Magnitude Bode Plots

Consider

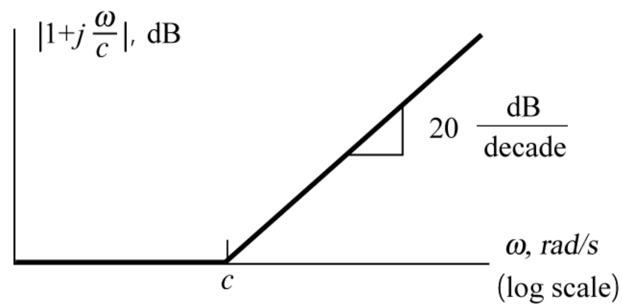
$$H(\omega) = k(j\omega)^n \frac{\prod_i \left(1 + j\frac{\omega}{z_i}\right)}{\prod_i \left(1 + j\frac{\omega}{p_i}\right)}$$

- For convenience, assume that *k*, all z_i and all p_i are positive real numbers.
- The exponent *n* is an integer which indicates the number of poles or zeros at the origin according to

$$n \geq 1 \quad \Rightarrow \quad n \text{ zeros at 0 rad/s}$$
$$n \leq -1 \quad \Rightarrow \quad |n| \text{ poles at 0 rad/s}$$
$$n = 0 \quad \Rightarrow \quad \text{no poles or zeros at 0 rad/s}$$

- The z_i are zeros, the p_i are poles and the z_i together with the p_i are corner frequencies. All have units of rad/s.

- $$\left|1 + j\frac{\omega}{c}\right| = \sqrt{1 + \left(\frac{\omega}{c}\right)^2} \simeq \begin{cases} 1 & \text{when } \omega < c \\ \frac{\omega}{c} & \text{when } \omega > c \end{cases}$$



- Let N be the number of corner frequencies. Arrange the corner frequencies in ascending order: $c_1,\ c_2,\ c_3,\ ..., c_N$. Doing so identifies *N*+1 frequency intervals:

$$\omega \leq c_1, \quad c_1 \leq \omega \leq c_2, \quad c_2 \leq \omega \leq c_3, \quad ..., \quad c_{N-1} \leq \omega \leq c_N$$

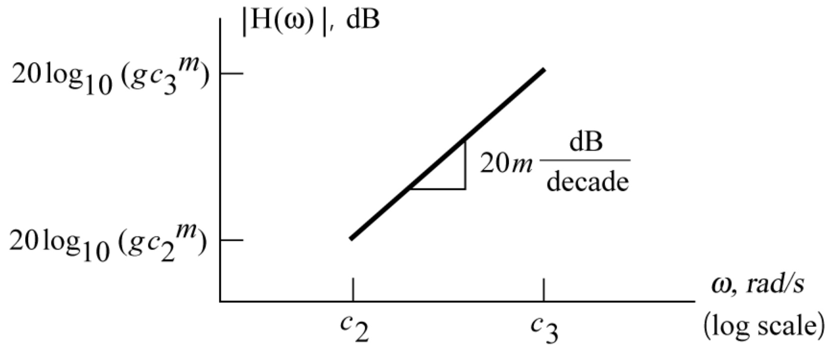
- The magnitude Bode plot of $H(\omega)$ is an approximate gain frequency response that consists of *N*+1 straight line segments – corresponding to the *N*+1 frequency intervals.

- For example, the 3rd line segment corresponds to $c_2 \leq \omega \leq c_3$. Pick a frequency ω_3 between c_2 and c_3. That is $c_2 < \omega_3 < c_3$ To obtain the equation of this straight line segment, replace each $\left|1 + j\frac{\omega}{c_i}\right|$ in $|H(\omega)|$ with 1 if $1 > \frac{\omega_3}{c_i}$ and with $\frac{\omega}{c_i}$ if $\frac{\omega_3}{c_i} > 1$. Represent the result as $g\,\omega^m$. That is

$$|H(\omega)| \simeq g\,\omega^m \quad \text{when } c_2 \le \omega \le c_3$$

Then

$$20\log_{10}|H(\omega)| \simeq 20\log_{10} g + m \times 20\log_{10}\omega \quad \text{when } c_2 \le \omega \le c_3$$



- Notice that the expression $g\,\omega^m$ gives the values of the end points of this line segment and that the exponent *m* gives the slope of the line segment.

Example:

$$H(\omega) = \frac{5(j\omega)\left(1+j\frac{\omega}{400}\right)}{\left(1+j\frac{\omega}{4}\right)\left(1+j\frac{\omega}{40}\right)} \Rightarrow |H(\omega)| \simeq \begin{cases} 5\,\omega^1 & \text{when } \omega \le 4 \\ 20\,\omega^0 & \text{when } 4 \le \omega \le 40 \\ 800\,\omega^{-1} & \text{when } 40 \le \omega \le 400 \\ 2\,\omega^0 & \text{when } 400 \le \omega \end{cases}$$

Notice that

- The expression representing each segment does indeed have the form $g\,\omega^m$
- $5\omega\big|_{\omega=4} = 20 = \frac{800}{\omega}\Big|_{\omega=40}$ and $\frac{800}{\omega}\Big|_{\omega=400} = 2$.
- The slopes of the 4 line segments are 1, 0, -1 and 0 multiplied by 20 dB/decade.

The magnitude Bode plot is: