Example
Find the Inverse Laplace transform of

$$F(s) = \frac{s+3}{s^3+3s^2+6s+4}$$

Solution:

$$F(s) = \frac{s+3}{s^3+3s^2+6s+4} = \frac{s+3}{(s+1)(s^2+2s+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+4}$$

where

$$A = \left.\frac{s+3}{s^2+2s+4}\right|_{s=-1} = \frac{2}{3}$$

Then

$$\frac{(s+3)}{(s+1)(s^2+2s+4)} = \frac{\frac{2}{3}}{s+1} + \frac{Bs+C}{s^2+2s+4} \Rightarrow (s+3) = (\frac{2}{3}+B)\, s^2 + \left(\frac{4}{3}+B+C\right)s + \frac{8}{3} + C$$

Equating coefficient yields

$$\text{s}^2 : \quad 0 = \frac{2}{3} + B \quad \Rightarrow \quad B = -\frac{2}{3}$$

$$\text{s} \; : \quad 1 = \frac{4}{3} - \frac{2}{3} + C \quad \Rightarrow \quad C = \frac{1}{3}$$

Then

$$F(s) = \frac{\frac{2}{3}}{s+1} + \frac{-\frac{2}{3}s+\frac{1}{3}}{s^2+2s+4} = \frac{\frac{2}{3}}{s+1} + \frac{-\frac{2}{3}s+\frac{1}{3}}{(s+1)^2+3} = \frac{\frac{2}{3}}{s+1} + \frac{-\frac{2}{3}(s+1)}{(s+1)^2+3} + \frac{\frac{1}{\sqrt{3}}\sqrt{3}}{(s+1)^2+3}$$

Taking the inverse Laplace transform yields

$$f(t) = \frac{2}{3}e^{-t} - \frac{2}{3}e^{-t}\cos\sqrt{3}t + \frac{1}{\sqrt{3}}e^{-t}\sin\sqrt{3}t, \quad t \ge 0$$

Example
Find the Inverse Laplace transform of

$$F(s) = \frac{s^2 - 2s + 1}{s^3 + 3s^2 + 4s + 2}$$

Solution:

$$F(s) = \frac{s^2 - 2s + 1}{s^3 + 3s^2 + 4s + 2} = \frac{s^2 - 2s + 1}{(s+1)(s+1-j)(s+1+j)} = \frac{a}{s+1-j} + \frac{a^*}{s+1+j} + \frac{b}{s+1}$$

where

$$b = \left. \frac{s^2 - 2s + 1}{(s+1)^2 + 1} \right|_{s=-1} = 4$$

$$a = \left. \frac{s^2 - 2s + 1}{(s+1)(s+1+j)} \right|_{s=-1+j} = \frac{3 - j4}{-2} = -\frac{3}{2} + j2$$

$$a^* = -\frac{3}{2} - j2$$

Then

$$F(s) = \frac{-\frac{3}{2} + j2}{s+1-j} + \frac{-\frac{3}{2} - j2}{s+1+j} + \frac{4}{s+1}$$

Next

$$m = \sqrt{(-3/2)^2 + (2)^2} = \frac{5}{2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{2}{-\frac{3}{2}}\right) = 126.9°$$

From Equation 14.5-8

$$f(t) = \left[5e^{-t}\cos(t + 127°) + 4e^{-t}\right]u(t)$$

Example
Find the Inverse Laplace transform of

$$F(s) = \frac{5s-1}{s^3 - 3s - 2}$$

Solution:

$$F(s) = \frac{5s-1}{(s+1)^2 (s-2)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2}$$

where $$B = \left.\frac{5s-1}{s-2}\right|_{s=-1} = 2 \quad \text{and} \quad C = \left.\frac{5s-1}{(s+1)^2}\right|_{s=2} = 1$$

Then $$A = \frac{d}{ds}\left[(s+1)^2 F(s)\right]\Big|_{s=-1} = \left.\frac{-9}{(s-2)^2}\right|_{s=-1} = -1$$

Finally $$F(s) = \frac{-1}{s+1} + \frac{2}{(s+1)^2} + \frac{1}{s-2} \quad \Rightarrow \quad f(t) = \left[-e^{-t} + 2te^{-t} + e^{2t}\right]u(t)$$

Example
Find the Inverse Laplace transform of

$$Y(s)=\frac{1}{s^3+3s^2+4s+2}$$

Solution:

$$Y(s)=\frac{1}{(s+1)\left(s^2+2s+2\right)}=\frac{1}{(s+1)\left[(s+1)^2+1\right]}=\frac{A}{s+1}+\frac{Bs+C}{(s+1)^2+1}$$

where $$A=\left.\frac{1}{s^2+2s+2}\right|_{s=-1}=1$$

Next

$$\frac{1}{(s+1)\left(s^2+2s+2\right)}=\frac{1}{s+1}+\frac{Bs+C}{s^2+2s+2}\quad\Rightarrow\quad 1=s^2+2s+2+(Bs+C)(s+1)$$

$$\Rightarrow\quad 1=(B+1)s^2+(B+C+2)s+C+2$$

Equating coefficients:

$$\text{s}^2:0=B+1\quad\Rightarrow\quad B=-1$$

$$s\ :0=B+C+2\quad\Rightarrow\quad C=-1$$

Finally $$Y(s)=\frac{1}{s+1}-\frac{s+1}{(s+1)^2+1}\quad\Rightarrow\quad y(t)=\left[e^{-t}-e^{-t}\cos t\right]u(t)$$

Example
Find the Inverse Laplace transform of

$$F(s)=\frac{2s+6}{(s+1)\left(s^2+2s+5\right)}$$

Solution:

$$F(s)=\frac{2(s+3)}{(s+1)\left(s^2+2s+5\right)}=\frac{1}{s+1}+\frac{-(s+1)}{(s+1)^2+4}+\frac{2}{(s+1)^2+4}$$

$$f(t)=\left[e^{-t}-e^{-t}\cos(2t)+e^{-t}\sin(2t)\right]u(t)$$

Example
Find the Inverse Laplace transform of

$$F(s) = \frac{2s+6}{s(s^2+3s+2)}$$

Solution:

$$F(s) = \frac{2(s+3)}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$$

where

$$A = sF(s)\Big|_{s=0} = \left.\frac{2(s+3)}{(s+1)(s+2)}\right|_{s=0} = 3, \quad B = (s+1)\,F(s)\Big|_{s=-1} = \left.\frac{2(s+3)}{s(s+2)}\right|_{s=-1} = -4$$

and

$$(s+2)\,F(s)\Big|_{s=-2} = \left.\frac{2(s+3)}{s(s+1)}\right|_{s=-2} = C = 1$$

Finally

$$F(s) = \frac{3}{s} + \frac{-4}{s+1} + \frac{1}{s+2} \quad \Rightarrow \quad f(t) = \left(3 - 4e^{-t} + e^{-2t}\right)u(t)$$