EE 221 Practice Problems for the Final Exam

1. The network function of a circuit is

$$\mathbf{H}(\omega) = \frac{-12.5}{1 + j\dfrac{\omega}{500}}.$$

This table records frequency response data for this circuit. Fill in the blanks in the table:

ω, rad/s	*A*, V	θ, °
0	12.5	180
100	12.26	168.7
200	11.61	158.2
500	8.84	135
1000	5.59	116.6

$$\mathbf{H}(100) = \frac{-12.5}{1 + j\dfrac{100}{500}} = \frac{-12.5}{1 + j0.2} = 12.26\angle 168.7^\circ \quad \text{and} \quad \mathbf{H}(200) = \frac{-12.5}{1 + j\dfrac{200}{500}} = \frac{-12.5}{1 + j0.4} = 11.61\angle 158.2^\circ$$

2. The network function of a circuit is

$$\mathbf{H}(\omega) = \frac{-k}{1 + j\dfrac{\omega}{p}}.$$

This table records frequency response data for this circuit. Determine the values of p and k:

p = ___500___ rad/s and k = __12.5__ V/V

ω, rad/s	*A*, V	θ, °
0	12.5	180
100	12.26	168.7
200	11.61	158.2
500	8.84	135
1000	5.59	116.6

$$\frac{-k}{1 + j\dfrac{500}{p}} = \frac{k}{\sqrt{1 + \left(\dfrac{500}{p}\right)^2}} \angle \left(180 - \tan^{-1}\left(\frac{500}{p}\right)\right) = 8.84\angle 135^\circ$$

so

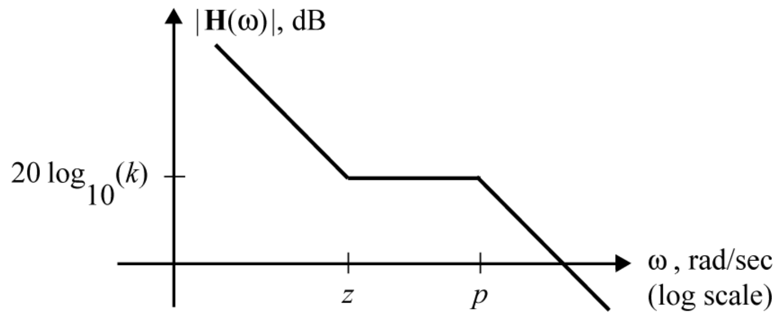
$$\tan^{-1}\left(\frac{500}{p}\right) = 45^\circ \quad \Rightarrow \quad \frac{500}{p} = \tan(45^\circ) = 1 \quad \Rightarrow \quad p = 500 \text{ rad/s}$$

and

$$\frac{k}{\sqrt{1 + \left(\dfrac{500}{p}\right)^2}} = \frac{k}{\sqrt{1 + (1)^2}} = \frac{k}{\sqrt{2}} = 8.84 \quad \Rightarrow \quad k = 12.5$$

3. Here's a network function and corresponding magnitude Bode plot:

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{50+\dfrac{1600}{j\omega}}{640+j4\omega}$$



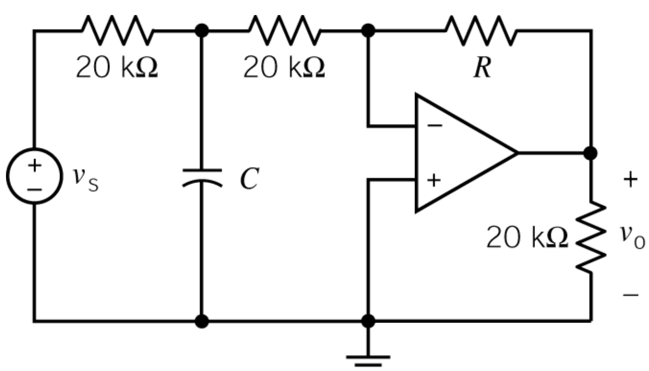
Determine the values of the constants *k*, *z* and *p* used to label the Bode plot:

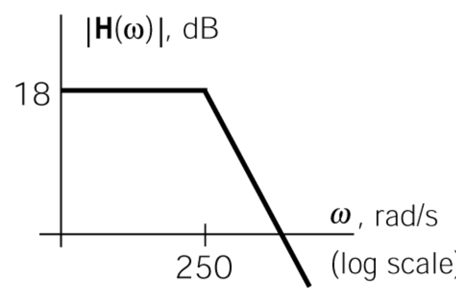
k = ___0.078125___, z = ___32___rad/s and p = __160___rad/s.

