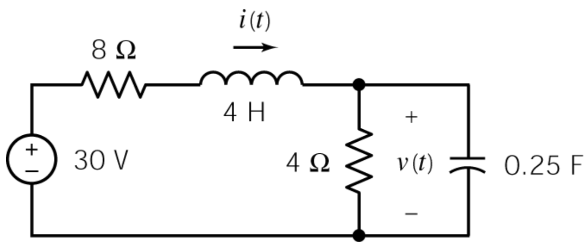
Circuit Types

DC Circuits

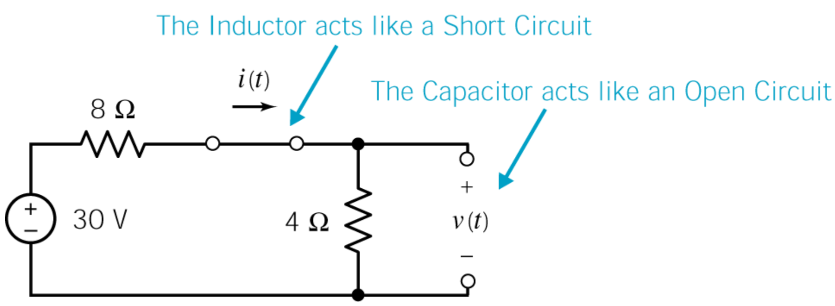
- Identifying features:
  - Constant inputs: the voltages of independent voltage sources and currents of independent current sources are all constant.
  - The circuit does not contain any switches.
- All voltages and currents in a dc circuit are constant.
- Capacitors act like open circuits and inductors act like short circuits.

Example:



Determine the current, $i(t)$, and voltage, $v(t)$, for this circuit.

Solution:
This is a dc circuit so the capacitor acts like an open circuit. The capacitor voltage, $v(t)$, is the voltage across that open circuit. The inductor acts like a short circuit. The inductor current, $i(t)$, is the current in that short circuit. Here's the circuit after replacing the capacitor by an open circuit and replacing the inductor by a short circuit:



Ohm's law gives

$$i(t) = \frac{30}{8+4} = 2.5 \text{ A}$$

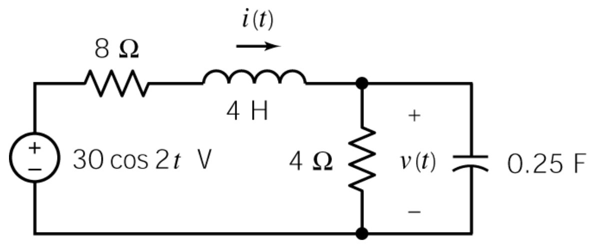
and

$$v(t) = 4\,i(t) = 4(2.5) = 10 \text{ V}$$

AC Circuits

- Identifying features:
  - Sinusoidal inputs: the voltages of independent voltage sources and currents of independent current sources are all sinusoidal at the same frequency.
  - The circuit does not contain any switches.
- All voltages and currents in a dc circuit are sinusoidal at the frequency of the sources.
- Capacitors **do not** act like open circuits and inductors **do not** act like short circuits.

Example:



Determine the current, $i(t)$, and voltage, $v(t)$, for this circuit.

Solution:
This is an ac circuit so the capacitor does not act like an open circuit and the inductor does not act like a short circuit.

Apply KCL at the top node of the 4 Ω resistor to get

$$i(t) = \frac{v(t)}{4} + 0.25\frac{d}{dt}v(t)$$

Apply KVL to the left mesh to get

$$8\,i(t) + 4\frac{d}{dt}i(t) + v(t) - 30\cos 2t = 0$$

We’re not ready to solve these equations. We’re considering this example now as a contrast to the previous example. Capacitors and inductors don’t always act like open and short circuits.

Later we’ll be able to show that

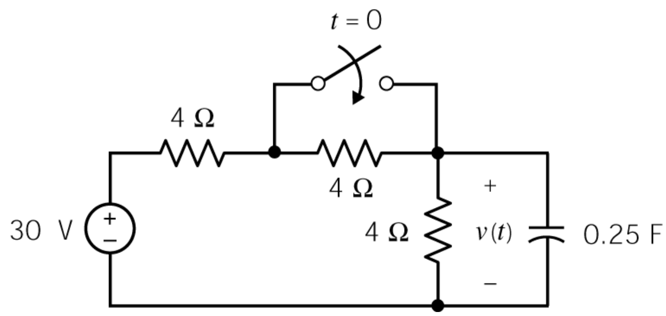
$$v(t) = 4.932\cos(2t - 99.5°)\ \text{V} \ \text{ and } \ i(t) = 2.757\cos(2t - 36°)\ \text{A}$$

For now, notice that even checking this solution by substituting $v(t)$ and $i(t)$ into the Kirchhoff‘s laws equations will require quite a bit of effort.

Switched Circuits

- Identifying feature: The circuit contains a switch.
- Open switches act like open circuits and closed switches act like short circuits.

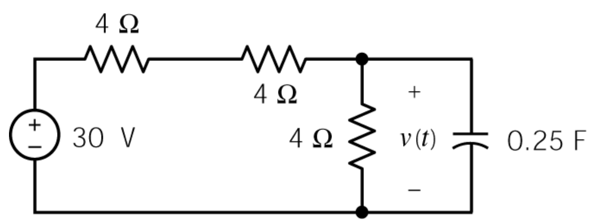
Example:



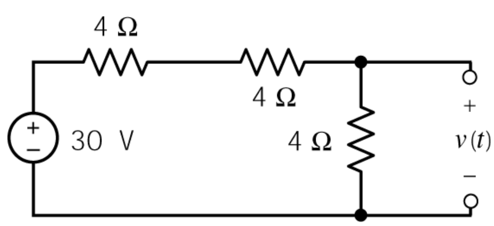
Determine the voltage, $v(t)$, for this circuit.

Solution:

Before time $t = 0$, the switch is open. An open switch acts like an open circuit. An open circuit in parallel with a resistor is equivalent to the resistor. (An open circuit is equivalent to an infinite resistance and $\infty \| R = R$.) Replacing the open switch by an open circuit gives:



This is a dc circuit, so the capacitor acts like an open circuit:



Using voltage division:

$$v(t) = \frac{4}{4+4+4}(30) = 10 \text{ V} \qquad \text{when } t < 0$$

Let $v(0-)$ denote the value of the voltage immediately before the switch closes at time $t = 0$ and let $v(0+)$ denote the value of the capacitor voltage immediately after the switch closes. In the absence of infinite currents, which are physically impossible, the capacitor voltage is continuous, so

$$v(0+) = v(0-) = 10 \text{ V}$$

We will see that after the switch closes the capacitor voltage is given by

$$v(t) = 15 - 5e^{-2t} \text{ V} \qquad \text{when } t \geq 0$$

The term $5e^{-2t}$ gets smaller and smaller as time goes on. Eventually it will be negligible compared to the other term. This observation is important enough that we have vocabulary that helps us talk about it

- 15 is the **steady state** (part of the) **response**

- $-5e^{-2t}$ is the **transient** part of the **response**

- $15 - 5e^{-2t}$ is the **complete response**

A dc circuit is said to be at steady state when all of its currents and voltages are constant.

An ac circuit is said to be at steady state when all of its currents and voltages are sinusoidal at the frequency of the sources.

With this new vocabulary, we can characterize dc and ac circuits more precisely:

DC Circuits

- Identifying features:
  - Constant inputs: the voltages of independent voltage sources and currents of independent current sources are all constant.
  - The circuit is at steady state.
- All voltages and currents in a dc circuit are constant.
- Capacitors act like open circuits and inductors act like short circuits.

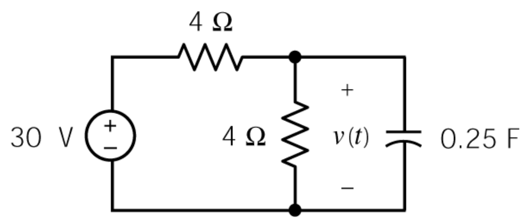
AC Circuits

- Identifying features:
  - Sinusoidal inputs: the voltages of independent voltage sources and currents of independent current sources are all sinusoidal at the same frequency.
  - The circuit is at steady state.
- All voltages and currents in a dc circuit are sinusoidal at the frequency of the sources.
- Capacitors **do not** act like open circuits and inductors **do not** act like short circuits.

Using this new vocabulary, the circuit is at steady state before the switch closes at time $t = 0$. Closing the switch disturbs circuit. The circuit is not a steady state immediately after the switch closes. Eventually the disturbance dies out and the circuit returns to a steady state, but probably a different steady state than before the switch closed.

After the switch closes, the circuit is not at steady state so the capacitor does not act like an open circuit.

