Example 1: Figure 8-N1*a* shows a plot of the voltage across the inductor in Figure 8-N1*b*.

a) Determine the equation that represents the inductor voltage as a function of time.
b) Determine the value of the resistance *R*.
c) Determine the equation that represents the inductor current as a function of time.

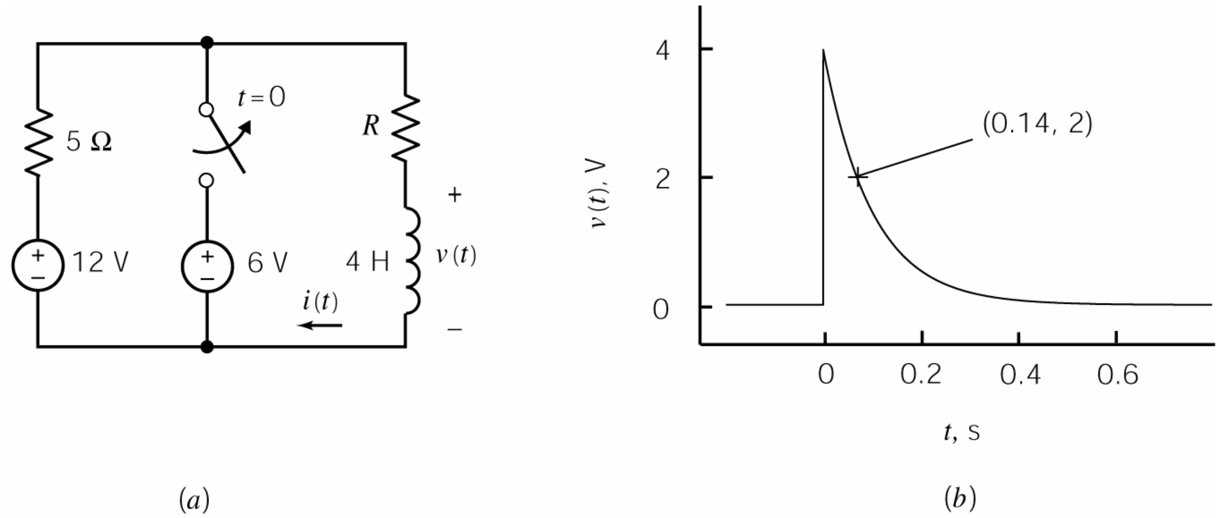

Figure 8-N1

Solution:

Part a)

The inductor voltage is represented by an equation of the form $v(t)=\begin{cases} D & \text{for } t<0 \\ E+F\,e^{-at} & \text{for } t\ge 0\end{cases}$

where *D*, *E*, *F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D=v(t)\ \text{ when } t<0,\quad E=\lim_{t\to\infty} v(t),\quad E+F=\lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D=0,\ E=0,\ \text{and}\ E+F=4\ \text{V}$$

Consequently,

$$v(t)=\begin{cases} 0 & \text{for } t<0 \\ 4\,e^{-at} & \text{for } t\ge 0\end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot in Figure 8-N1. We see $v(0.14)=2\ \text{V}$, that is, the value of the voltage is 2 volts at time 0.14 seconds. Substituting these into the equation for $v(t)$ gives

$$2 = 4e^{-a(0.14)} \quad \Rightarrow \quad a = \frac{\ln(0.5)}{-0.14} = 5$$

Consequently

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 4\,e^{-5t} & \text{for } t \ge 0 \end{cases}$$

Part b)

Figure 8-N2a shows the circuit immediately after the switch opens. In Figure 8-N2b, the part of the circuit connected to the inductor has been replaced by its Thevenin equivalent circuit.

The time constant of the circuit is given by

$$\tau = \frac{L}{R_t} = \frac{4}{R+5}$$

Also, the time constant is related to the exponent in $v(t)$ by $-5t = -\frac{t}{\tau}$. Consequently

