80)$$

17. The network function of a circuit is $\mathbf{H}(\omega) = -10\dfrac{j\omega}{1 + j\dfrac{\omega}{20}}$. The table below tabulates frequency response data for this circuit. Fill in the blanks in the table:

ω, rad/s	Gain, V/V	Phase Shift, °
10	89.44	___−116.6__
40	__178.9__	−153.4

$$\mathbf{H}(10) = -10\frac{j(10)}{1 + j\dfrac{10}{20}} = -10\frac{j(10)}{1 + j0.5} = \frac{100}{\sqrt{1.25}}\angle\left(-180 + 90 - \tan^{-1}(0.5)\right) = 89.44\angle -116.6°$$

$$\mathbf{H}(40) = -10\frac{j(40)}{1 + j\dfrac{40}{20}} = -10\frac{j(40)}{1 + j2} = \frac{400}{\sqrt{5}}\angle\left(-180 + 90 - \tan^{-1}(2)\right) = 178.9\angle -153.4°$$

18. The network function of a circuit is $\mathbf{H}(\omega)=\dfrac{k}{1+j\dfrac{\omega}{p}}$. The table below tabulates frequency response data for this circuit.

ω, rad/s	Gain, V/V	Phase Shift, °
10	17.18	−17.4
40	11.25	−51.3

Determine the values of *p* and *k*: $p=$ _____32___ rad/s and $k=$ ____18____ V/V

$$\frac{k}{1+j\dfrac{10}{p}}=\frac{k}{\sqrt{1+\left(\dfrac{10}{p}\right)^2}}\angle-\tan^{-1}\left(\frac{10}{p}\right)=17.18\angle-17.4°$$

so

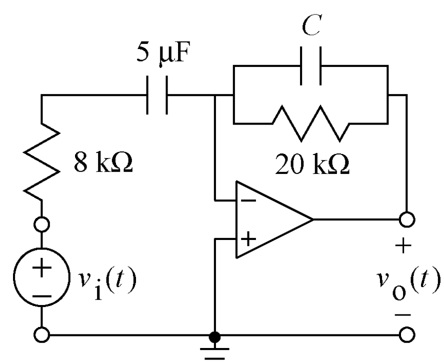
$$-\tan^{-1}\left(\frac{10}{p}\right)=-17.4° \;\Rightarrow\; \frac{10}{p}=\tan(17.4°)=0.3134 \;\Rightarrow\; p=\frac{10}{0.3134}=31.9\text{ rad/s}$$

and

$$\frac{k}{\sqrt{1+\left(\dfrac{10}{p}\right)^2}}=\frac{k}{\sqrt{1+(0.3134)^2}}=\frac{k}{1.048}=17.18 \;\Rightarrow\; k=18$$

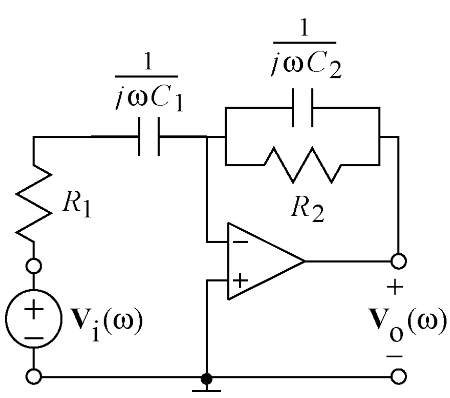
19. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

$$\mathbf{H}(\omega)=\frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)}=\frac{(-0.1)\,j\omega}{\left(1+j\dfrac{\omega}{p}\right)\left(1+j\dfrac{\omega}{125}\right)}$$



Determine the values of the capacitance, *C*, and the pole, *p*.

