Example:
The input to a linear circuit is the voltage, v_i. The output is the voltage, v_o. The step response of the circuit is

$$v_o(t) = \left(40 - 41.03e^{-8t} + 1.03e^{-320t}\right)u(t) \text{ V}$$

Determine the network function, $\mathbf{H}(\omega) = \dfrac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$, of the circuit. Sketch the asymptotic magnitude Bode plot.

Solution:

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(40 - 41.03\,e^{-8t} + 1.03\,e^{-320t}\right)u(t)\right] = \frac{40}{s} - \frac{41.03}{s+8} + \frac{1.03}{s+320} = \frac{102400}{s(s+8)(s+320)}$$

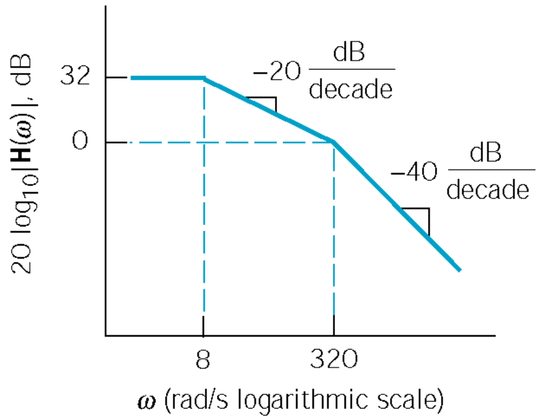
so

$$H(s) = \frac{102400}{(s+8)(s+320)}$$

The poles of the transfer function are $s_1 = -8$ rad/s and $s_2 = -320$ rad/s, so circuit is stable. Consequently,

$$\mathbf{H}(\omega) = H(s)\big|_{s=j\omega} = \frac{102400}{(j\omega+8)(j\omega+320)} = \frac{40}{\left(1 + j\dfrac{\omega}{8}\right)\left(1 + j\dfrac{\omega}{320}\right)}$$

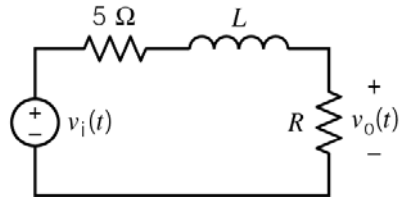
The network function has poles at 8 and 320 rad/s and has a low frequency gain equal to 32 dB = 40. Consequently, the asymptotic magnitude Bode plot is



Example:

The input to a circuit is the voltage source voltage, v_i. The step response of the circuit is

$$v_o(t) = \frac{3}{4}\left(1 - e^{-100t}\right)u(t) \text{ V}$$



(a) Determine the value of the inductance, *L*, and of the resistance, *R*.

(b) Determine the impulse response of this circuit.

(c) Determine the steady-state response of this circuit when the input is

$$v_i(t) = 5\cos(100t) \text{ V}.$$

Solution:

(a) From the given step response:

$$\frac{H(s)}{s} = \mathcal{L}\left[\frac{3}{4}\left(1 - e^{-100t}\right)u(t)\right] = \frac{75}{s(s+100)}$$

From the circuit:

$$H(s) = \frac{R}{R+5+Ls} \quad \Rightarrow \quad \frac{H(s)}{s} = \frac{\frac{R}{L}}{s\left(s + \frac{R+5}{L}\right)}$$

Comparing gives

$$\left.\begin{array}{l} \frac{R}{L} = 75 \\ \frac{R+5}{L} = 100 \end{array}\right\} \Rightarrow \begin{array}{l} R = 15\ \Omega \\ L = 0.2\text{ H} \end{array}$$

(b) The impulse response is

$$h(t) = \mathcal{L}^{-1}\left[\frac{75}{s+100}\right] = 75\, e^{-100t} u(t)$$

(c) The steady-state response is

$$\mathbf{H}(\omega)\Big|_{\omega=100} = \frac{75}{j100+100} = \frac{3}{4\sqrt{2}}\angle -45^\circ$$

$$\mathbf{V}_o(\omega) = \left(\frac{3}{4\sqrt{2}}\angle 45^\circ\right)(5\angle 0^\circ) = \frac{15}{4\sqrt{2}}\angle -45^\circ \text{ V}$$

$$v_o(t) = 2.652\cos(100t - 45^\circ) \text{ V}$$

Example:
The input to a circuit is the voltage source voltage, v_{i}. The step response of the circuit is

$$v_{\text{o}}(t) = 5\left(1-(1+2t)e^{-2t}\right)u(t) \quad \text{V}$$

Determine the steady-state response of this circuit when the input is

$$v_{\text{i}}(t) = 5\cos(2t+45°) \quad \text{V}$$

Solution:
The transfer function of this circuit is given by

$$\frac{H(s)}{s} = \mathcal{L}\left[\left(5-5e^{-2t}(1+2t)\right)u(t)\right] = \frac{5}{s} + \frac{-5}{s+2} + \frac{-10}{(s+2)^2} = \frac{20}{(s+2)^2} \quad \Rightarrow \quad H(s) = \frac{20}{(s+2)^2}$$

This transfer function is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \left.\frac{20}{(s+2)^2}\right|_{s=j\omega} = \frac{20}{(2+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_{\text{o}}(\omega) = \frac{20}{(2+j2)^2}(5\angle 45°) = \frac{20}{\left(2\sqrt{2}\angle 45°\right)^2}(5\angle 45°) = 12.5\angle -45° \quad \text{V}$$

The steady-state response is

$$v_{\text{o}}(t) = 12.5\cos(2t-45°) \quad \text{V}$$

Example

The input to a linear circuit is the voltage, v_{i}. The output is the voltage, v_{o}. The impulse response of the circuit is

$$h(t) = 30\,t\,e^{-5t}\,u(t) \quad \mathrm{V}$$

Determine the steady-state response of this circuit when the input is

$$v_{\mathrm{i}}(t) = 10\cos(3t) \quad \mathrm{V}$$

Solution:

The transfer function of the circuit is $H(s) = \mathcal{L}^{-1}\left[30\,t\,e^{-5t}u(t)\right] = \dfrac{30}{(s+5)^2}$. The circuit is stable so we can determine the network function as

$$\mathbf{H}(\omega) = H(s)\Big|_{s=j\omega} = \left.\frac{30}{(s+5)^2}\right|_{s=j\omega} = \frac{30}{(5+j\omega)^2}$$

The phasor of the output is

$$\mathbf{V}_{\mathrm{o}}(\omega) = \frac{30}{(5+j3)^2}(10\angle 0°) = \frac{30}{(5.83\angle 31°)^2}(10\angle 0°) = 8.82\angle -62° \quad \mathrm{V}$$

The steady-state response is

$$v_{\mathrm{o}}(t) = 8.82\cos(3t - 62°) \quad \mathrm{V}$$