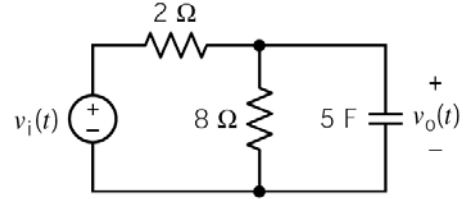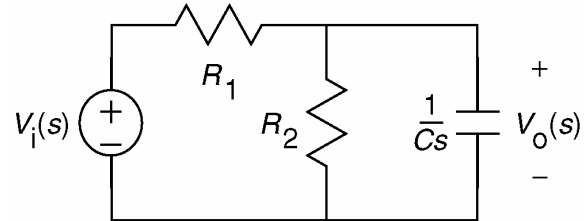**Example:**

The input to this circuit is the voltage of the voltage source, $v_i(t)$. The output is the capacitor voltage, $v_o(t)$. Determine the transfer function, impulse response and step response of this circuit.

Solution:

The transfer function is

$$H(s) = \frac{\dfrac{R_2}{R_2 C s + 1}}{R_1 + \dfrac{R_2}{R_2 C s + 1}} = \frac{\dfrac{1}{R_1 C}}{s + \dfrac{R_1 + R_2}{R_1 R_2 C}}$$

Using $R_1 = 2\ \Omega$, $R_2 = 8\ \Omega$ and $C = 5$ F gives

$$H(s) = \frac{0.1}{s + 0.125}$$

The impulse response is $h(t) = \mathcal{L}^{-1}[H(s)] = 0.1 e^{-0.125t} u(t)$ V.

The step response is

$$\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \mathcal{L}^{-1}\left[\frac{0.1}{s(s + 0.125)}\right] = \mathcal{L}^{-1}\left[\frac{0.8}{s} - \frac{0.8}{s + 0.125}\right] = 0.8\left(1 - e^{-0.125t}\right) u(t) \text{ V}$$

Example:

The input to a linear circuit is the voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. The impulse response of the circuit is

$$h(t) = 12t e^{-4t} u(t) \text{ V}$$

Determine the step response of this circuit.

Solution:

The transfer function is:

$$H(s) = \mathcal{L}\left[12t\,e^{-4t}u(t)\right] = \frac{12}{(s+4)^2} = \frac{12}{s^2+8s+16}$$

The Laplace transform of the step response is:

$$\frac{H(s)}{s} = \frac{12}{s(s+4)^2} = \frac{\frac{3}{4}}{s} + \frac{-3}{(s+4)^2} + \frac{k}{s+4}$$

The constant k is evaluated by multiplying both sides of the last equation by $s(s+4)^2$.

$$12 = \frac{3}{4}(s+4)^2 - 3s + ks(s+4) = \left(\frac{3}{4}+k\right)s^2 + (3+4k)s + 12 \quad \Rightarrow \quad k = -\frac{3}{4}$$
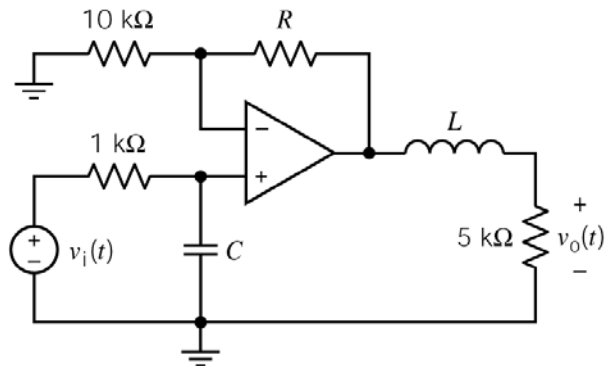
The step response is