$C=$ _____0.4_______ μF and $p=$ ______25______ rad/s .



$$\mathbf{H}(\omega)=-\frac{R_2\,\|\,\dfrac{1}{j\omega C_2}}{R_1+\dfrac{1}{j\omega C_1}}=-\frac{\dfrac{R_2}{1+j\omega C_2R_2}}{\dfrac{j\omega C_1R_1+1}{j\omega C_1}}$$

$$=\frac{(-C_1R_2)\,j\omega}{(1+j\omega C_1R_1)(1+j\omega C_2R_2)}$$

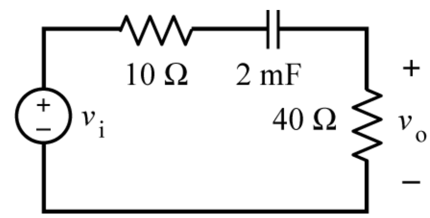
$$\frac{\left(-C_1 R_2\right) j\omega}{\left(1+j\omega C_1 R_1\right)\left(1+j\omega C_2 R_2\right)} = \frac{(-0.1)\, j\omega}{\left(1+j\dfrac{\omega}{p}\right)\left(1+j\dfrac{\omega}{125}\right)} \quad \Rightarrow \quad \begin{cases} -C_1 R_2 = -0.1 \\ C_1 R_1 = \dfrac{1}{p} \ \text{ or } \ \dfrac{1}{125} \\ C_2 R_2 = \dfrac{1}{125} \ \text{ or } \ \dfrac{1}{p} \end{cases}$$

Since $C_1 = 5\ \mu\text{F}$, $R_1 = 8\ \text{k}\Omega$ and $R_2 = 20\ \text{k}\Omega$

$$C_1 R_1 = \left(5\times10^{-6}\right)\left(8\times10^{3}\right) = \frac{40}{1000} = \frac{1}{25} \neq \frac{1}{125} \quad \Rightarrow \quad p = 25\ \text{rad/s}$$

$$\frac{1}{125} = C_2 R_2 \quad \Rightarrow \quad C_2 = \frac{1}{125 R_2} = \frac{1}{125\left(20\times10^{3}\right)} = 0.4\times10^{-6} = 0.4\ \ \mu\text{F}$$

20. The network function of this circuit is:



$$\mathbf{H}(\omega) = \frac{\mathbf{V}_\text{o}(\omega)}{\mathbf{V}_\text{s}(\omega)} = (k)\frac{j\omega}{1+j\dfrac{\omega}{p}}$$

Determine the values of k and p:

$k =$ ___0.08___ , and $p =$ ___10____rad/s **.**

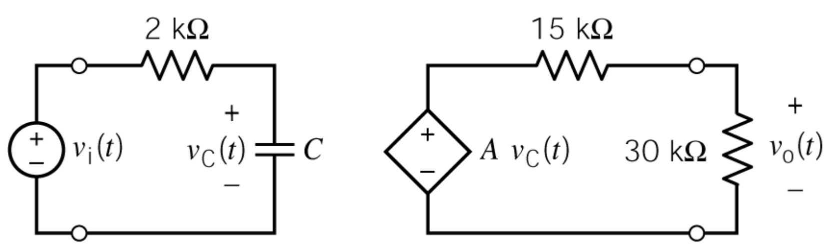
$$\mathbf{V}_\text{o}(\omega) = \left(\frac{R_2}{R_1+R_2+\dfrac{1}{j\omega C}}\right)\mathbf{V}_\text{s}(\omega) \quad \Rightarrow \quad \mathbf{H}(\omega) = \frac{\mathbf{V}_\text{o}(\omega)}{\mathbf{V}_\text{s}(\omega)} = \frac{R_2}{R_1+R_2+\dfrac{1}{j\omega C}} = \frac{j\omega C R_2}{1+j\omega C\left(R_1+R_2\right)} = (k)\frac{}{1\text{-}}$$

Consequently

$$k = C R_2 = (0.002)(40) = 0.08\ \text{s} \quad \text{and} \quad p = \frac{1}{C\left(R_1+R_2\right)} = \frac{1}{(0.002)(10+40)} = 10\ \text{rad/s}$$

21. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The network function of this circuit is

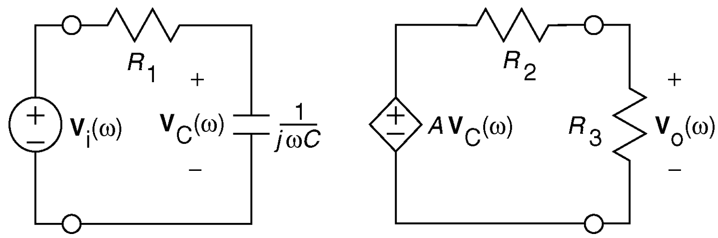
$$\mathbf{H}(\omega) = \frac{\mathbf{V}_\mathbf{o}(\omega)}{\mathbf{V}_\mathbf{i}(\omega)} = \frac{4}{1+j\dfrac{\omega}{100}}$$



Determine the values of the capacitance, C, and the VCVS gain, A.

$C =$ _____5____ μF and $A =$ ___6____ V/V .

In the frequency domain, use voltage division on the left side of the circuit to get:



$$\mathbf{V}_C(\omega)=\frac{\frac{1}{j\omega C}}{R_1+\frac{1}{j\omega C}}\mathbf{V}_i(\omega)=\frac{1}{1+j\omega C R_1}\mathbf{V}_i(\omega)$$

Next, use voltage division on the right side of the circuit to get:

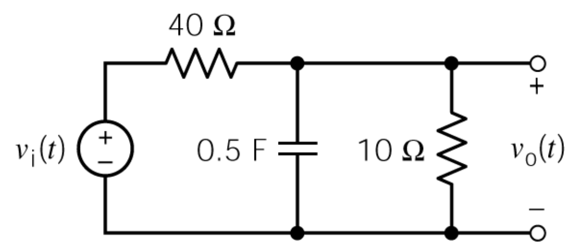
$$\mathbf{V}_o(\omega)=\frac{R_3}{R_2+R_3}A\,\mathbf{V}_C(\omega)=\frac{2}{3}A\,\mathbf{V}_C(\omega)=\frac{\frac{2}{3}A}{1+j\omega C R_1}\mathbf{V}_i(\omega)$$

Compare the specified network function to the calculated network function:

$$\frac{4}{1+j\frac{\omega}{100}}=\frac{\frac{2}{3}A}{1+j\omega C R_1}=\frac{\frac{2}{3}A}{1+j\omega C\,2000}\quad\Rightarrow\quad 4=\frac{2}{3}A\ \text{ and }\ \frac{1}{100}=2000\,C$$

Thus, C = 5 μF and A = 6 V/V.

22. The network function of this circuit is:

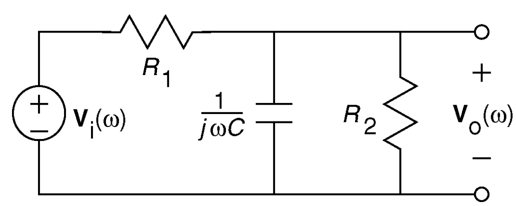


$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{k}{1+j\frac{\omega}{p}}$$

Determine the values of k and p:

k = ___0.2___ , and p = ___0.25____rad/s **.**

Represent the circuit in the frequency domain. It's convenient to calculate:



$$R_2 \,||\, \frac{1}{j\omega C}=\frac{R_2}{1+j\omega C\,R_2}$$

Then, using voltage division

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}=\frac{\frac{R_2}{1+j\omega C R_2}}{R_1+\frac{R_2}{1+j\omega C R_2}}=\frac{\frac{R_2}{R_1+R_2}}{1+j\omega C R_p}$$

where $R_p = R_1 \,||\, R_2$. When R_1 = 40 Ω, R_2 = 10 Ω and C = 0.5 F

$$\mathbf{H}(\omega)=\frac{0.2}{1+j4\omega}$$