$$\mathbf{H}(\omega)=\frac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}=\frac{50+\dfrac{1600}{j\omega}}{640+j4\omega}=\frac{50\,j\omega+1600}{(j\omega)(640+j4\omega)}=\left(\frac{1600}{640}\right)\frac{1+j\omega\dfrac{50}{1600}}{(j\omega)\left(1+j\omega\dfrac{4}{640}\right)}=(2.5)\frac{1+j\dfrac{\omega}{32}}{(j\omega)\left(1+j\dfrac{\omega}{160}\right)}$$

When $z<\omega<p$, $\left|\mathbf{H}(\omega)\right|=\left(\frac{1600}{640}\right)\frac{\left|1+j\omega\frac{50}{1600}\right|}{\left|j\omega\right|\left|1+j\omega\frac{4}{640}\right|}\cong\left(\frac{1600}{640}\right)\frac{\left|j\omega\frac{50}{1600}\right|}{\left|j\omega\right|\left|1\right|}=\frac{50}{640}=0.078125$

4. Here's a circuit and corresponding Bode plot. The network function of this circuit is $\mathbf{H}(\omega)=\dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_s(\omega)}$.





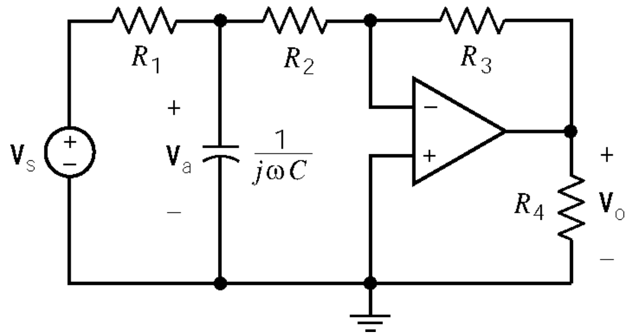
Determine the values of the resistance, R and capacitance, C :

R = ___320___ kΩ and C = ___0.4___ μF

From Figure P13.3-25*b*, $\mathbf{H}(\omega)$ has a pole at 500 rad/s and a low frequency gain of 18 dB = 8. Consequently, the network function corresponding to the Bode plot is

$$\mathbf{H}(\omega)=\frac{\pm 8}{\left(1+j\dfrac{\omega}{250}\right)}$$

Next, we find the network function corresponding to the circuit. Represent the circuit in the frequency domain.



The node equations are

$$\frac{\mathbf{V}_a - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}_a}{\frac{1}{j\omega C}} + \frac{\mathbf{V}_a}{R_2} = 0 \quad \Rightarrow \quad \mathbf{V}_a = \frac{R_2}{R_1 + R_2 + j\omega C\, R_1\, R_2}\mathbf{V}_s$$

and

$$\frac{\mathbf{V}_a}{R_2} + \frac{\mathbf{V}_o}{R_3} = 0 \quad \Rightarrow \quad \mathbf{V}_o = -\frac{R_3}{R_2}\mathbf{V}_a$$

The network function is

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\frac{R_3}{R_2}R_2}{R_1 + R_2 + j\omega C\, R_1\, R_2} = \frac{-\frac{R_3}{R_1 + R_2}}{1 + j\omega C\frac{R_1\, R_2}{R_1 + R_2}}$$

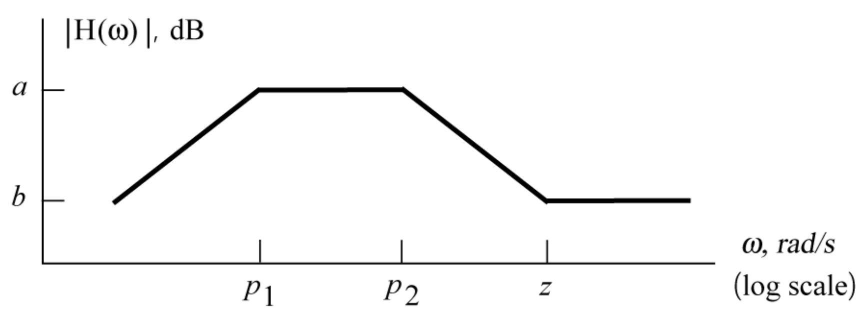
Using the given values for R_1 and R_2 and letting $R_3 = R$ gives

$$\mathbf{H} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{-\frac{R}{4\times 10^4}}{1 + j\omega C\left(10^4\right)}$$

Comparing this network function to the specified network function gives

$$C\left(10^4\right) = \frac{1}{250} \quad \Rightarrow \quad C = 0.4\ \mu\text{F} \text{ and } \frac{R}{4\times 10^4} = 8 \quad \Rightarrow \quad R = 320\ \text{k}\Omega$$

5. Here's a magnitude Bode plot and corresponding network function:



$$\mathbf{H}(\omega) = \frac{j\frac{\omega}{4}\left(100 + j\frac{\omega}{4}\right)}{\left(1 + j\frac{\omega}{4}\right)\left(5 + j\frac{\omega}{8}\right)}$$

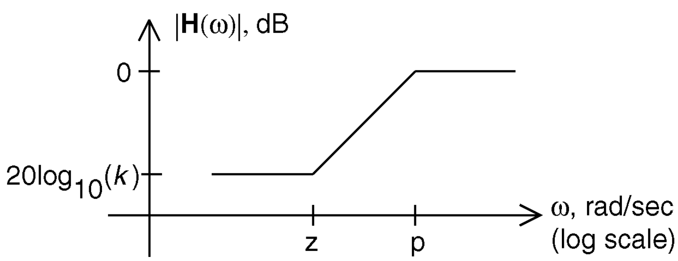
Determine the values of the constants a, b, p_1, p_2 and z used to label the Bode plot:

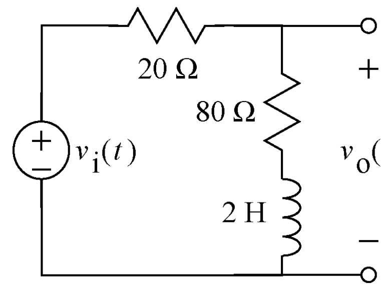
a = __26__dB, b = __6__dB, p_1 = __4__rad/s, p_2 = __40__rad/s and z = __400__rad/s.

$$\mathbf{H}(\omega)=\frac{j\frac{\omega}{4}\left(100+j\frac{\omega}{4}\right)}{\left(1+j\frac{\omega}{4}\right)\left(5+j\frac{\omega}{8}\right)}=\frac{5(j\omega)\left(1+j\frac{\omega}{400}\right)}{\left(1+j\frac{\omega}{4}\right)\left(1+j\frac{\omega}{40}\right)}\quad\Rightarrow\quad\left|\mathbf{H}(\omega)\right|\simeq\begin{cases}5\omega & \text{when } \omega\le 4\\ 20 & \text{when } 4\le\omega\le 40\\ \dfrac{800}{\omega} & \text{when } 40\le\omega\le 400\\ 2 & \text{when } 400\le\omega\end{cases}$$

$$a=20\log_{10}(20)=26 \text{ db} \quad \text{and} \quad b=20\log_{10}(2)=6 \text{ db}$$

6. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. $\mathbf{H}(\omega)=\dfrac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)}$ is the network function. The magnitude bode plot that represents this circuit is



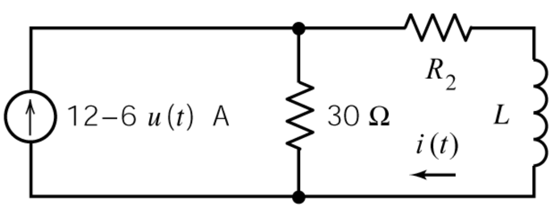


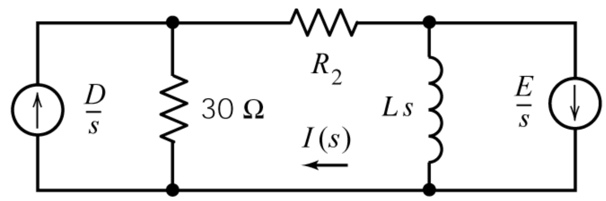
The values of the corner frequencies are z = _____40_______ rad/sec and p = ______50______ rad/sec.

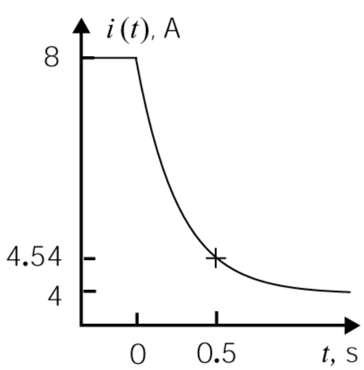
The value of the low frequency gain is k = _______0.8_____ V/V.

$$\mathbf{V_o}(\omega)=\left(\frac{80+j2\omega}{20+80+j2\omega}\right)\mathbf{V_i}(\omega)\quad\Rightarrow\quad\mathbf{H}(\omega)=\frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)}=(0.8)\frac{1+j\frac{\omega}{40}}{1+j\frac{\omega}{50}}$$

7. Here is the same circuit represented in the time domain and also in the complex frequency domain.







Here's a plot of the inductor current. Determine the values of D and E used to represent the circuit in the complex frequency domain:

D = ___6___V and E = ___8___ V

Determine the values of the resistance R_2 and the inductance L:

R_2 = ___15___Ω and L = ___11.25___ H

a) $\mathcal{L}[12-6u(t)]=\mathcal{L}[6u(t)]=\frac{6}{s} \Rightarrow D=6$ A.

b) E is the initial inductor current = 8 A from the plot.

c) The circuit is at steady state before $t = 0$, so the inductor acts like a short circuit. Using current division, $8=\left(\frac{30}{30+R_2}\right)12 \Rightarrow R_2=15\ \Omega$. Similarly, the circuit will at steady state for $t \to \infty$.