$$\mathcal{L}^{-1}\left[\frac{H(s)}{s}\right] = \left(\frac{3}{4} - e^{-4t}\left(3t+\frac{3}{4}\right)\right)u(t) \text{ V}$$
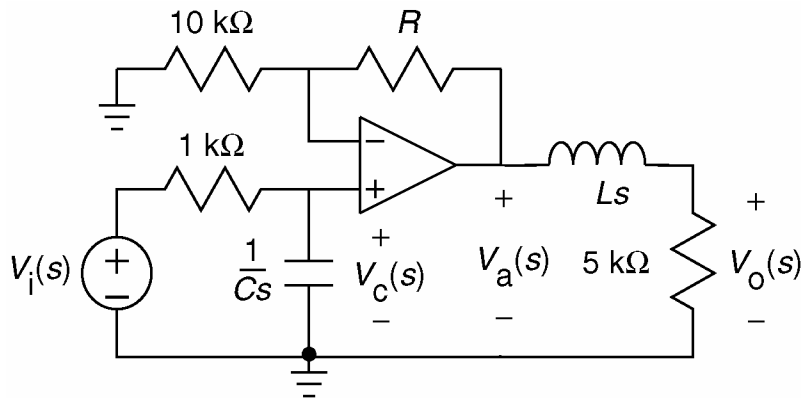
Example:

The input to this circuit is the voltage source voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. Specify value of R, C and L that cause the transfer function of this circuit to be

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{15\times10^6}{(s+2000)(s+5000)}$$

Solution:
The transfer function can also be calculated form the circuit itself. The circuit can be represented in the frequency domain as

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We can save ourselves some work be noticing that the 10000 ohm resistor, the resistor labeled R and the op amp comprise a non-inverting amplifier. Thus

$$V_a(s) = \left(1 + \frac{R}{10000}\right) V_c(s)$$

Now, writing node equations,

$$\frac{V_c(s) - V_i(s)}{1000} + CsV_c(s) = 0 \quad \text{and} \quad \frac{V_o(s) - V_a(s)}{Ls} + \frac{V_o(s)}{5000} = 0$$

Solving these node equations gives

$$H(s) = \frac{\dfrac{1}{1000\,C}\left(1 + \dfrac{R}{10000}\right)\dfrac{5000}{L}}{\left(s + \dfrac{1}{1000C}\right)\left(s + \dfrac{5000}{L}\right)}$$

Comparing these two equations for the transfer function gives

$$\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{1}{1000C}\right) = (s + 5000)$$

$$\left(s + \frac{5000}{L}\right) = (s + 2000) \quad \text{or} \quad \left(s + \frac{5000}{L}\right) = (s + 5000)$$

$$\frac{1}{1000C}\left(1 + \frac{R}{10000}\right)\frac{5000}{L} = 15 \times 10^6$$

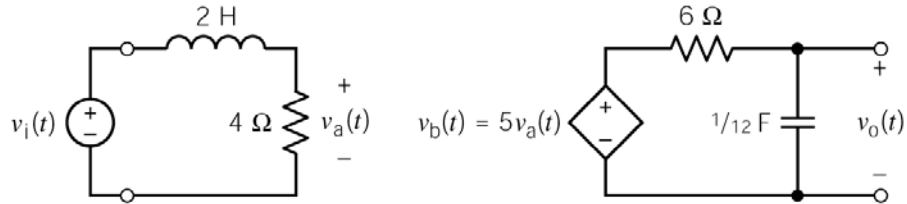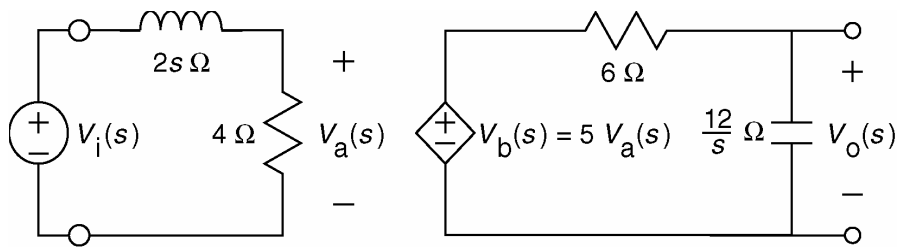The solution isn't unique, but there are only two possibilities. One of these possibilities is

$$\left(s + \frac{1}{1000C}\right) = (s + 2000) \quad \Rightarrow \quad C = 0.5\,\mu\text{F}$$

$$\left(s + \frac{5000}{L}\right) = (s + 5000) \quad \Rightarrow \quad L = 1\,H$$

$$\frac{1}{1000\left(0.5 \times 10^6\right)}\left(1 + \frac{R}{10000}\right)\frac{5000}{1} = 15 \times 10^6 \quad \Rightarrow \quad R = 5\,\text{k}\Omega$$

Example:

The input to this circuit is the voltage of the voltage source, $v_i(t)$. The output is the capacitor voltage, $v_o(t)$. Determine the step response of this circuit.

