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Polarized social media networks: a novel approach to quantify the polarization level of individual users

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ABSTRACT
This paper presents a novel methodology to quantify users' polarization within social media networks, focusing on network structure instead of content. We model polarized networks as directed graphs, collecting users' follower/following connections data, and propose a simple algorithm to compute a Connectivity Score for each node in sub-graphs associated with identified contrary groups of the polarized network. Our method has several advantages: it relies solely on users' follower/following connections data, which is objective, often public and easy to obtain. Also, this input data is usually memory-efficient, unlike the input data of content-based methods which may require a whole corpus. The algorithm's performance is assessed through application to Twitter's 116th Congressmembers network, comparing against a content analysis method: Sentiment score (using VADER), and two political behavior-based methods: Ideology score (based on co-sponsorship frequency) and Roll Call score (based on bill-voting similarity). The results demonstrate that: (1) users' choice of connections on the social media can represent polarization behavior; (2) a meaningful correlation between Connectivity Scores and Ideology or Roll Call Scores shows that the political behavior of users is reflected in their social media connections; (3) Democrats' Twitter following behavior and their bill-voting and bill co-sponsorship behavior (represented by Roll Call and Ideology) are all significantly more correlated than that of Republicans. We believe applying our algorithm in conjunction with other methods is a valuable contribution resulting in more comprehensive analysis of the social media polarization space.

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1. Introduction

Social media is a significant component of society; at its core, it provides users with a virtual outlet to share opinions on various topics (Weinberger, 2014). The goal of many social media platforms is to increase user engagement, such as suggesting

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connections with like-minded individuals and tailoring content to users' historical interactions. Unfortunately, the nature of these platforms has contributed to the emergence of polarized discourse and online echo chambers (Guo et al., 2020).

Polarization refers to the sharp division of opinions or beliefs into multiple contrary groups, and is often described as a key root of the extreme segregation, radicalism, and racism seen in contemporary society (Colleoni et al., 2014; Pew Research Center, 2019). This phenomenon is especially apparent in the United States (US) where 84% of Americans believe the country has significant political divisions (Duffy & Gottfield, 2018). The reach of social media platforms is growing considerably, boasting a cumulative total of over 4.59 billion users worldwide (Statista, 2022). As a result, investigating and analyzing the link between polarization and social media has never been more urgent.

This paper presents a new method for quantifying polarization within social media networks where group divisions of the polarized network are known a priori. Our method is simple, focuses solely on the objective network structure (following/follower relationships between users), and quantifies each individual user's polarization. We have no need to assess the content of users' posts or evaluate their typical social media interactions over content (e.g., retweets, likes, share, etc.). To test the algorithm, we evaluate it against three other analysis techniques utilized in existing research. All four methods are deployed on the 116th US Congressmembers and compared.

While the algorithm proposed in this paper can be independently utilized for quantifying the polarization level of each user in social media networks, it can also be used to complement existing content-based techniques. This paper's approach will prove most beneficial when analyzing platforms with limited access to users' content. Ultimately, this paper will serve the greater good and will hopefully assist in the development of methods for reducing polarization and promoting constructive dialogue in society.

2. Literature review and background

Polarization studies are often associated with politics, especially in the context of the US (Judge et al., 2023; Otala et al., 2021a). The two main political parties in the US are the Democrats and Republicans, and the country's main legislative body is the US Congress (Christensen, 2013). The ideological divide between Democrats and Republicans has increased significantly since the 1970s (Dimock et al., 2014), especially when compared to countries with similar political structures (Boxell et al., 2020).

In recent years, the rise in political polarization in the US has been attributed significantly to social media platforms and their echo chambers, which have intensified the divide between Democratic and Republican viewpoints (Bail et al., 2018). Out of all social media platforms, Twitter (recently renamed as X)[1] is considered to have the greatest influence on this divide, and it is seen as an easy and effective way for politicians to interact with their constituents (Beveridge & Tran, 2023; Hong & Kim, 2016; Matsilele & Nkoala, 2023).

Graph theory is a simple and flexible tool to model different types of networks including polarized networks (Bales & Johnson, 2006; Musco et al., 2017; Otala et al., 2021b; Stoica et al., 2018). Polarized networks can be modeled by graphs such that nodes represent people and edges denote the connections between people. We used directed graphs to model a polarized network with follower/following connections on Twitter. In general, a given user's followers are exposed to their posts and activities (such as 'retweets' and 'likes') (Falkenberg et al., 2022). While the direct effect of consuming a user's content cannot be known, it can be assumed that they have some amount of influence over their followers.

The literature on polarization problems on social media can be categorized as follows:

(1) Exploring the societal impacts of political polarization (Garcia et al., 2015; Kearney, 2019; Stewart et al., 2018; Tsfati & Nir, 2017; Tufekci & Wilson, 2012; Valenzuela et al., 2018; Warner & McKinney, 2013)
(2) Predicting the alignment of users in a potentially polarized network (Conover et al., 2011) or detecting polarization in a network where the divisions of users are unknown. In these studies, usually methods based on both content and network structure have been used such as homophily and segregation measures, random walks (Garimella et al., 2018; Jiang et al., 2021), balance theory (Interian et al., 2023), community detection (Blondel et al., 2008), and antagonism quantification (Guerra et al., 2021). Guerra et al. (2021) used modularity as an optimization metric to analyze both homophily and antagonism behaviors. They focused on the structural antagonism displayed by users who also have connections to the opposite group (they referred to them as boundary nodes). Their study results in the development of a network-wide and community-wide polarization metric (Guerra et al., 2021). These approaches often focus on quantifying the network at the community level and not node level (Phillips et al., 2023). This paper further captures nuanced homophily within the communities and quantifies at the user level for each node.
(3) Quantifying polarization in an already known polarized network on social media. This category is the most relevant one to our study (Moernaut et al., 2022; Santoro et al., 2023; Simon et al., 2022; Urman & Katz, 2022). Most studies in this category require users' content (e.g., tweets, Facebook posts, etc.) for a content-based analysis technique such as sentiment analysis, natural language processing, and other text mining methods. If not content-based, the links between users are often retweets. Relying on retweet networks does assume that a retweet equates endorsement. While the Associated Press and NPR agree with this (Haden, 2023), many users on Twitter still have the disclaimer in their biographies: 'RTs (Retweets) ≠ endorsements'. These techniques can be effective, but they often either require access to users' content and their metadata or rely on the 'retweet equals endorsement' assumption. Additionally these methods often produce a network level or community level polarization metric (Guerra et al., 2021), while this paper will provide a polarization score for each individual node.

As our proposed method falls within the third category, it does not have the restrictions of the existing methods. It features with the following unique advantages:

(1) It is solely focused on the social network structure (the relationships between users). This is a critical feature since user connection data is often public while access to some data related to users' content is often restricted by platforms.

(2) The user connection data itself is objective and relatively more stable, unlike using retweet networks or the content data, which tend to change frequently in a short period of time (Conover et al., 2011).
(3) Our analysis is at the user level (rather than community or network level), so a nuanced polarization score is assigned to each user. This provides a detailed view of individual users, which is especially interesting when applied to politicians or pundits.
(4) Our approach necessitates a smaller computational expense, as it often requires smaller size of input data than existing content-based methods. It is also important to note that the size and complexity of collecting our input data (the connection between users) are dependent on the density of the connections within the network. In turn, this allows larger social media networks to be analyzed on an accelerated timetable.

These unique features highlight our method's significance in comparison to previous studies.

In scenarios where group divisions of polarized networks are not known a priori, our method along with an existing method capable of identifying poles in a polarized network, such community detection (second category), can be applied. The next section extensively explains the unique features of our method.

3. Our proposed algorithm to quantify polarization

We modeled a polarized network on social media using a directed graph $G(N, E)$ where the set of nodes N and the set of edges E represent users and their following/follower connections, respectively. This graph generally contains $z \geq 2$ mutually exclusive sets of nodes referring to the contrary groups of users (Bright, 2018). Therefore, $G(N, E)$ can be divided into z sub-graphs $G = \cup_{i=1}^{z} G_i(N_i, E_i)$, such that $N_i, \forall i = 1, 2, \ldots, z$, are mutually exclusive. However, most traditional polarized networks typically contain two contrary groups. This study uses the following assumptions:

- Each user (node) has at least one following (successor).
- There is at least one connection between each pair of contrary groups $G_i(N_i, E_i)$. As a result, isolated groups do not exist. Previous literature has made similar assumptions (Guerra et al., 2021).
- A user is influenced by their followings (successors), not followers (predecessors) referring to the flow of information that usually occurs on Twitter and other platforms.
- Sets $N_i, \forall i = 1, 2, \ldots, z$ and $G_i(N_i, E_i)$ are given and known. In other words, it is predefined which user belongs to which contrary group.

In the literature, there are methods like community detection (Blondel et al., 2008) to identify group divisions if they are not known in advance. However, it is important to note that this study does not serve as a tool for this purpose.

Now, let us define functions $\mathrm{Succ}(n)$ and $\mathrm{Pred}(n)$ returning the sets of followings (successor nodes) and followers (predecessor nodes) of node n, respectively. Also, let the notation "‖" represents the size of a set. For example, $|\mathrm{Pred}(n)|$ represents the number of n's followers.

The successors and predecessors of a node can belong to either of the contrary groups N_i. Let $n \in N_i$, then, set D_n is defined such that $D_n \subseteq \mathrm{Succ}(n)$ and it contains all *similar* *successors* of a node n, meaning, for all $m \in D_n$, then $m \in N_i$. Next, we define the classification of nodes:

- If $|D_n| < |\text{Succ}(n)|$, node n is defined as a *central node*, meaning that $n \in N_i$ follows at least one user from another group (n has a heterogeneous set of successors). In previous literature, this group has been referred to as 'boundary nodes' or the community boundary (Guerra et al., 2021). While the labels 'central' and 'boundary' may appear contradictory for the same set of nodes, they are not. This discrepancy likely arises from differing viewpoints: that of the community versus that of the individual node. From the perspective of a community, Guerra et al. (2021) accurately consider these nodes to define an edge between the contrary groups. However, here, when considering the ideology of individual nodes, we expect these nodes to represent the ideological center.
- If $|D_n| = |\text{Succ}(n)|$, node n is defined as an *outlying node*, meaning that n only follows users in the same group as n (n has a homogeneous set of successors). These are occasionally referred to as internal nodes in previous literature (Guerra et al., 2021). Similarly to 'central' versus 'boundary', these opposing labels ('outlying' vs 'internal') stem from the same difference in orientation. From the perspective of a community, these are the internal community members. From the perspective of a node's political ideology, these nodes represent the more extreme end of a political spectrum.

The concept of central and outlying nodes is illustrated in Figure 1. In the case where $|D_n| < |\text{Succ}(n)|$, as $|D_n|$ grows toward $|\text{Succ}(n)|$, user n is less likely to be exposed to information shared by a contrary group. Therefore, it can be assumed that outlying nodes are more polarized than central nodes.

The activeness of user m in set $\text{Succ}(n)$ affects the amount of information received by user n. Because of this, the number of followers, $|\text{Pred}(m)|$, is an appropriate measure to address the activeness of m. This is a common assumption on social media: the more followers user m has, the more active m is, and the more influence m has (this is the definition of 'Influencer' and blue check mark). For simplicity, influence, activeness, and number of followers are all the same.