$$5 = \frac{1}{\tau} = \frac{R+5}{4} \quad \Rightarrow \quad R = 15\ \Omega$$

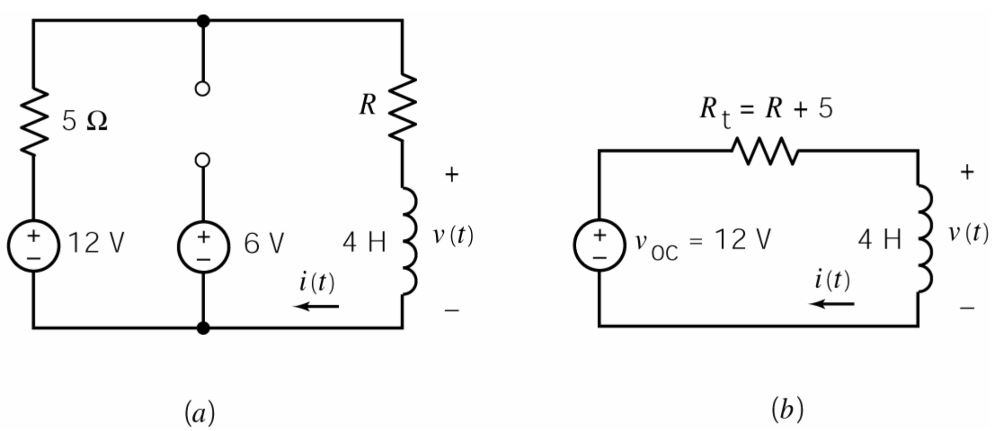

Figure 8-N2

Part c)

The inductor current is related to the inductor voltage by

$$i(t) = \frac{1}{L}\int_0^t v(\tau)\,d\tau + i(0)$$

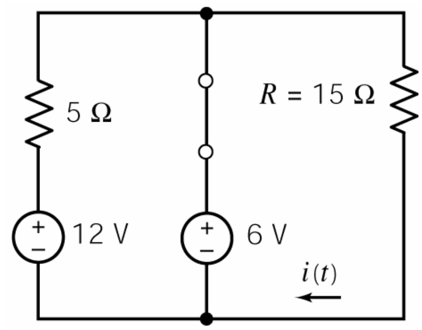

Figure 8-N3

Figure 8-N3 show the circuit before the switch opens. The closed switch is represented by a short circuit. The circuit is at steady state and the voltage sources have constant voltages so the inductor acts like a short circuit. The inductor current is given by

$$i(t) = \frac{6}{15} = 0.4 \text{ A}$$

In particular, $i(0-) = 0.4 \text{ A}$. The current in an inductor is continuous, so $i(0+) = i(0-)$. Consequently,

$$i(0) = 0.4 \text{ A}$$

Returning to the equation for the inductor current, we have

$$i(t) = \frac{1}{4}\int_0^t 4e^{-5\tau}\, d\tau + 0.4 = \frac{1}{-5}\left(e^{-5t} - 1\right) + 0.4 = 0.6 - 0.2\, e^{-5t}$$

In summary,

$$i(t) = \begin{cases} 0.4 & \text{for } t < 0 \\ 0.6 - 0.2\, e^{-5t} & \text{for } t \ge 0 \end{cases}$$

Example 2: Figure 8-N4*a* shows a plot of $v(t)$, the voltage across the 24 kΩ resistor in Figure 8-N4*b*.

a) Determine the equation that represents $v(t)$ as a function of time.
b) Determine the value of the capacitance *C*.

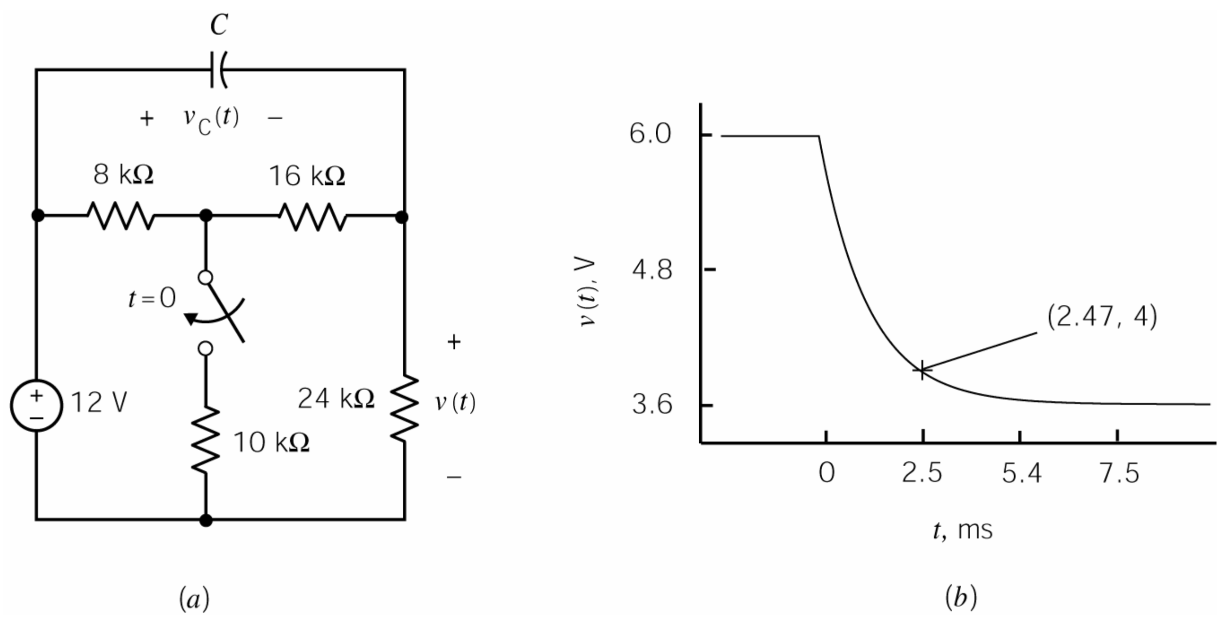

Figure 8-N4

Part a)