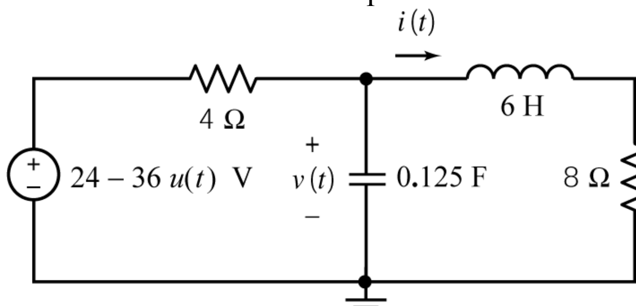
Again, the inductor acts like a short circuit. Using current division, $4=\left(\frac{30}{30+R_2}\right)6 \Rightarrow R_2=15\ \Omega$.

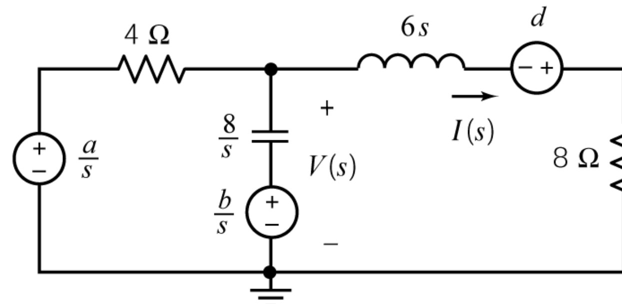
d) The inductor current can be represented as $v(t)=4+4e^{-at}$ for $t \geq 0$. From the plot,

$4.54=4+4e^{-a(0.5)}$ so $a=\frac{\ln\left(\frac{4.54-4}{4}\right)}{-0.5}=4.005\cong 4\ \ 1/\text{s}$. Then

$\frac{1}{4}=\tau=\frac{L}{15+30} \Rightarrow L=\frac{45}{4}=11.25\text{ H}$.

8. Here is the same circuit represented in the time domain and also in the complex frequency domain.



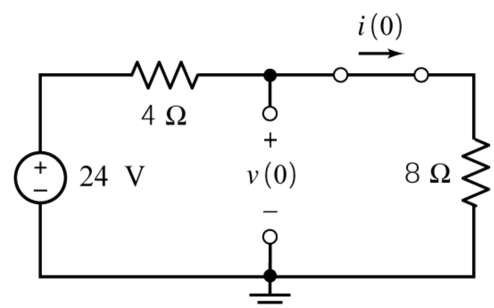


Determine the values of *a, b* and *d* used to represent the circuit in the complex frequency domain:

a = ___–12___ b = ___16___ and d = ___12___

$24-36u(t)=-12$ for $t>0$ $\mathcal{L}[-12]=\frac{-12}{s} \Rightarrow a=-12$ V.

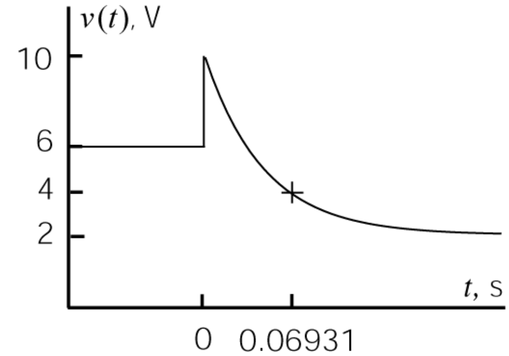
The circuit is at steady state before $t = 0$, and the input is constant, so the capacitor acts like an open circuit and the inductor acts like a short circuit.



$v(0)=\left(\frac{8}{4+8}\right)24=16$ V and $i(0)=\frac{24}{4+8}=2$ A

Consequently, $b=v(0)=16$ V and $d=L\,i(0)=(6)(2)=12$.

9. Given that $\mathcal{L}\left[v(t)\right]=\dfrac{a\,s+b}{2\,s^2+40\,s}$ where $v(t)$ is the voltage shown to the right, determine the values of *a* and *b*.



a = __20___V and b = __80___ V

From the plot, $v(0+) = 10$ V and $\lim_{t\to\infty} v(t) = 2$ V. From the final value theorem,

$$\lim_{t\to\infty} v(t) = \lim_{s\to 0} sV(s) = \lim_{s\to 0} s\frac{a\,s+b}{2\,s^2+40\,s} = \lim_{s\to 0}\frac{a\,s+b}{2\,s+40} = \frac{b}{40}.$$

Consequently, $2=\dfrac{b}{40} \quad\Rightarrow\quad b=80$. From the initial value theorem

$$\lim_{t\to 0+} v(t) = \lim_{s\to\infty} sV(s) = \lim_{s\to\infty} s\frac{a\,s+b}{2\,s^2+40\,s} = \lim_{s\to\infty}\frac{a\,s+b}{2\,s+40} = \frac{a}{2}$$

Consequently $10=\dfrac{a}{2} \quad\Rightarrow\quad a=20$.

10. The Laplace transform of a voltage $v(t)=\left[b\,e^{-at}\sin(c\,t)\right]u(t)$ is $V(s)=\dfrac{80}{s^2+8\,s+25}$. Determine the values of the constant coefficients *a*, *b*, and *c*:

a = ___4___1/s, b = ___26.67___V, and c = ___3___V.

The step response is given by

$$v(t)=\mathcal{L}^{-1}\left[\frac{80}{s^2+8\,s+25}\right]=\mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]=\frac{80}{3}e^{-4t}\sin(3t)u(t)\ \text{ V}$$

11. The Laplace transform of a voltage $v(t)=\left[b-e^{-at}(c+d\,t)\right]u(t)$ is $V(s)=\dfrac{12}{s\left(s^2+8s+16\right)}$. Determine the values of the constant coefficients *a*, *b*, *c* and *d*:

a = ___4__1/s, b = ___0.75___V, c = ___0.75___V and d = ___3___V.

$$V(s)=\frac{12}{s\left(s^2+8s+16\right)}=\frac{12}{s(s+4)^2}=\frac{\frac{3}{4}}{s}+\frac{-3}{(s+4)^2}+\frac{k}{s+4}$$

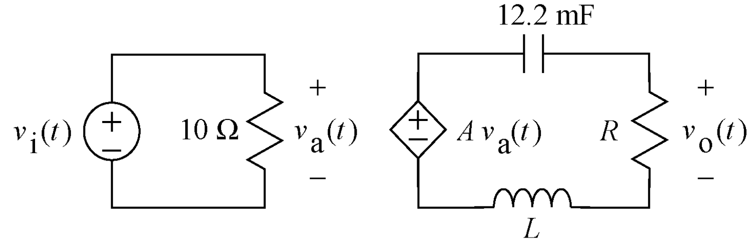
The constant *k* is evaluated by multiplying both sides of the last equation by $s(s+4)^2$.

$$12=\frac{3}{4}(s+4)^2-3s+ks(s+4)=\left(\frac{3}{4}+k\right)s^2+(3+4k)s+12 \quad\Rightarrow\quad k=-\frac{3}{4}$$

Finally

$$v(t)=\mathfrak{L}^{-1}\left[\frac{\frac{3}{4}}{s}+\frac{-3}{(s+4)^2}+\frac{k}{s+4}\right]=\left(\frac{3}{4}-e^{-4t}\left(3t+\frac{3}{4}\right)\right)u(t)\ \text{V}$$

12. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage $v_o(t)$. The step response is $v_o(t)=6e^{-4t}\sin(5t)u(t)$.



Determine the values of the gain, *A*, of the VCVS, the resistance, *R*, and the inductance, *L*.

$A=$ ______3.75______ V/V, $R=$ _____16_____ Ω and $L=$ ______2______ H .

Equating the transfer function of the circuit to Laplace transform of the given step response yields:

$$\frac{\frac{AR}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}=\frac{6(5)s}{(s+4)^2+25}=\frac{30s}{s^2+8s+41}$$

Equating coefficients:

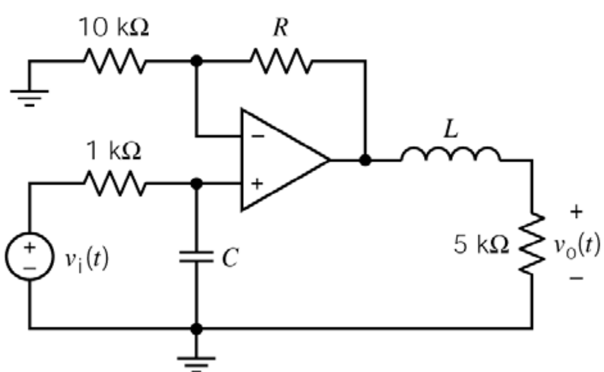
$$\frac{1}{LC}=\frac{1}{L(0.0122)}=41 \quad\Rightarrow\quad L=2\ \text{H},$$

$$\frac{R}{L}=\frac{R}{2}=8 \quad\Rightarrow\quad R=16\ \Omega \quad \text{and} \quad \frac{AR}{L}=\frac{A16}{2}=30 \quad\Rightarrow\quad A=3.75\ \text{V/V}$$