3.1. Our polarization metrics

Central nodes are directly exposed to the contrary group's opinions. If a central user $n \in N_i$ follows more users belonging to N_i compared to their total number of successors, then n has less chance to be exposed to the contrary opinions and might get polarized (Interian & Ribeiro, 2018). However, this claim is not quite accurate since the influential power of each successor is disregarded. Thus we introduce a novel metric to address this. Generally, users are influenced by their successors, so if a user has more followers, they have more influence. The *Polarization Centrality*, r_n, is defined as the ratio of the cumulative influence of D_n over the influence of user n's total successors, $0 \leq r_n \leq 1$. This metric is calculated using Equation (1):

$$r_n = \frac{\sum_{i \in D_n} |\text{Pred}\ (i)|}{\sum_{i \in \ \text{Succ}(n)} |\text{Pred}\ (i)|}\ . \tag{1}$$

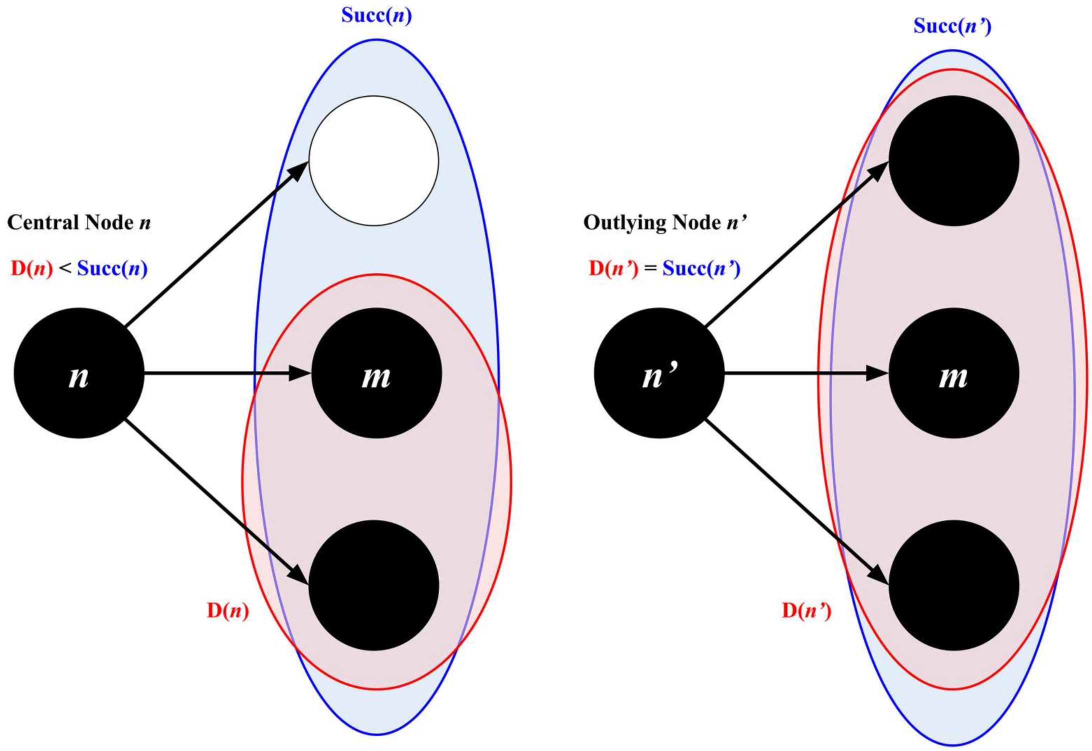

Figure 1. In a symbolic black/white polarized network, n is central and n' is outlying.

The r_n measures the total influence n receives from its like-minded successors in D_n, compared to the total influence of all its successors. The greater r_n is, the higher n's polarization level is expected to be. It is worth noting that r_n can be calculated for all nodes (central or outlying). However, it is only useful for central nodes because r_n of all outlying nodes equals 1.

Although outlying nodes do not follow any users from a contrary group, they can still be indirectly exposed to a contrary group's information. Paths exist between an outlying node n and nodes in a contrary group. If all users on these paths reshare a post from the contrary group, then user n will eventually be exposed to it via the domino effect. Therefore, the shortest path between outlying node n and any of the contrary group's nodes can be considered as the most probable path through which n can be influenced by the users in the contrary group. We propose the *Shortest Path* metric, s_n, to measure the length of the shortest path from node $n \in N_i$ to a node which does not belong to the same sub-graph (such as $m \notin N_i$). s_n can be defined for both central and outlying nodes. Given that all central nodes are directly connected to at least one node from a contrary group, s for all central nodes equals 1. To find s_n for any outlying node n, the central nodes will be the starting points. The greater s_n is, the more polarized n is.

We propose a final metric along with s_n for outlying nodes to quantify their polarization: the *Percentage of Sub-graph Connectivity* metric, t_n calculated by Equation (2). t_n is defined for only outlying nodes and shows the proportion of group (sub-graph) N_i which is followed by $n \in N_i$.

$$t_n = \frac{|Succ(n)|}{|N_i|} \tag{2}$$

Outlying nodes with the highest polarization level are not only far away from center (high s_n) but also follow more users from their group (high t_n). It should be noted that in a larger or potentially sparser network, this metric may skew heavily toward zero simply given account-following limits often imposed by platforms (i.e., the total sub-graph population is 100,000 but Instagram's following limit is 7500 (How Many Accounts You Can Follow on Instagram | Instagram Help Center, n.d.). In the case of a restrictive platform, the following cap could be utilized in place of $|N_i|$.

3.2. The Connectivity Algorithm

The Connectivity Algorithm is executed on each sub-graph individually to quantify the polarization level of each individual node in the sub-graph. This refers to the node's current polarization state based on its connection within and outside of the sub-graph. The polarization in the context of this paper does not represent any polarization process. The intuition behind this algorithm is that users are considered as polarized if they are far from people with different views (farther away, more polarized). Additionally, we incorporate the distance influence must flow over in our metric to further quantify the polarization of each user.

In more details, the functionality of the Connectivity Algorithm can be described as follows: the algorithm identifies the central nodes then traverses the sub-graph starting from the central nodes, continuing toward outlying nodes, until all nodes in the sub-graph are visited. The direction of visiting nodes (during the traversal) is aligned with the influence flow (the opposite of the edge direction). Every time the algorithm visits a node, it calculates the *Connectivity Score* (CS) for that node. The farther the node is, the larger the CS, which estimates a higher level of polarization for the node. To keep track of visited nodes, we used a dynamic sorted set of nodes, Q which has a FIFO queue data structure. This means nodes are added and removed from the Q in a first-in-first-out order.

First, the Connectivity Algorithm identifies set D_n for each node $n \in N_i$. A 'for' loop is used to evaluate the entire set of successors of n (Lines 3–5). $|D_n| \leq |\mathrm{Succ}(n)|$ is always true, since $D_n \subseteq \mathrm{Succ}(n)$, so based on the size of set D_n two scenarios are possible:

(1) $|D_n| < |\mathrm{Succ}(n)|$ (Lines 6–11) determines that n is a central node. Thus the polarization centrality metric r_n is calculated using Equation (1) (Line 7). The length of the shortest path of n is 1, $s_n \leftarrow 1$ (n is one edge away from the contrary group) (Line 8). Since n is already visited and the value of all relevant metrics are finalized, it should be inserted in Q (Line 9) and CS_n should be calculated (Line 11). A binary variable C_n is used to avoid visiting a node more than once. This mechanism guarantees the termination of the algorithm (this is mathematically proved in the next section).

(2) $|D_n| = |\mathrm{Succ}(n)|$ determines that n is an outlying node and should wait to be visited later, so it is not inserted in Q and other metrics cannot be finalized for n yet.

Now, Q is populated by only central nodes (the start points of traversal towards outlying nodes). The algorithm removes one node at a time from Q (e.g., $n \in Q$) and finds n's proper metrics. Then, only those predecessors of n that have not been visited yet will be inserted into Q. This process iteratively continues until Q empties (using a 'while' loop in Line 12 for this purpose). All nodes are eventually inserted and removed from Q once (see the proof in Section 3.3).

Every time a node $n \in N_i$ is removed from Q (Line 13), two conditions are checked for all $y \in \text{Pred}(n)$ (Line 15):

(1) Whether y is in the same sub-graph ($y \in N_i$)
(2) Whether y has not been already visited ($C_y = 0$).

If both conditions are 'True', then y is an outlying node (see the proof in Section 3.3) and algorithm can find s_y and t_y metrics. In the traversal, node y is reached through node n and $y \in \text{Pred}(n)$ (y is one edge away from n), so the shortest path is $s_y = s_n + 1$ (mathematically proved in Section 3.3) (Line 16). Metric t_y is also calculated through Equation (2) (Line 17). Therefore, $CS_y = s_y + t_y$ is determined and y is inserted in Q and flagged as visited by setting $C_y \leftarrow 1$ (Lines 18–20).

When Q empties, C_n for all nodes is already set as 1 (showing that all nodes were visited).

- The Connectivity Score range for all central nodes n is $1 \leq CS_n < 2$, because s_n is set as 1 and $r_n < 1$.
- CS_m for all outlying nodes m is greater than 2, because $s_m \geq 2$, $t_m \leq 1$.

The final nodes inserted and removed from Q are users who are likely more polarized than earlier visited nodes, since they have the farthest distance from the center. CS is used to sort the sub-graph based on polarization level of users in that sub-graph.

The Connectivity Algorithm [2]

Input $G_i(N_i, E_i)$

Output CS_n: The Connectivity Score for node n for all $n \in N_i$

*** Initialization and calculation of r_n***

1. For all $n \in N_i$
2. $s_n \leftarrow +\infty$ (initializing the value of shortest path for all nodes)
3.   For all $m \in \text{Succ}(n)$ (this 'for' loop identifies D_n for n)
4.   If $m \in N_i$
5.   Insert m into D_n
6.   If $|D_n| \neq |\text{Succ}(n)|$ (if n is a central node)
7. $r_n \leftarrow \frac{\sum_{i \in D_n} |\text{Pred}\ (i)|}{\sum_{i \in \text{Succ}(n)} |\text{Pred}\ (i)|}$
8. $s_n \leftarrow 1$ (assigning 1 as the shortest path of the central node)
9.   Insert n into Q (initially populating set Q by the central node)
10. $C_n \leftarrow 1$ (flagging n as a visited node which is inserted into Q)
11. $CS_n \leftarrow s_n + r_n$

***Calculation of the shortest path ***

12. While Q is not empty
13.   Remove n from Q (marking n as visited, then calculating its metrics)
14.   For all $y \in \text{Pred}(n)$
15.   If $y \in N_i$ and $C_y = 0$ (checking to not visit an already visited node)
16. $s_y \leftarrow (s_n + 1)$ (calculating the shortest path for node y)
17. $t_y \leftarrow \frac{|\text{Succ}(y)|}{|N_i|}$
18.   Insert y into Q
19. $C_y \leftarrow 1$ (flagging y as a visited node which is inserted into Q)
20. $CS_y \leftarrow s_y + t_y$

3.3. Proof of the connectivity algorithm

We will prove that (1) the Connectivity Algorithm eventually terminates and (2) it is functionally correct, meaning that the algorithm's input/output behavior follows valid mathematical logic.