The voltage is represented by an equation of the form $v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\,e^{-at} & \text{for } t \ge 0 \end{cases}$

where *D*, *E*, *F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D = v(t) \ \text{ when } t < 0, \quad E = \lim_{t\to\infty} v(t), \quad E + F = \lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D = 6,\ \ E = 3.6,\ \text{ and }\ E + F = 6\ \text{ V}$$

Consequently,

$$v(t) = \begin{cases} 6 & \text{for } t < 0 \\ 3.6 + 24\,e^{-at} & \text{for } t \ge 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot in Figure 8-N4. We see $v(0.00247) = 4\ \text{V}$, that is, the value of the voltage is 2 volts at time 0.00247 seconds or 2.47 ms. Substituting these into the equation for $v(t)$ gives

$$4 = 3.6 + 2.4\, e^{-a(0.00247)} \quad \Rightarrow \quad a = \frac{\ln(0.1667)}{-0.00247} = 725$$

Consequently

$$v(t) = \begin{cases} 6 & \text{for } t < 0 \\ 3.6 + 2.4\, e^{-725t} & \text{for } t \ge 0 \end{cases}$$

Part b)

Figure 8-N5a shows the circuit immediately after the switch closes. In Figure 8-N5b, the part of the circuit connected to the capacitor has been replaced by its Thevenin equivalent circuit.

The time constant of the circuit is given by

$$\tau = R_t\, C = \left(11 \times 10^3\right) C$$

Also, the time constant is related to the exponent in $v(t)$ by $-725t = -\dfrac{t}{\tau}$. Consequently

$$\frac{1}{725} = \left(11 \times 10^3\right) C \quad \Rightarrow \quad C = \frac{1}{725\left(11 \times 10^3\right)} = 125 \times 10^{-9} = 125 \text{ nF}$$

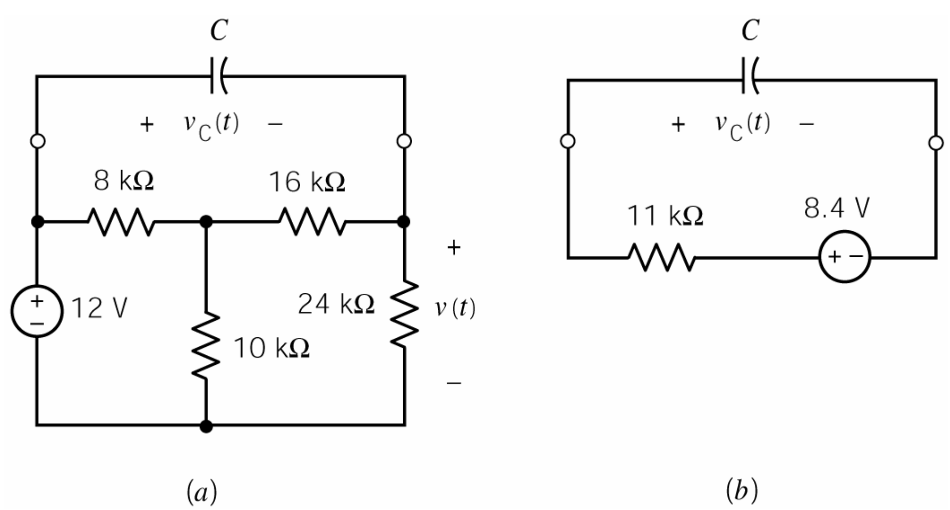

Figure 8-N5

Example 3: Figure 8-N6*a* shows a plot of $v(t)$, the voltage across one of the 5 Ω resistors in Figure 8-N6*b*.

c) Determine the equation that represents $v(t)$ as a function of time.
d) Determine the value of the capacitance *C*.