Solution:

$$V_a(s) = \left(\frac{4}{4+2s}\right)V_i(s) = \left(\frac{2}{s+2}\right)V_i(s)$$

$$V_o(s) = \left(\frac{\frac{12}{s}}{6+\frac{12}{s}}\right)V_b(s) = \left(\frac{2}{s+2}\right)V_b(s) = \left(\frac{2}{s+2}\right)5V_a(s) = \left(\frac{2}{s+2}\right)5\left(\frac{2}{s+2}\right)V_i(s)$$

The transfer function is:

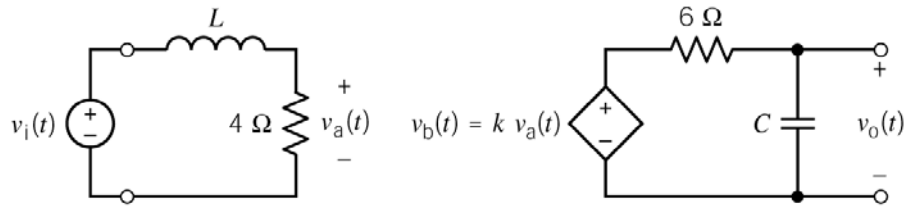$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{20}{(s+2)^2}$$

The Laplace transform of the step response is:

$$V_o(s) = \frac{20}{s(s+2)^2} = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2}$$

Taking the inverse Laplace transform:

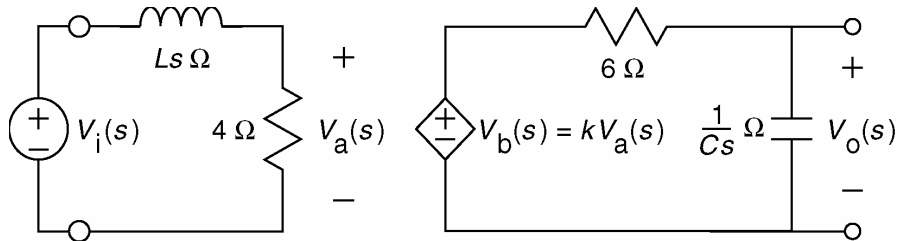$$v_o(t) = \left[5 - 5e^{-2t}(1+2t)\right]u(t) \text{ V}$$

Example:

The input to this circuit is the voltage source voltage, $v_i(t)$. The output is the voltage, $v_o(t)$. Specify value of k, C and L that cause the step response of this circuit to be

$$v_o(t) = \left[2 + 4e^{-3t} - 6e^{-2t}\right]u(t) \text{ V}$$

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From the circuit:

$$H(s) = \left(\frac{4}{4+Ls}\right)(k)\left(\frac{\dfrac{1}{Cs}}{6+\dfrac{1}{Cs}}\right) = \left(\dfrac{\dfrac{4}{L}}{s+\dfrac{4}{L}}\right)(k)\left(\dfrac{\dfrac{1}{6C}}{s+\dfrac{1}{6C}}\right)$$

From the given step response:

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(2+4e^{-3t}-6e^{-2t}\right)u(t)\right] = \frac{2}{s} + \frac{4}{s+3} - \frac{6}{s+2} = \frac{12}{s(s+3)(s+2)}$$

so

$$H(s) = \frac{12}{s(s+3)(s+2)}$$

Comparing the two representations of the transfer functions let $\dfrac{1}{6C} = 3 \implies C = \dfrac{1}{18}$ F,

$\dfrac{4}{L} = 2 \implies L = 2$ H and $2 \times 3 \times k = 12 \implies k = 2$ V/V.