Let's return to the example. After time $t = 0$, the switch is closed. A closed switch acts like a short circuit. A short circuit in parallel with a resistor is equivalent to the a short circuit. (A short circuit is equivalent to an zero resistance and $0 \| R = 0$.) Replacing the closed switch by a short circuit gives:



Apply KCL at the top node of the capacitor to get

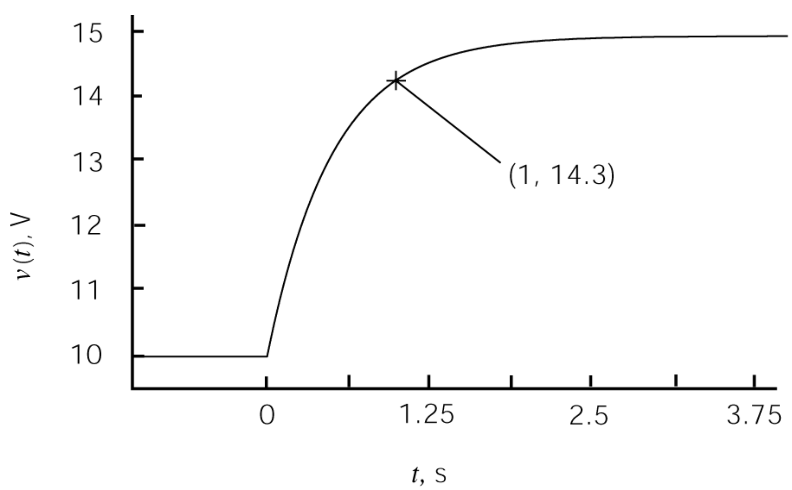
$$\frac{30 - v(t)}{4} = \frac{v(t)}{4} + 0.25 \frac{d}{dt} v(t) \quad \Rightarrow \quad \frac{d}{dt} v(t) + 2v(t) = 30$$

Solving this differential equation (using the initial condition $v(0+) = 10$ V) gives

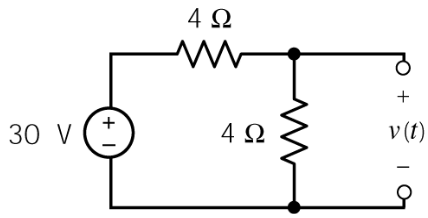
$$v(t) = 15 - 5e^{-2t} \text{ V} \qquad \text{when } t \geq 0$$

In summary, we have

$$v(t) = \begin{cases} 10 & t \leq 0 \\ 15 - 5e^{-2t} \text{ V} & t \geq 0 \end{cases}$$



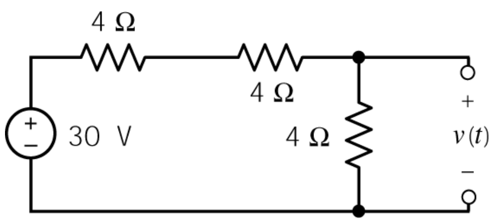
One more observation, eventually the transient part of the response dies out and the circuit reaches steady state. It is, once again, a dc circuit. The capacitor acts like an open circuit, so we have



Consequently,

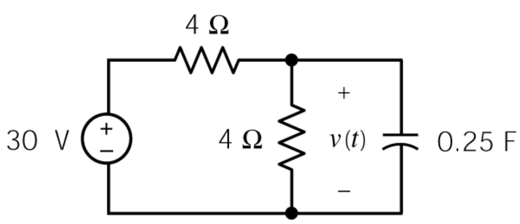
$$v(t) = 15 \text{ V} \qquad \text{when } t \to \infty$$

As a final summary, here are the circuits that we considered, together with the information that we obtained from each circuit:



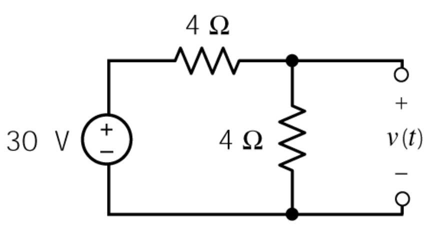
$$v(t) = 10 \text{ V} \qquad \text{when } t < 0$$

$$v(0+) = v(0-) = 10 \text{ V}$$



$$\frac{d}{dt}v(t) + 2v(t) = 30 \quad \Rightarrow$$

$$v(t) = 15 - 5e^{-2t} \text{ V} \qquad \text{when } t \geq 0$$



$$v(t) = 15 \text{ V} \qquad \text{when } t \to \infty$$