13. The input to this circuit is the voltage source voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. The transfer function of this circuit is

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{15\times10^6}{(s+2000)(s+5000)}$$

Determine the values of *R*, *L* and *C*:

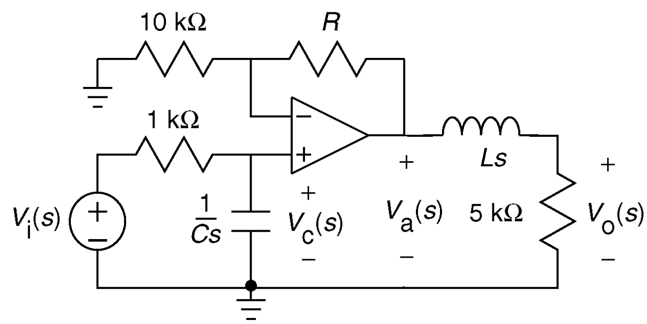


$R=$ ___5___kΩ, $L=$ ___1___H and $C=$ ___0.5___**μF**.

or

$R=$ ___5___kΩ, $L=$ __2.5__H and $C=$ ___0.2___**μF**.

The transfer function can also be calculated from the circuit itself. The circuit can be represented in the frequency domain as



We can save ourselves some work be noticing that the 10000 ohm resistor, the resistor labeled R and the op amp comprise a non-inverting amplifier. Thus

$$V_a(s) = \left(1 + \frac{R}{10000}\right) V_c(s)$$

Now, writing node equations,

$$\frac{V_c(s) - V_i(s)}{1000} + CsV_c(s) = 0 \quad \text{and} \quad \frac{V_o(s) - V_a(s)}{Ls} + \frac{V_o(s)}{5000} = 0$$

Solving these node equations gives

$$H(s) = \frac{\frac{1}{1000\,C}\left(1 + \frac{R}{10000}\right)\frac{5000}{L}}{\left(s + \frac{1}{1000C}\right)\left(s + \frac{5000}{L}\right)}$$

Comparing these two equations for the transfer function gives

$$\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{1}{1000C}\right) = (s + 5000)$$

$$\left(s + \frac{5000}{L}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{5000}{L}\right) = (s + 5000)$$

$$\frac{1}{1000C}\left(1 + \frac{R}{10000}\right)\frac{5000}{L} = 15 \times 10^6$$

The solution isn't unique, but there are only two possibilities. One of these possibilities is

$$\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \Rightarrow \quad C = 0.5\,\mu\text{F}$$

$$\left(s + \frac{5000}{L}\right) = (s + 5000) \quad \Rightarrow \quad L = 1\text{ H}$$

$$\frac{1}{1000\left(0.5 \times 10^6\right)}\left(1 + \frac{R}{10000}\right)\frac{5000}{1} = 15 \times 10^6 \quad \Rightarrow \quad R = 5\text{ k}\Omega$$

The other is

$$\left(s+\frac{1}{1000C}\right)=(s+5000) \quad\Rightarrow\quad C=0.2\ \mu\text{F}$$

$$\left(s+\frac{5000}{L}\right)=(s+2000) \quad\Rightarrow\quad L=2.5\text{ H}$$

$$\frac{1}{1000\left(0.2\times10^{6}\right)}\left(1+\frac{R}{10000}\right)\frac{5000}{2.5}=15\times10^{6} \quad\Rightarrow\quad R=5\text{ k}\Omega$$

14. The transfer function of a circuit is $H(s)=\dfrac{12}{s^2+8s+16}$. The step response of this circuit is: $step\ \ response=\left[b-e^{-at}(c+d\,t)\right]u(t)$. Determine the values of the constant coefficients *a*, *b*, *c* and *d*:

a = ___4__1/s, *b* = ___0.75___V, *c* = ___0.75___V and *d* = ___3___V.

The Laplace transform of the step response is:

$$\frac{H(s)}{s}=\frac{12}{s\left(s^2+8s+16\right)}=\frac{12}{s(s+4)^2}=\frac{\frac{3}{4}}{s}+\frac{-3}{(s+4)^2}+\frac{k}{s+4}$$

The constant *k* is evaluated by multiplying both sides of the last equation by $s(s+4)^2$.

$$12=\frac{3}{4}(s+4)^2-3s+ks(s+4)=\left(\frac{3}{4}+k\right)s^2+(3+4k)s+12 \quad\Rightarrow\quad k=-\frac{3}{4}$$

The step response is

$$\mathfrak{L}^{-1}\left[\frac{H(s)}{s}\right]=\left(\frac{3}{4}-e^{-4t}\left(3t+\frac{3}{4}\right)\right)u(t)\ \text{ V}$$

15. The transfer function of a circuit is $H(s)=\dfrac{80\,s}{s^2+8\,s+25}$. The step response of this circuit is: $step\ \ response=\left[b\,e^{-at}\sin(c\,t)\right]u(t)$. Determine the values of the constant coefficients *a*, *b*, *c* and *d*:

a = ___4___1/s, *b* = ___26.67___V, and *c* = ___3___V.

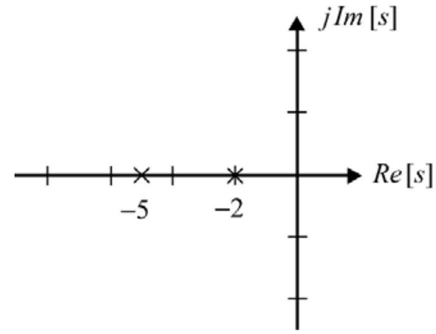
The step response is given by

$$v_o(t)=\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right]=\mathcal{L}^{-1}\left[\frac{80\,s}{s\left(s^2+8\,s+25\right)}\right]=\mathcal{L}^{-1}\left[\frac{80}{s^2+8\,s+25}\right]=\mathcal{L}^{-1}\left[\frac{80}{3}\times\frac{3}{(s+4)^2+3^2}\right]$$

$$=\frac{80}{3}e^{-4t}\sin(3t)u(t)\ \text{ V}$$

16. The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The transfer function of the circuit is

$$H(s) = \frac{V_o(s)}{V_i(s)}$$



The poles and zeros of $H(s)$ are shown on this pole-zero diagram. (There are no zeros.) The dc gain of the circuit is

$$\mathbf{H}(0) = 5$$

The step response of the circuit is $v_o(t) = \left(a + b e^{-5t} - c e^{-2t}\right) u(t)$ V. Determine the values of the constants *a*, *b* and *c*.

a = _____5____ V, b = ___10/3__ V and c = ___25/3__ V.

The transfer function of the circuit is

$$H(s) = \frac{a}{(s+2)(s+5)}$$

where *a* is a constant to be determined. The circuit is stable because all its poles lie in the right half of the *s*-plane. Consequently,

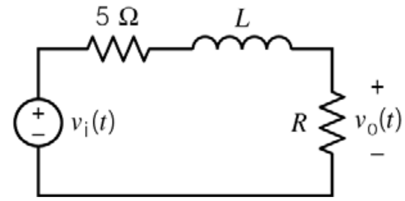
$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{a}{(2+j\omega)(5+j\omega)} = \frac{\frac{a}{10}}{\left(1+j\frac{\omega}{2}\right)\left(1+j\frac{\omega}{5}\right)}$$

At dc ($\omega = 0$) $\quad 5 = \mathbf{H}(0) = \frac{a}{10} \quad \Rightarrow \quad a = 50$

The step response is given by

$$v_o(t) = \mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{50}{s(s+2)(s+5)}\right] = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{\frac{10}{3}}{s+5} - \frac{\frac{25}{3}}{s+2}\right] = \left(5 + \frac{10}{3}e^{-5t} - \frac{25}{3}e^{-2t}\right)u(t) \text{ V}$$

17. The input to a circuit is the voltage source voltage, v_{i}. The step response of the circuit is



$$v_{\text{o}}(t) = \frac{3}{4}\left(1 - e^{-100t}\right)u(t) \text{ V}$$

Determine the value of the inductance, *L*, and of the resistance, *R*

$R =$ ___15___ Ω and $L =$ ___0.2___ H.

From the step response: $\dfrac{H(s)}{s} = \mathcal{L}\left[\frac{3}{4}\left(1 - e^{-100t}\right)u(t)\right] = \dfrac{75}{s(s+100)}$

From the circuit $H(s) = \dfrac{R}{R+5+Ls} \Rightarrow \dfrac{H(s)}{s} = \dfrac{\frac{R}{L}}{s\left(s + \frac{R+5}{L}\right)}$

Comparing gives

$$\left.\begin{aligned}\frac{R}{L} &= 75 \\ \frac{R+5}{L} &= 100\end{aligned}\right\} \Rightarrow \begin{aligned}R &= 15\ \Omega \\ L &= 0.2\text{ H}\end{aligned}$$

18. The input to a circuit is the voltage source voltage, v_{i}. The step response of the circuit is

$$v_{\text{o}}(t) = 5\left(1 - (1+2t)e^{-2t}\right)u(t) \text{ V}$$

When the input is $v_{\text{i}}(t) = 5\cos(2t + 45°)$ V

the steady-state response is $v_{\text{i}}(t) = A\cos(2t + \theta)$ V

Determine the values of *A* and *θ*.

$A =$ ___12.5__ V and $\theta =$ ___–45___°.

The transfer function of this circuit is given by

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(5 - 5e^{-2t}(1+2t)\right)u(t)\right] = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2} = \frac{20}{(s+2)^2} \Rightarrow H(s) = \frac{20}{(s+2)^2}$$

This transfer function is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \left.\frac{20}{(s+2)^2}\right|_{s=j\omega} = \frac{20}{(2+j\omega)^2}$$

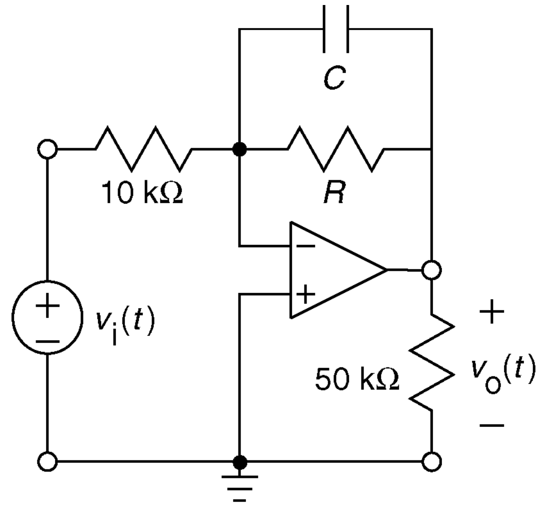
The phasor of the output is

$$\mathbf{V}_o(\omega) = \frac{20}{(2+j2)^2}(5\angle 45°) = \frac{20}{\left(2\sqrt{2}\angle 45°\right)^2}(5\angle 45°) = 12.5\angle -45° \text{ V}$$

The steady-state response is

$$v_o(t) = 12.5\cos(2t - 45°) \text{ V}$$

19. The input to a circuit is the voltage $v_i(t)$. The output is the voltage $v_o(t)$.



When the input is:

$$v_i(t) = 2 + 4\cos(100t) + 5\cos(200t + 45°) \quad \text{V}$$

the corresponding output is:

$$v_o(t) = -5 + 7.071\cos(100t + 135°) + c_2\cos(200t + \theta_2) \quad \text{V}$$

Determine the value of R, C, c_2, and θ_2:

$R =$ ___25___ kΩ , $C =$ ___0.4___ μF, $c_2 =$ __5.59__V and $\theta_2 =$ __161.6__°

The network function of the circuit is

$$\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = -\frac{\frac{R}{1+j\omega RC}}{10^4} = -\frac{\frac{R}{10^4}}{1+j\omega RC}$$

At dc:
$$\frac{-5}{2} = -\frac{R}{10^4} \quad \Rightarrow \quad R = 25 \text{ k}\Omega$$

At ω = 100 rad/s
$$-\frac{2.5}{1+j(100)(25\times 10^3)C} = \frac{7.071\angle 135°}{4\angle 0°}$$

$$180 - \tan^{-1}\left((100)(25\times 10^3)C\right) = 135° \quad \Rightarrow \quad C = \frac{\tan(45°)}{(100)(25\times 10^3)} = 0.4\ \mu\text{F}$$

Finally, at ω = 200 rad/s
$$-\frac{2.5}{1+j(200)(25\times 10^3)\left(0.4\times 10^{-6}\right)} = -\frac{2.5}{1+j2} = 1.118\angle 116.6°$$

so
$$c_2 = (1.118)(5) = 5.59 \text{ and } \theta_2 = 45° + 116.6° = 161.6°$$

20. The transfer function of a circuit is $H(s)=\dfrac{20}{s+8}$. When the input to this circuit is sinusoidal, the output is also sinusoidal. Let ω_1 be the frequency at which the output sinusoid is twice as large as the input sinusoid and let ω_2 be the frequency at which output sinusoid is delayed by one tenth period with respect to the input sinusoid. Determine the values of ω_1 and ω_2.

ω_1 = _____6____ rad/s and ω_2 = ___5.8123___ rad/s

The circuit is stable so $\mathbf{H}(\omega)=H(s)\big|_{s\leftarrow j\omega}=\dfrac{20}{8+j\omega}$.

The gain is 2 at the frequency ω_1 so $2=\dfrac{20}{\sqrt{8^2+\omega_1^{\,2}}}$ and $\omega_1=\sqrt{\left(\dfrac{20}{2}\right)^2-8^2}=6$ rad/s.

When the frequency is ω_2, the period is $\dfrac{2\pi}{\omega_2}$. Also a delay t_o corresponds to a phase shift $-\omega_2 t_o$. In this case, $t_0=0.1\left(\dfrac{2\pi}{\omega_2}\right)$ so the phase shift is -0.2 π. Then $-0.2\pi=-\tan^{-1}\left(\dfrac{\omega_2}{8}\right)$ so

$\omega_2=8\tan(0.2\pi)=5.8123$ rad/s.