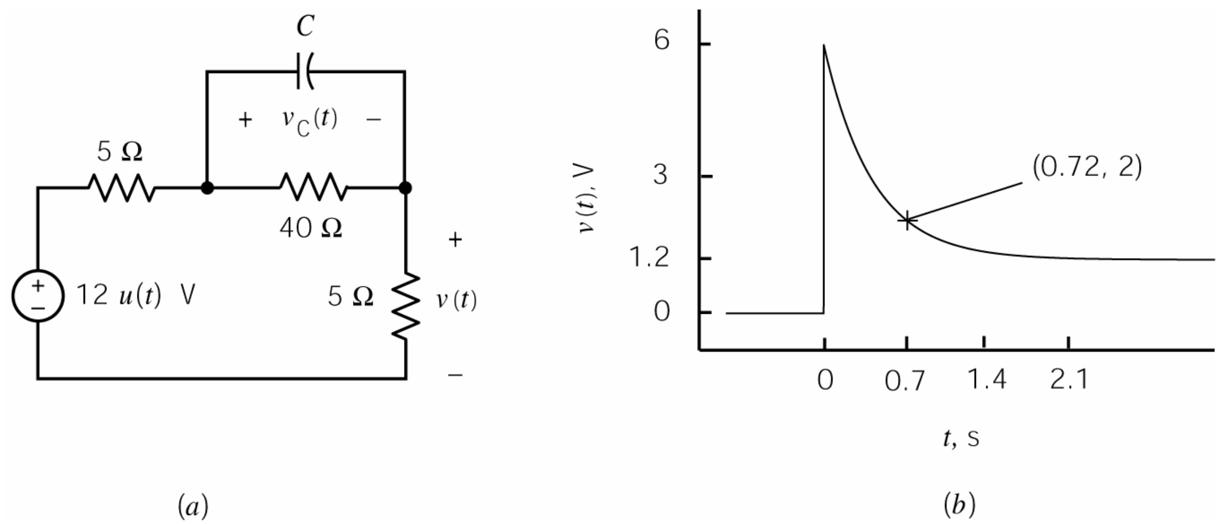

Figure 8-N6

Part a)

The voltage is represented by an equation of the form $v(t)=\begin{cases} D & \text{for } t<0 \\ E+F\,e^{-at} & \text{for } t\ge 0 \end{cases}$

where *D*, *E*, *F* and *a* are unknown constants. The constants *D*, *E* and *F* are described by

$$D=v(t) \text{ when } t<0,\quad E=\lim_{t\to\infty} v(t),\quad E+F=\lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D = 0,\ E = 1.2,\ \text{and}\ E + F = 6 \text{ V}$$

Consequently,

$$v(t)=\begin{cases} 0 & \text{for } t<0 \\ 1.2+4.8\,e^{-at} & \text{for } t\ge 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot in Figure 8-N6. We see $v(0.72)=2$ V, that is, the value of the voltage is 2 volts at time 0.7.2 seconds. Substituting these into the equation for $v(t)$ gives

$$2 = 1.2 + 4.8\,e^{-a(0.72)} \quad \Rightarrow \quad a = \frac{\ln(0.1667)}{-0.72} = 2.5$$

Consequently

$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1.2 + 4.8\,e^{-2.5t} & \text{for } t \ge 0 \end{cases}$$

Part b)

Figure 8-N7a shows the circuit immediately after time $t = 0$. In Figure 8-N7b, the part of the circuit connected to the capacitor has been replaced by its Thevenin equivalent circuit.

The time constant of the circuit is given by

$$\tau = R_t\,C = 8C$$

Also, the time constant is related to the exponent in $v(t)$ by $-725t = -\frac{t}{\tau}$. Consequently

$$\frac{1}{2.5} = 8\,C \quad \Rightarrow \quad C = \frac{1}{2.5(8)} = \frac{1}{20} = 0.05 \text{ F}$$

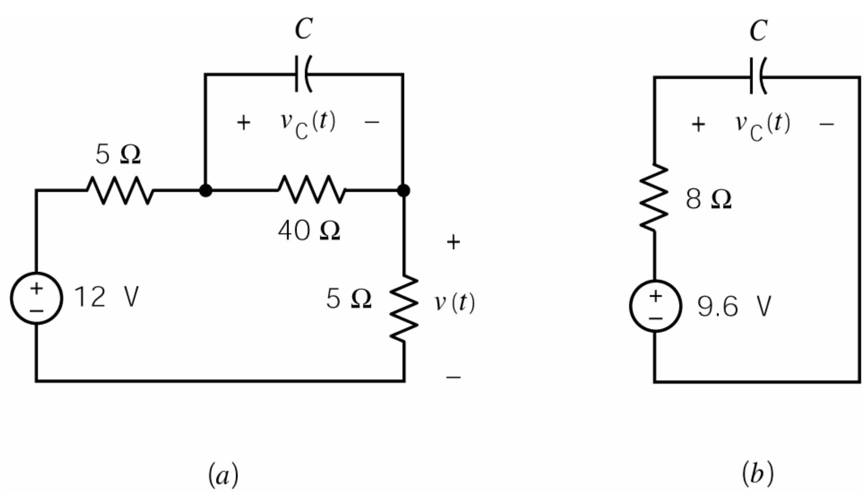

Figure 8-N7