Lemma 1. *Suppose a polarized Graph, $G(N, E)$ with at least two mutually exclusive sets of nodes (contrary groups of users), such as N_i, $N_i' \subset N$. In the case of existence of more than two contrary groups, $N_i' = N - N_i$. Furthermore, let Q be a queue where nodes $n \in N_i$ can be inserted in and removed from, and assume that:*

(1) *Set Q is initially populated by a subset of N_i (second assumption in Section 3).*
(2) *After a node n was inserted into Q, it is flagged by a binary variable, $C_n \leftarrow 1$.*
(3) *A node n can be inserted into Q, if $n \in N_i$ and $C_n = 0$ (note that these conditions are necessary, but not sufficient).*

Then the set Q eventually empties.

Proof. Each node can be inserted into Q at most once. This can be proved by contradiction: Suppose n was inserted into Q for the second time. According to the third hypothesis, the following are true about any node that was inserted in Q: $n \in N_i$ and $C_n = 0$ (the Connectivity Algorithm, Line 17). On the other hand, based on the second hypothesis, the first time that n was inserted into Q, it was already flagged, so $C_n = 1$ which contradicts $C_n = 0$. Therefore, n cannot be inserted into Q more than once.

Given that N_i is a finite set of nodes, there cannot be any more than $|N_i|$ nodes added/removed from Q, so Q eventually empties. ■

Lemma 1 proves that the Connectivity Algorithm eventually terminates (the 'while' loop in Line 12 stops).

Lemma 2. *Suppose a polarized Graph, $G(N, E)$ with all terms and assumptions given in Lemma 1. Moreover, assume that:*

(1) *The initial elements of set Q are central nodes in N_i ($n \in N_i$ such that $|D_n| \neq |Succ(n)|$) and the length of the shortest path from n to contrary group equals to 1 ($s_n \leftarrow 1$).*
(2) *As nodes $n \in Q$ are removed from Q one at a time, a subset of predecessors of n, ($Y \subset Pred(n)$) is inserted into Q such that $Y \subset N_i$ and $C_y = 0$, $\forall y \in Y$.*

Then the Connectivity Algorithm correctly calculates $s_y = s_n + 1$ as the length of the shortest path from node $y \in N_i$ to a node $v \notin N_i$.

Proof. The hypothesis is proved by induction over the distance of nodes $n \in N_i$ from a node $m \notin N_i$ in a contrary group. First, consider the base case, x_1 which is one edge away from other groups. This means x_1 is a central node by definition and has a successor y which does not belong to N_i (i.e., $y \in \text{Succ}(x_1)$, $y \notin N_i$). Therefore, by the definition, the shortest path of x_1 is 1 ($s_{x_1} = 1$). Also, the Connectivity Algorithm assigns $s_{x_1} \leftarrow 1$ (Line 8), which is correct. This proof can be extended to all central nodes n in N_i.

Next, for the induction step, consider node $x_k \in N_i$, such that there exists a shortest path consisting of $k > 1$ edges between x_k and a node in a contrary group. Assume that the Connectivity Algorithm correctly calculated s_{x_j} for all $x_j \in N_i$, $s_{x_j} \leq k$, $\forall j \in \mathbb{N}_k$.

Now, we need to prove that the Algorithm correctly calculates the shortest path for nodes whose distance from the contrary groups is $k+1$.

Using the induction assumption, x_k had been visited ($C_{x_k} = 1$) and inserted in Q once. Also, its shortest path is already calculated. According to the second hypothesis of Lemma 2, when a node such as $x_k \in Q$ is removed from Q, only a subset of its predecessors such as $x_y \in \text{pred}(x_k)$ is inserted into Q which have not been visited before (based on the second hypothesis of Lemma 2, $C_{x_{k+1}} = 0$). x_y is only one edge further from the contrary groups than node x_k ($x_y \in \text{pred}(x_k)$). Hence, the shortest path of x_y equals $s_{x_y} = s_{x_{k+1}} = s_{x_k} + 1$. Connectivity Algorithm (Line 16) also calculates $s_{x_y} \leftarrow s_{x_k} + 1$ which is correct. ■

Now, Theorem 1 uses Lemmas 1 and 2 to prove that the Connectivity Algorithm is correct.

Theorem 1. *Given a polarized graph, $G(N, E)$ with at least two mutually exclusive sets of nodes N_i', $N_i \subset N$. Then, after execution of the Connectivity Algorithm on N_i, the Connectivity Scores (CS_n) for all nodes $n \in N_i$ are correctly calculated.*

Proof. As for the central nodes $n \in N_i$, the algorithm uses $CS_n = s_n + r_n$ (Line 11) to calculate the Connectivity Score of n. In this formula, $s_n \leftarrow 1$ (Line 8) which is correct according to Lemma 1; additionally, r_n is correctly computed through Line 7 of the algorithm. Concerning the outlying nodes $m \in N_i$, the algorithm utilizes $CS_m = s_m + t_m$ (Line 20). As per Lemma 2, the Connectivity Algorithm accurately calculates s_m. Furthermore, every time a node m is visited, the algorithm computes the t_m metric (Line 17). By Lemma 1, each node is visited only once, ensuring s_m and t_m are calculated once and correctly. Hence, the Connectivity Algorithm correctly determines CS_n for all nodes, both central and outlying. ■

4. Numerical example

Figure 2 shows a polarized network with two subsets of red and blue nodes. To find the Connectivity scores for red nodes, our algorithm will be executed on the red sub-graph $\{A, B, C, D, E, F\}$.

The Connectivity Algorithm starts with processing all nodes randomly (Line 1), seen in Table 1. Initially, C_n for all nodes are 0, and set Q is empty. The algorithm classifies nodes either as central or outlying; and, accordingly, set Q is populated by all central nodes.

The outputs of Table 1 (Q and C_n) are used to execute the 'while' loop (Lines 12–20) in Table 2 to traverse the remaining nodes. Since Q has a FIFO queue data structure, nodes are removed in the order they were inserted in Q. Finally, the Connectivity Algorithm finds the total Connectivity Scores of nodes and sorts the sub-graph accordingly in Figure 3.

It should be noted that the dashes in a cell indicate that no changes are necessary, and no calculations have been done by the algorithm at that step. Also, the previous value for the corresponding element remains valid.

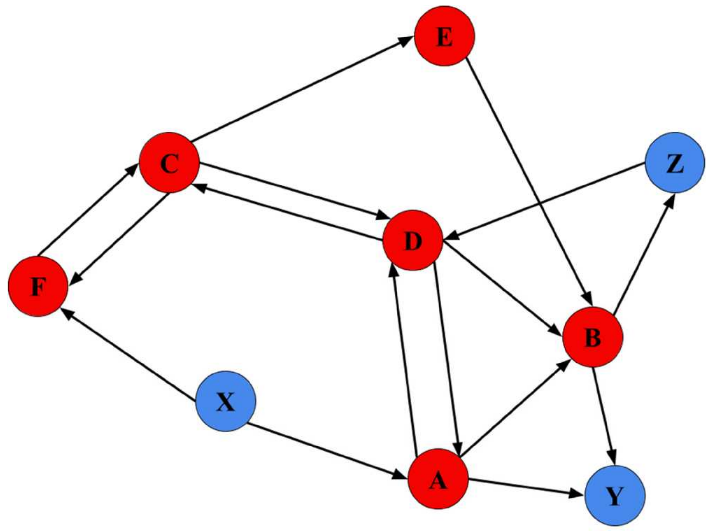

Figure 2. A polarized network with red and blue contrary groups.

Table 1. Executing the connectivity algorithm, the 'for' loop, lines 1–11.

Line 1 'For' loop over the node	Line 2 Initial s_n	Lines 3–5 D_n	Line 7 r_n	Line 8 Updated s_n for Central nodes	Line 9 Q	Line 10 C_n	Line 11 CS_n
A	$+\infty$	$\{B, D\}$	0.75	1	$\{A\}$	$C_A = 1$	$CS_A = 1.75$
B	$+\infty$	$\{\ \}$	0	1	$\{A, B\}$	$C_B = 1$	$CS_B = 1$
D	$+\infty$	$\{A, B, C\}$	–	–	–	–	–
E	$+\infty$	$\{B\}$	–	–	–	–	–
C	$+\infty$	$\{E, D, F\}$	–	–	–	–	–
F	$+\infty$	$\{C\}$	–	–	–	–	–

Table 2. Execution of the connectivity algorithm, the 'while' loop, Lines 12–20.

Line 12–13 'While' loop over Q - Removed node (n)	Line 14–15 For all $y \in \mathrm{Pred}(n)$	Line 16 s_y	Line 17 t_y	Line 18 Q	Line 19 C_y	Line 20 CS_y
A	D	2	0.5	$\{B, D\}$	$C_D = 1$	$CS_D = 2.5$
	X	–	–	–	–	–
B	E	2	0.167	$\{D, E\}$	$C_E = 1$	$CS_E = 2.167$
	D	–	–	–	–	–
	A	–	–	–	–	–
D	C	3	0.5	$\{E, C\}$	$C_C = 1$	$CS_C = 3.5$
	A	–	–	–	–	–
	Z	–	–	–	–	–
E	C	–	–	–	–	–
C	D	–	–	–	–	–
	F	4	0.16	$\{F\}$	$C_F = 1$	$CS_F = 4.167$
F	C	–	–	$\{\ \}$	–	–
	X	–	–	–	–	–

5. Real case studies on US Congressmembers Twitter network

We applied the Connectivity Algorithm to the polarized network of US 116th Congressmembers' users (2019–2021)[3] on Twitter. If the algorithm identifies multiple verified accounts for a particular Congressmember, the account with more followers and

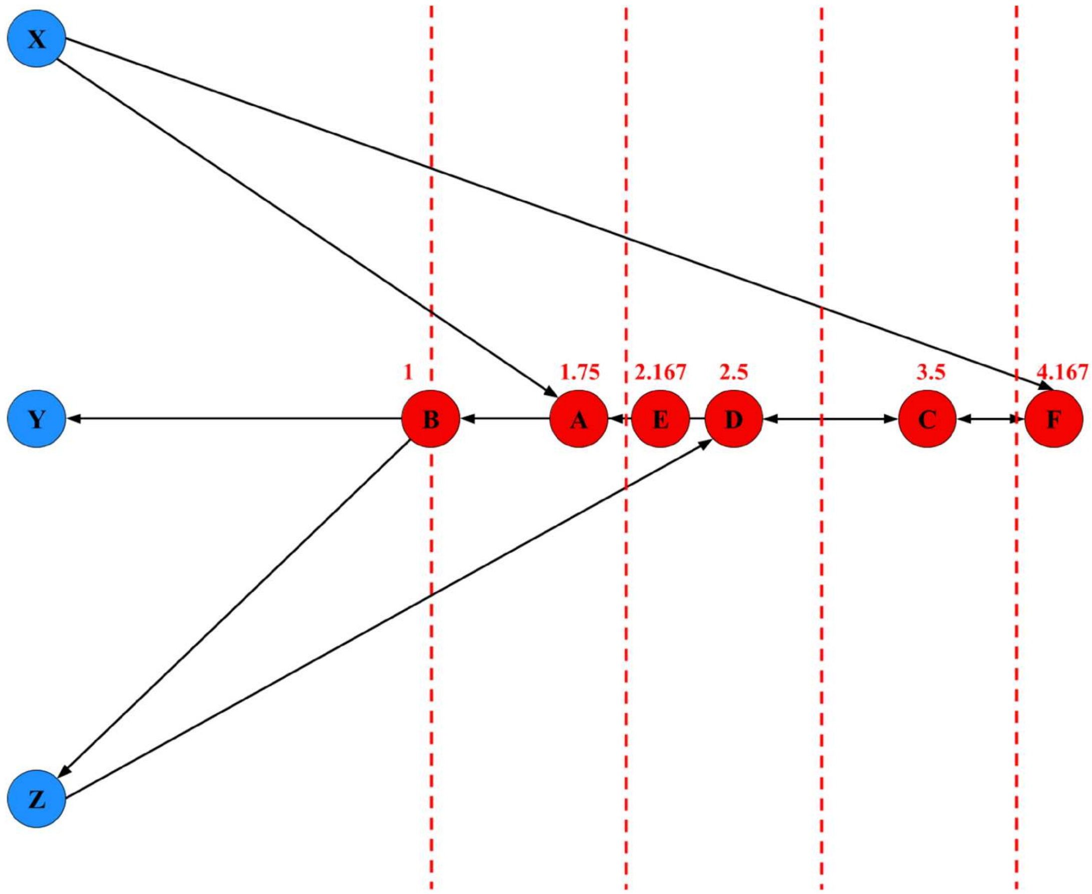

Figure 3. The sorted red sub-graph based on the *CS* of the red users.

following is used. After filtering, our network consists of 527 nodes ($n = 527$) representing verified Congressmember Twitter accounts. This includes 269 Democrats and 258 Republicans. This sample makes a real-world polarized network since each Congressmember's political party (sub-graph) is publicly known (Democrat vs. Republican). Independent politicians were manually labeled based on their political party history because in practice, those registered as Independent often do not caucus independently. They may have a historical preference for or caucus with one of the major parties. For instance, Senator Bernie Sanders, initially registered as a Democrat, has a well-documented left-leaning ideology and currently caucuses with Democrats. Grouping Sanders with an Independent whose behaviors lean Republican introduces bias into that group's scores.

Three other methods for quantifying polarization are applied to the same dataset for comparison: *sentiment analysis* (Section 5.2), *ideology scores* (Section 5.3.1), and *roll call scores* (Section 5.3.2). The last two methods measure politicians' polarization based on their political behavior in real world.

5.1. The connectivity algorithm applied to the politicians Twitter network

The distribution of Connectivity Scores is depicted in Figure 4. Scores range from +1 to +2.74, where scores below 2 indicate central users and above 2 indicate outlying users.

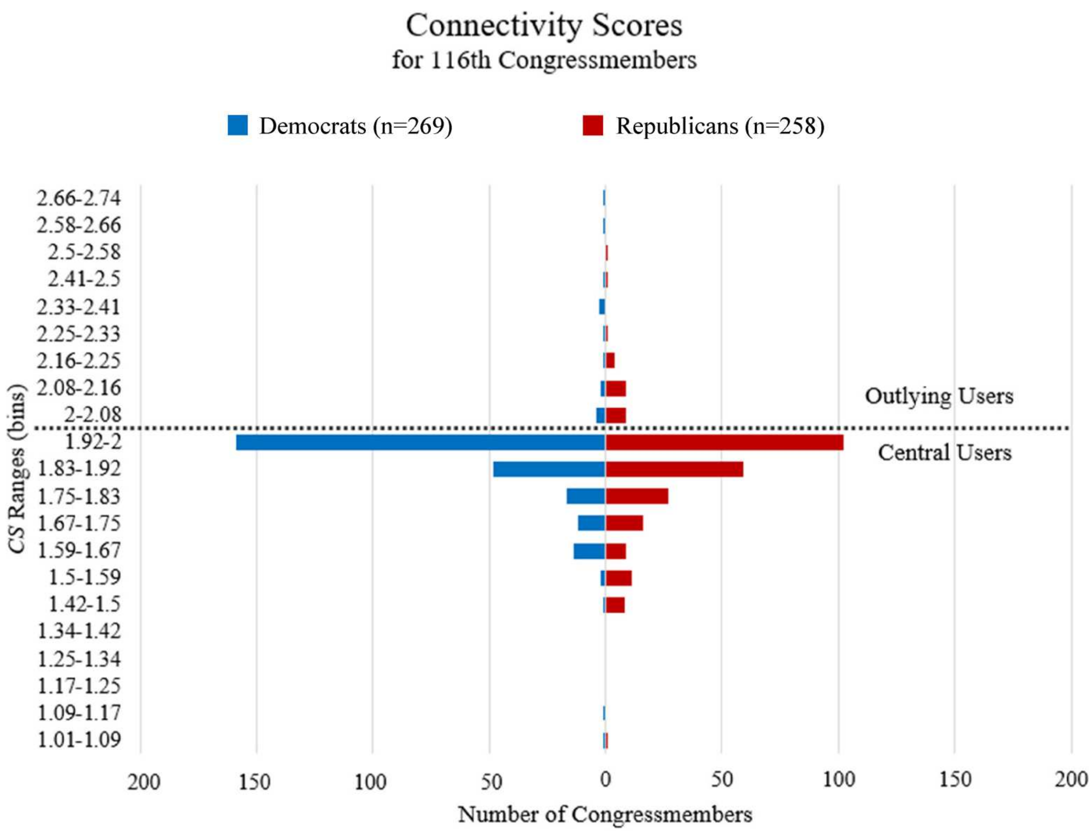

Figure 4. Distribution of connectivity scores (*CS*) of 527 Congressmembers.

Figure 4 illustrates that most Republicans (90.3%) and Democrats (94.5%) are classified as central users who follow at least one user from the opposing party. However, scores for central users heavily lean towards +2, suggesting that they follow significantly fewer users from the opposing party compared to users from their party. This is further explored later in the section. This observation aligns with our intuition: even if a Democrat follows Republicans, they are likely to follow more Democrats than Republicans (and vice versa).

The remaining users are identified as outlying users as listed in Table 3. These politicians (25 Republicans and 14 Democrats) are more polarized by not following any politicians from the opposing party. The *CS* of outlying nodes (in Table 3) are all between 2 and 3 calculated by $CS_y = s_y + t_y$, i.e., in this particular example, all these users are 2 edges away from an opposing Congressmember, so $s_y = 2$ for them; and t_y (the fraction part of the score) measures how well-connected the user is within their own subgroup (Percentage of Sub-graph Connectivity). Specific accounts of interest in Table 3 include Senate Majority Leader Mitch McConnell ($CS = 2.154$), Speaker of the House Nancy Pelosi ($CS = 2.597$), and well-known left-leaning Senator Bernie Sanders ($CS = 2.044$).

There are also central users of interest: both Senator Kyrsten Sinema and Senator Joe Manchin, both well known as centrists, were identified as central via the algorithm, with *CS* of 1.596 and 1.569 (within the 10 most central Democrats). Sinema most recently switched political affiliation to Independent but still often caucuses with Democrats (Collins & Wise, 2022). Manchin is a self-proclaimed centrist and potentially also

Table 3. The connectivity scores for the outlying nodes.

Outlying Republicans (Twitter Handle, Score)		Outlying Democrats (Twitter Handle, Score)	
RepBrianBabin	2.563707	RepThompson	2.74359
RepLizCheney	2.432432	SpeakerPelosi	2.59707
SenToddYoung	2.254826	JoaquinCastrotx	2.483516
RepFranklin	2.239382	Repblumenauer	2.369963
RepBlaine	2.212355	RepMarkTakano	2.347985
RepBobGood	2.204633	RepPressley	2.344322
RepTroyNehls	2.177606	RepRaulGrijalva	2.252747
LeaderMcConnell	2.15444	RepJahanaHayes	2.179487
SenJohnThune	2.15444	RepRichardNeal	2.128205
RepBurgessOwens	2.15444	RepSaraJacobs	2.087912
SenTomCotton	2.15444	BernieSanders	2.043956
RepGrothman	2.135135	RepSchakowsky	2.029304
Jbletlow	2.127413	Repdelgado	2.018315
Lancegooden	2.111969	RepLouCorrea	2.007326
RepClayHiggins	2.104247		
KenCalvert	2.092664		
RepJacobs	2.07722		
RepChipRoy	2.065637		
CarlosGimenezFL	2.061776		
ByronDonalds	2.057915		
LindseyGrahamSC	2.054054		
MittRomney	2.023166		
HawleyMO	2.023166		
RepMcClintock	2.007722		
RepDevinNunes	2.003861		

switching to an Independent as well (Alfaro, 2023). Senate Minority Leader Chuck Schumer was also identified as a central user, ($CS = 1.855$).

To facilitate a comparison with other polarization methods, CS_n are normalized using Equation (3) to range from 0 to +1, representing the least and most polarized levels, respectively. The distribution of these normalized scores is depicted in Figure 5.

$$x' = \frac{x - x_{min}}{x_{max} - x_{min}} \tag{3}$$

In the rest of Section 5, only the normalized Connectivity Scores are used, so, from now on, the '*Connectivity Scores*' refers to '*normalized Connectivity Scores*'.

We believe the heavy skew toward the boundary between central and outlying nodes is based on a nuanced feature of the users in the network: over 90% of the politicians in our dataset are central nodes and follow other politicians from the opposite party, placing them just one edge away from the opposing group (shortest path s_n equals to 1). Despite this, they follow far fewer politicians from the opposing party than those from their own (resulting in larger r_n).

Since CS is highly reliant on a node's degree count and the shortest path to the opposite subgroup, correlation coefficients and corresponding confidence intervals (using 95% and the Fisher transformation, (Fisher, 1915)) are calculated between key network metrics and CS, as seen below in Table 4. This is to ensure that CS is producing a novel ranking that cannot be demonstrated with common existing network metrics such as degree.

The correlation coefficients involving edge counts are worth noting, but not substantial. For the shortest path to opposing subgroup, this correlation coefficient is much larger but makes sense within context of CS. The largest portion of each node's CS is its

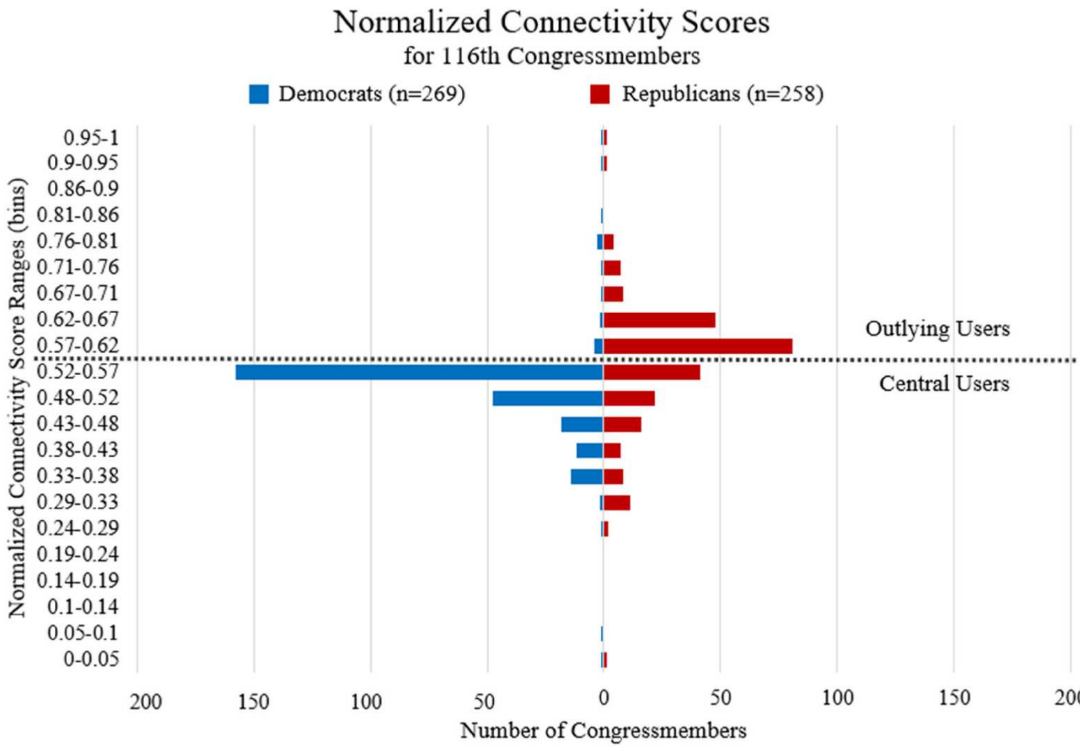

Figure 5. Distribution of normalized connectivity scores, for 527 Congressmembers.

shortest path to opposing information. The narrow confidence intervals further validate the limited correlation coefficients between key network metrics and *CS*. This highlights the uniqueness of *CS* as a metric for determining polarization based on network features.

Additionally, the degree distribution of the network was generated, Figure 6, to compare to the *CS* distribution seen in Figure 5. This is to further confirm that *CS* is not being skewed in the same direction as degree.

By comparing Figure 5 and Figure 6, we can see that the density distributions for degree versus *CS* are also very different. We believe these calculations and visuals confirm that *CS* is producing a novel ranking in comparison to well-known network metrics. *CS* is not replicable simply by using well-known network metrics.

5.2. Comparing results of sentiment analysis vs connectivity scores

Sentiment analysis has been widely used in political polarization studies (Alonso et al., 2021; Del Vicario et al., 2017; Haselmayer & Jenny, 2017). Despite its popularity,

Table 4. Correlation coefficients (and corresponding confidence intervals using 95%) between relevant network metrics and *CS*. The only noteworthy correlation is between Shortest Path and *CS*, which is understandable given the large portion that Shortest Path contributes to each *CS*.

Network metric vs *CS*	Correlation coefficient	Confidence interval (95%)
Incoming edge count vs *CS*	0.0263	[−0.0592, 0.111]
Outgoing edge count vs *CS*	−0.1796	[−0.261, −0.0956]
Degree (Incoming + Outgoing) vs *CS*	−0.1238	[−0.207, −0.0388]
Shortest path to opposing subgroup vs *CS*	0.4940	[0.427, 0.556]

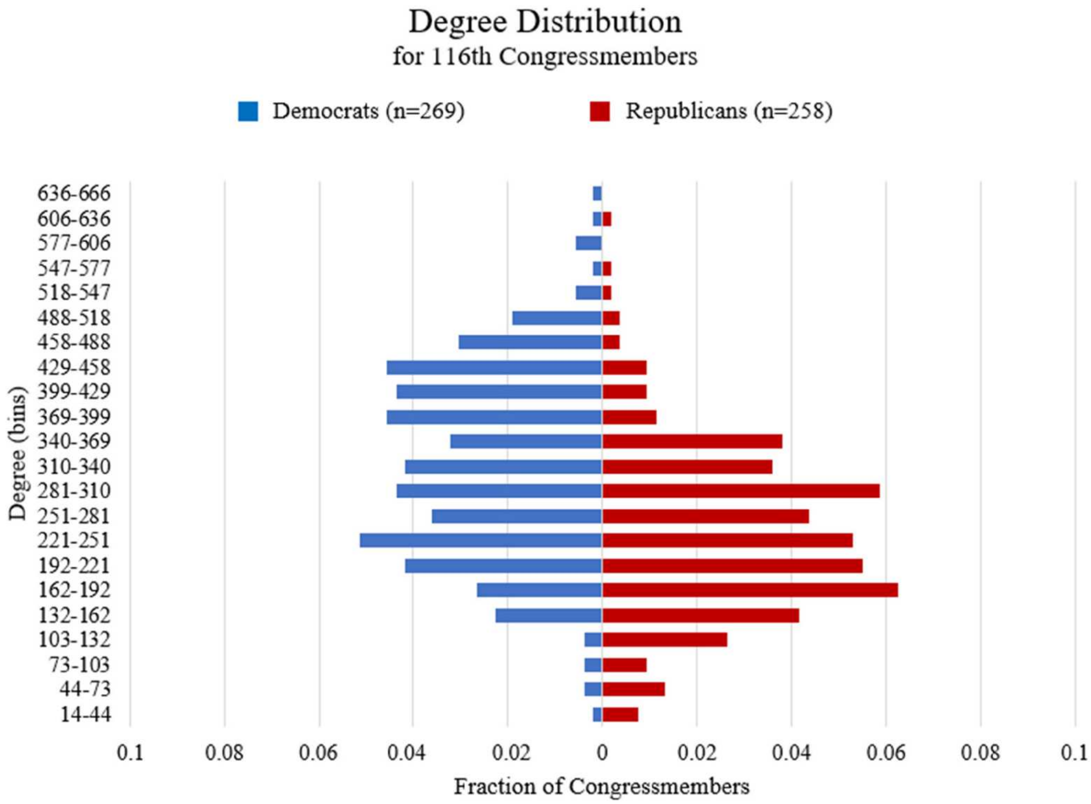

Figure 6. Degree distribution for 116th Congressmembers.

sentiment analysis has some major shortcomings, e.g., biases in classification of text, contextual ambiguity, oversimplification of complex opinions to a restrictive scale, etc. (Medhat et al., 2014). Considering both its popularity and controversial shortcomings, we believe it is worthwhile to compare the results of the Connectivity method to the sentiment analysis since it can capture another aspect of a Congressmember's behavior on Twitter: the content of their Tweets.

This study pulls the most recent 100 Tweets of 558 Congressmembers from the 116th Congress dataset using the Twitter API. The data is then exported and organized into the data frames, resulting in a total of 55,800 Tweets collected on 02/03/2023. We utilize the Python Natural Language Toolkit (NLTK) sentiment package and apply VADER (Valence Aware Dictionary and Sentiment Reasoner), a widely used tool for identifying the polarity and intensity of emotions. VADER analyzes each Tweet and returns scores in four categories: negative, neutral, positive, and compound.

The compound score, which ranges from −1 to +1, is assigned to each Tweet of each user in our dataset. A compound score closer to +1 signifies a higher level of positive sentiment, while closer to −1 indicates a higher level of negative sentiment. A compound score around 0 suggests a more neutral sentiment.

A 'positivity' average was calculated for all Tweets with compound scores greater than 0 and a 'negativity' average was calculated for all Tweets with compound scores less than 0. These average compound scores can be considered as a politician's tendency toward extreme positivity or extreme negativity. The two average compound scores for each Congressmember are then normalized with respect to the dataset to facilitate the comparison. Finally, the difference between the normalized 'positivity average' and 'negativity average' is considered for each Congressmember to measure their polarization level based on their recent 100 Tweets. This difference ranges from 0 to 2, so for consistency and meaningful comparison between metrics, another round of normalization has been applied. This resulting measure will be referred to as '*Sentiment Score*' and ranges between 0 and 1. A high Sentiment Score for a Congressmember means that s/he had tweeted some harsh contents (within their recent 100 Tweets) with extreme sentiments (positive and/or negative).

It should be noted that there exists a potential bias arising from imbalanced usage of positive and negative sentiments among individual users. Based on the state of the art in the literature, bias seems like a common challenge within the context of sentiment analysis (as a subfield of text mining and Natural Language Processing) (Ebrahimi et al., 2017; Hutchinson et al., 2020; Parvin et al., 2021; Wankhade et al., 2022; Yahav et al., 2019). However, given that our study is not centered around sentiment analysis, but rather employs Sentiment Score as a comparative metric, we tolerate this bias. This study chose this simple sentiment metric (the Sentiment Score) under this assumption that the frequency and likelihood of usage of positive and negative sentiment are equivalent; while in the reality, users might have more reservation regarding using negative sentiment compared to positive.

These Sentiment Scores are represented in Figure 7 for a total of 507 Congressmembers who are accounted for after data processing and filtering. The Sentiment Scores are then compared to the Connectivity Scores in Figure 8 and Figure 9 for 258 Democrats, 249 Republicans in our dataset, respectively.

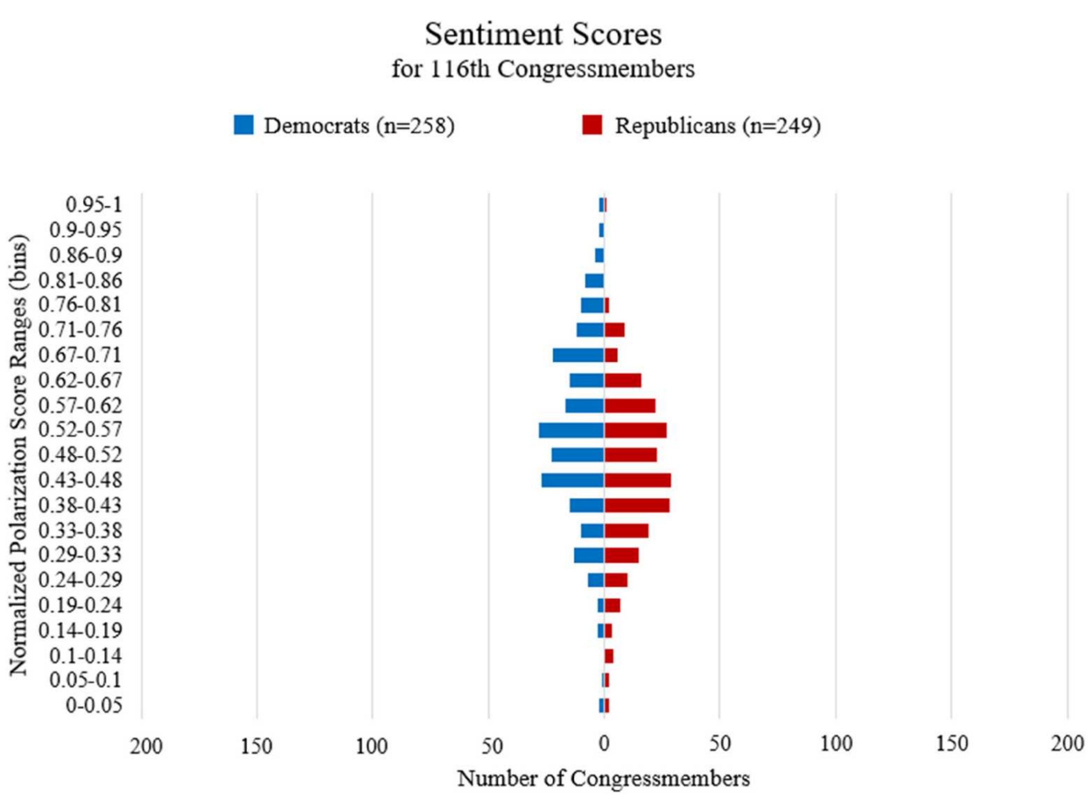

Figure 7. Distribution of Sentiment Scores of 507 members of 116th Congress.

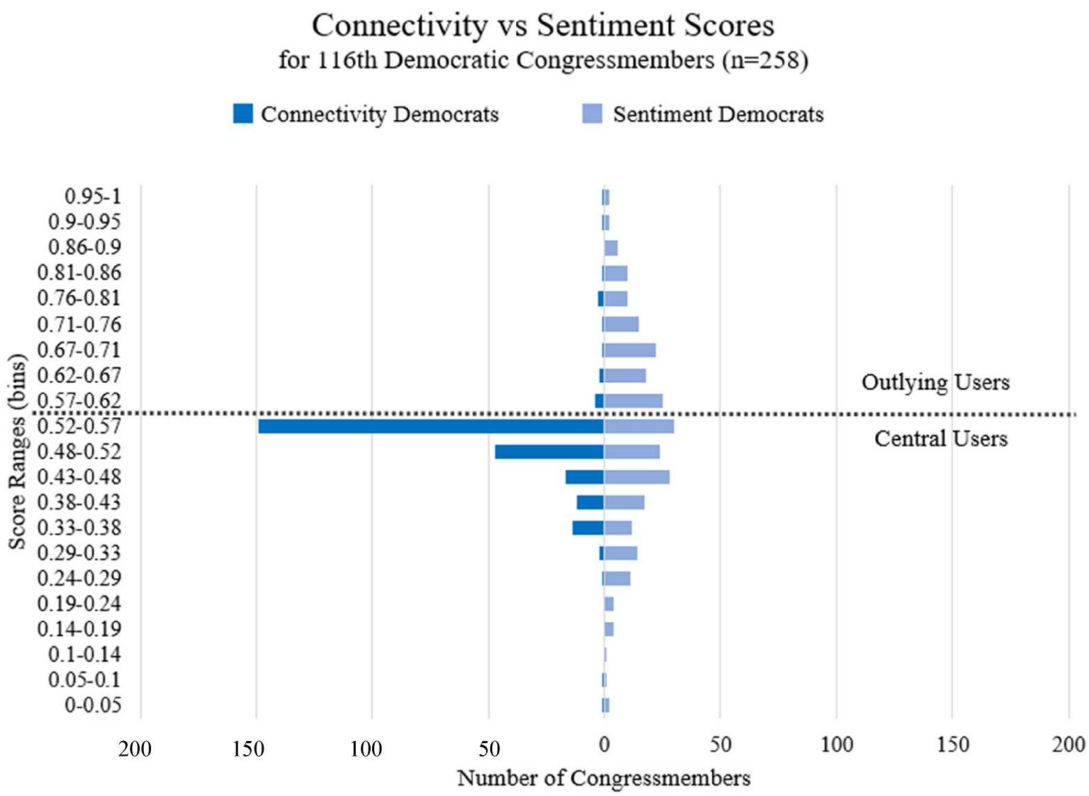

Figure 8. Comparing Sentiment to Connectivity Scores for 258 Democrats in the dataset.

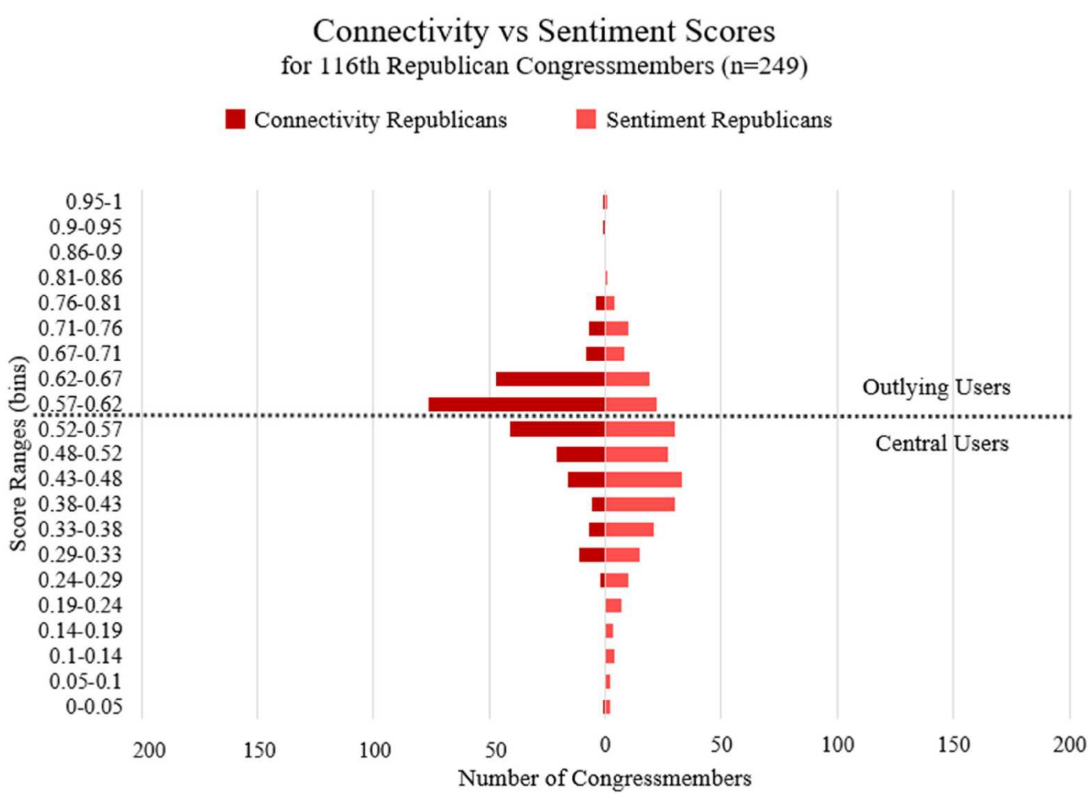

Figure 9. Comparing Sentiment to Connectivity Scores for 258 Republicans in the dataset.

The most apparent difference between Sentiment and Connectivity is their distribution: Sentiment Scores, especially for Republicans, follow a near perfect normal distribution. However, the Connectivity analysis shows that most of these politicians are moderately polarized with the Connectivity Scores within $0.48 - 0.57$. Part of this substantial difference may be the results of controversial limitations of sentiment analysis methods. For instance, the semi-normal distribution of the Sentiment Scores may derive from the amount of data points used (100 most recent Tweets for each Congressmember). Also, sentiment analysis results are often related to the temporal dynamics of social media. Sentiments can be transient and rapidly changing in response to events, news, or public discourse, making it challenging to accurately measure polarization over time. Finally, sentiment analysis focuses on sentiment of words individually instead of phrases and sentences which may result in analysis out of context.

5.3. Comparing connectivity scores versus real-world political polarization scores

The users in our dataset are well-known politicians and their real-world political polarization behavior has been analyzed and quantified in the literature. Here, we will discuss how these politicians are measured for polarization based on real-world political Congressional actions and compare it with their polarization behavior on social media using the Connectivity Score.

5.3.1. Ideology scores versus connectivity scores

GovTrack, publishing data on Congressmembers, has an interesting metric known as an *Ideology value*. This value shows a member's frequency of sponsoring and co-sponsoring overlapping sets of bills and resolutions with other Congressmembers (GovTrack.us, 2023). GovTrack performs Singular Value Decomposition on the co-sponsorship matrix to determine the major dimensions which best describe the data (Wall et al., 2003).

After exporting the Ideology value for 116th Congress from GovTrack, we normalized them by Equation (3) for 526 Congressmembers. We refer to these normalized GovTrack scores as *Ideology Scores*. The higher the Ideology Score for a user shows more polarization level.

Figure 10 showcases the difference in concentration between Democrats' and Republicans' Ideology Scores. Republican scores are concentrated above the average. The higher average Ideology Scores for Republicans reveal that they are more likely to co-sponsor each other's bills in comparison to Democrats who may not be as unified in their Congressional behavior.

The Ideology Scores are then compared to the Connectivity Scores in Figure 11 and Figure 12 for Democratic and Republican Congressmembers, respectively. Figure 11 and Figure 12 show an obvious difference in the distribution of Ideology and Connectivity Scores. In both figures, only a few Congressmembers have high Connectivity Scores (greater than 0.67), while there are more politicians with Ideology Scores greater than 0.67. This means that Ideology method identifies more politicians as 'extreme' in terms of their polarization level compared to our method. This difference can be explained through the nature of input data of these two methods: Ideology Scores use

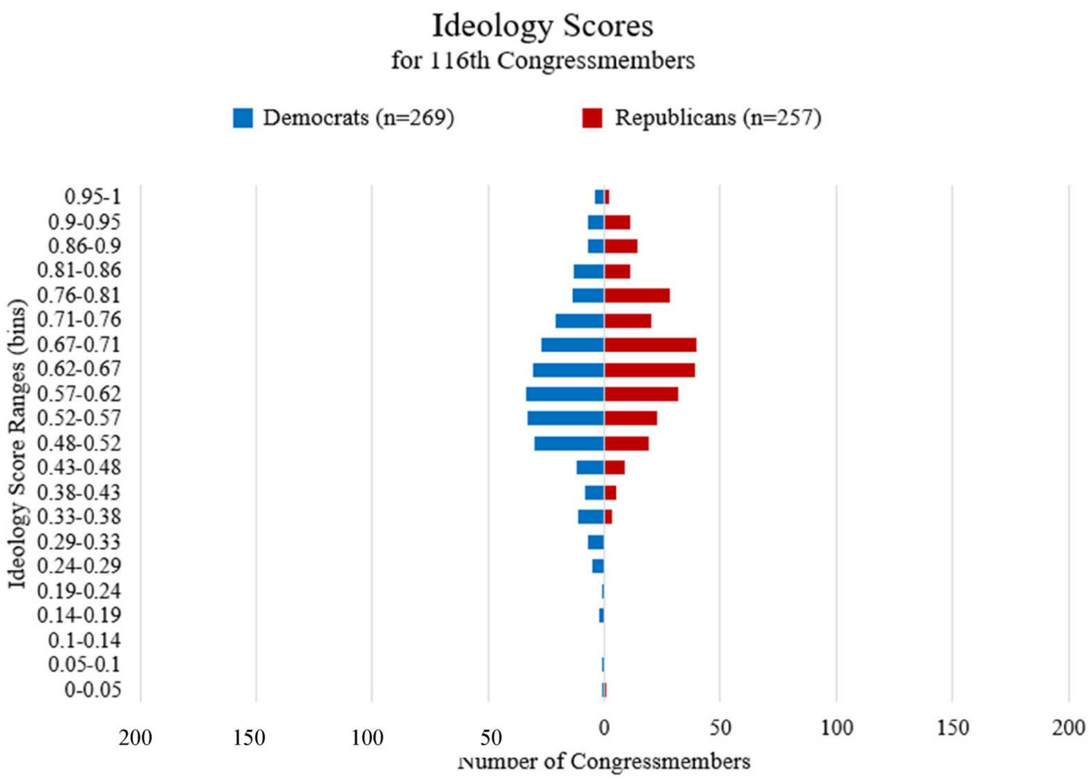

Figure 10. Distribution of Ideology Scores for 526 members of 116th Congress.

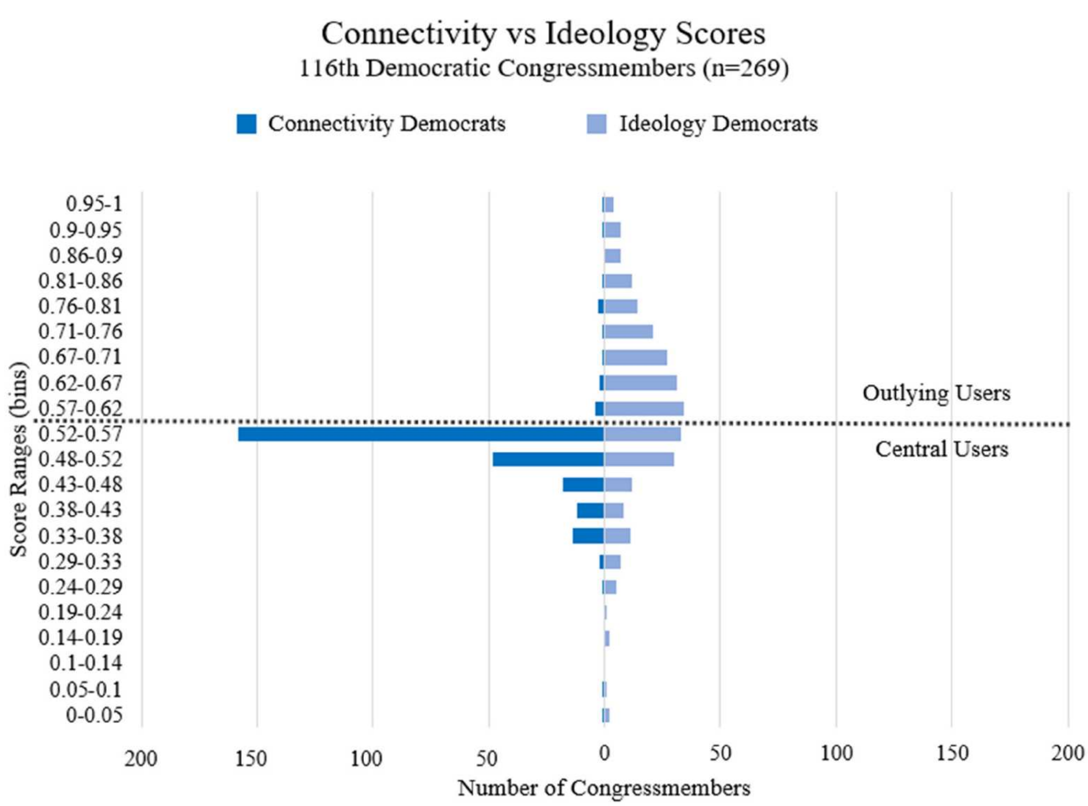

Figure 11. Comparison of the Ideology and Connectivity Scores for 269 Democratic Congressmembers.

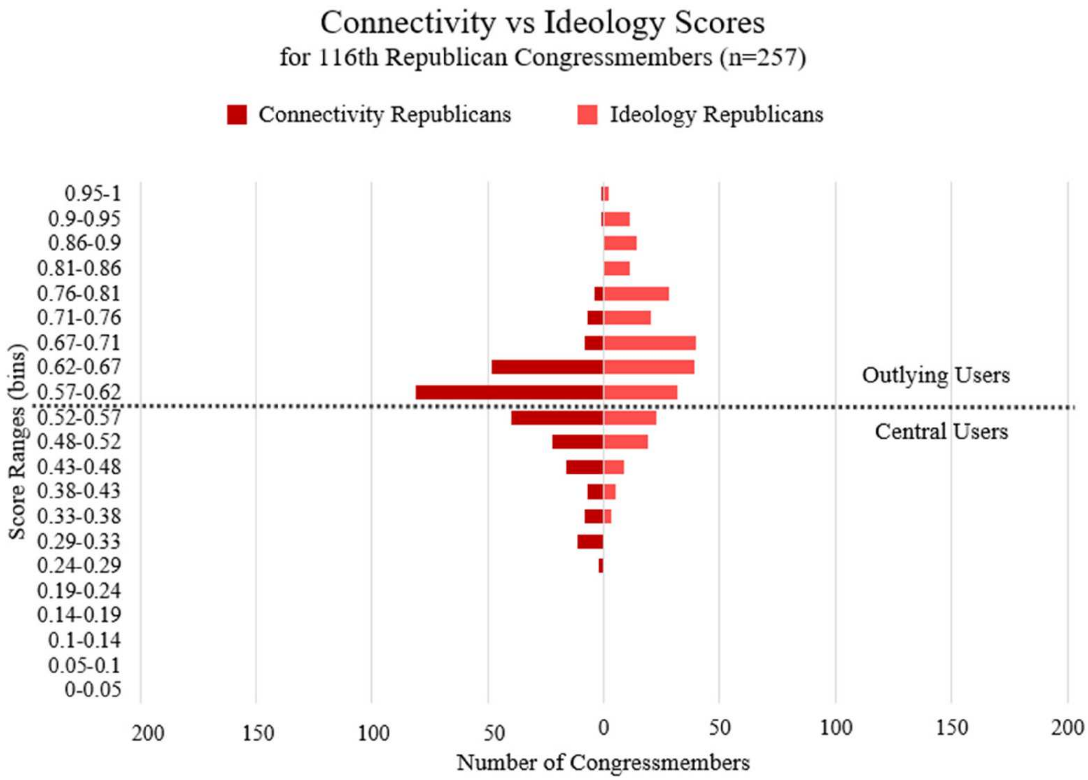

Figure 12. Comparison the Ideology and Connectivity Scores for 257 Republican Congressmembers.

the co-sponsorship of bills data which likely has a larger political impact than connections data on Twitter, so polarized behavior of politicians in real world (co-sponsorship matrix) is bolder than social media.

5.3.2. Roll call scores versus connectivity scores

Voteview proposed a model to analyze politicians' polarization level based on their roll call vote behavior referring to the 'yea' or 'nay' vote a Congressmember casts when presented with a measure (Desilver, 2022). The roll call votes are used within the *Dynamic Weighted NOMINAI Three-step Estimation* (*DW-NOMINATE*) model to create a two-dimensional spatial map. Closeness between Congressmembers in this spatial map represents a similarity in their voting records. The model has some shortcomings such as the 'mis-categorization' of canonically Liberal Representatives Alexandria Ocasio-Cortez, Lauren Underwood, Rashida Tlaib, and Ilhan Omar as moderates (Lewis, 2022b). They explain that when a member votes against most of their party, their position will be greatly affected (Lewis, 2022a). Additionally, new Congressmembers have significantly less historical data which would otherwise stabilize their position.

We exported the DW-NOMINATE scores calculated based on roll call votes for 116th Congressmembers in October 2022, then normalized them by Equation (3) for accurate comparison. We will refer to these normalized values as *Roll Call Scores*. Figure 13 shows the Roll Call Scores for 527 Congressmembers. Higher scores (closer to 1) refer to higher levels of polarization.

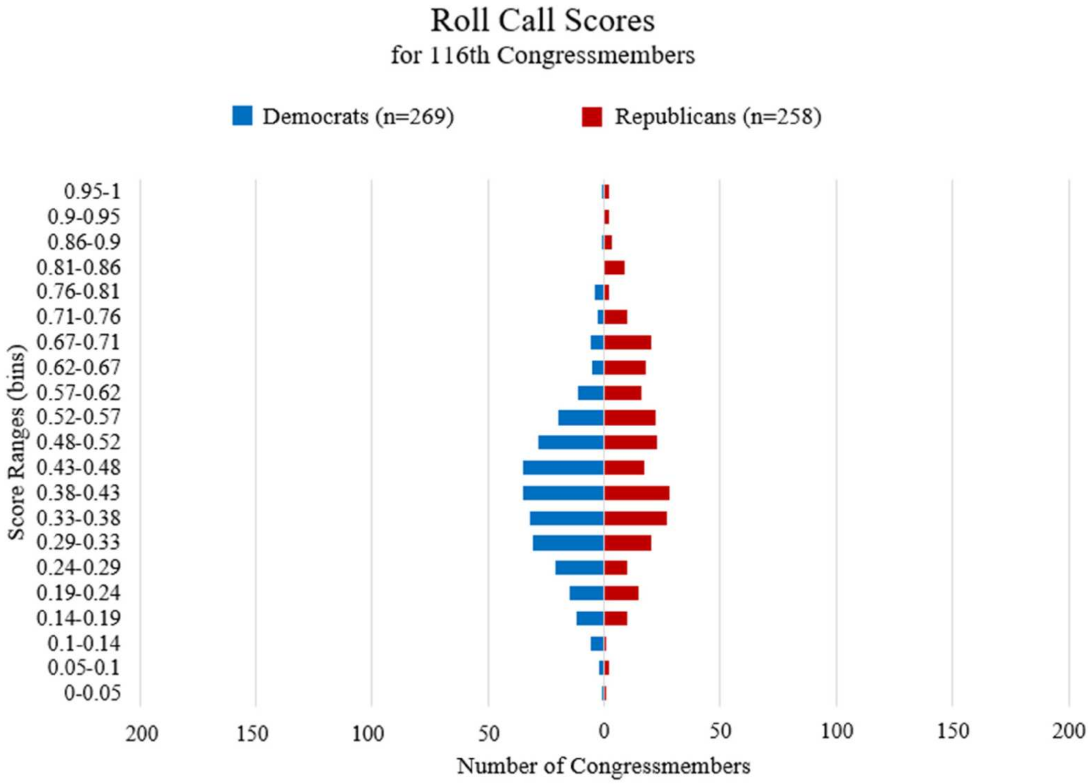

Figure 13. Distribution of the Roll Call Scores of the 527 Congressmembers.

Figure 13 reveals that the Roll Call Scores skew higher for Democrats than Republicans. This is an interesting observation in comparison to Ideology Scores which said the opposite. Also, Republicans' Roll Call Scores do not taper very much when approaching the extremes of polarization scores. This may imply a wide distribution of roll call voting behavior in Republicans in comparison to Democrats.

The Roll Call Scores are compared to the Connectivity Scores in Figure 14 and Figure 15, for Democratic and Republican Congressmembers. In Figure 14, over 44% of Democrats show higher polarization behavior in their Twitter following behavior (Connectivity Score). However, using these politicians' real-world Congressional behavior, far fewer members show high level of polarization (with Roll Call Score greater than 0.52).

In Figure 15, although distributions of Republican Roll Call Scores and Connectivity Scores are very different, both for most members are between 0.24 and 0.71. However, similar analysis cannot be observed for Democrats (Figure 14). Therefore, Republicans' polarization behavior is slightly more consistent within their social media and their real-world political role, compared to Democrats.

5.4. Discussion and analysis of results

Now, let's compare all scores (Connectivity, Sentiment, Ideology, and Roll Call) for 116th Congressmembers to investigate potential correlations. This comparison is possible since all scores range between 0 and 1, with higher values indicating greater polarization and

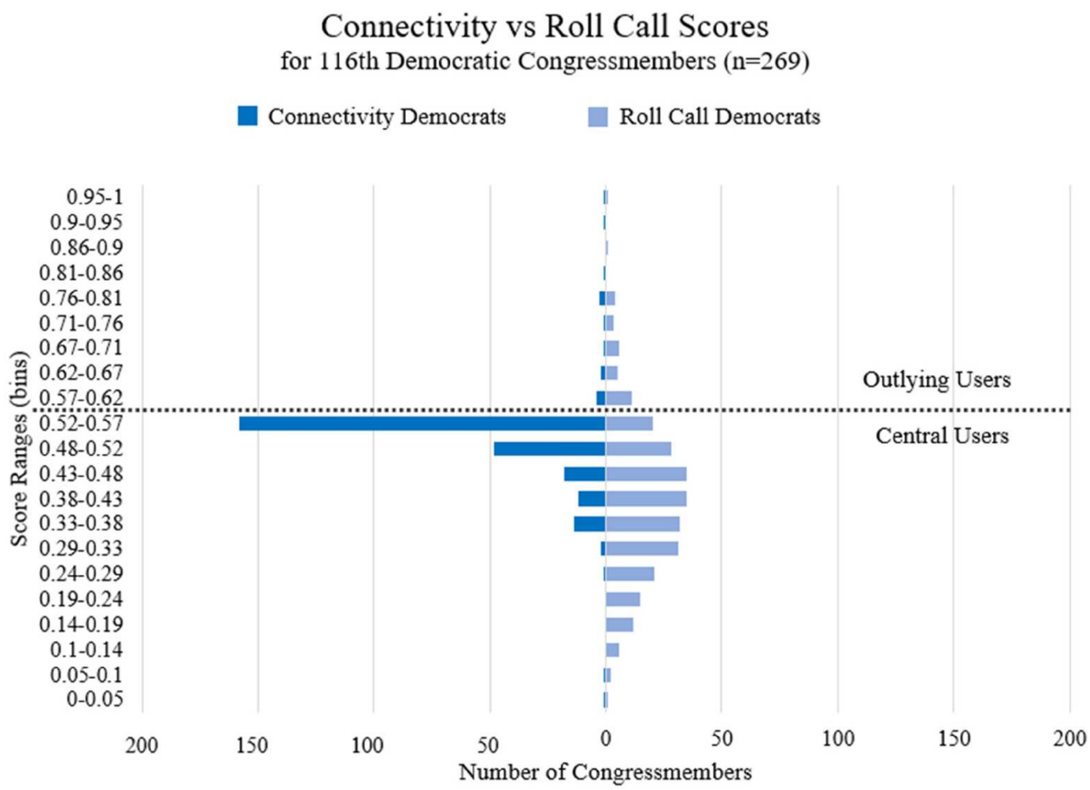

Figure 14. Comparing Roll Call Scores and Connectivity Scores for 269 Democrats.

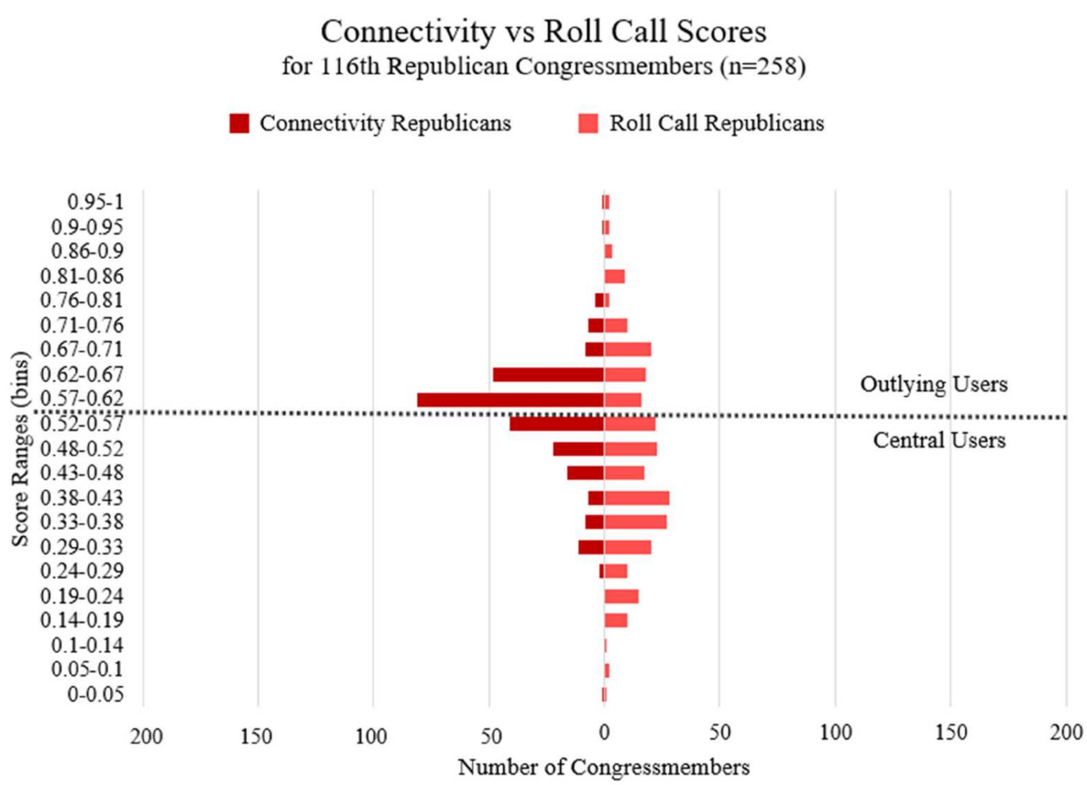

Figure 15. Comparing Roll Call Scores and Connectivity Scores for 269 Republicans.

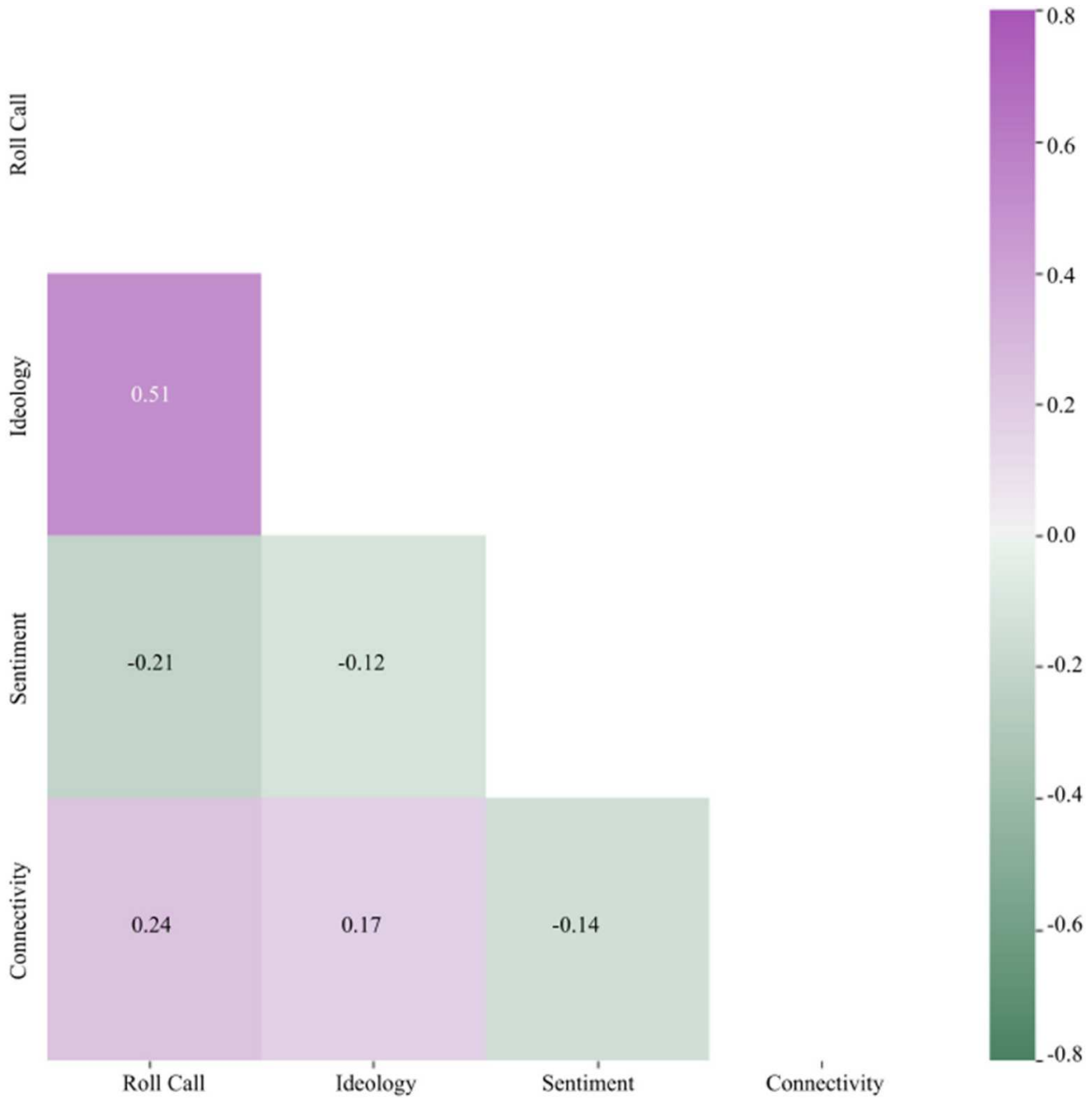

Figure 16. Pearson correlation matrix for connectivity, sentiment, ideology, and roll call scores.

lower values indicating neutrality. We generated a Pearson correlation matrix with heat map coloring in Figure 16.

An interesting observation in Figure 16 is that the Connectivity Scores have a positive correlation with both Roll Call and Ideology Scores. This implies a similarity between politicians' social media following and their real-world political interactions (e.g., bill voting and co-sponsoring a bill). The highest correlation exists between Roll Call and Ideology (0.51), which is intuitive given both focus on politicians' real-world behavior. Politicians' Roll Call Score (bill-voting behavior) has more correlation to the Connectivity Score (social media following), 0.24, than the Ideology Score (bill co-sponsorship behavior), 0.17.

On the other hand, the Sentiment Scores have negative correlation coefficients with each of the other scores, meaning that the sentiment expressed in politicians' Tweets results in a very different polarization analysis compared to other methods. This may be due to sentiment analysis limitations discussed in Section 5.2.

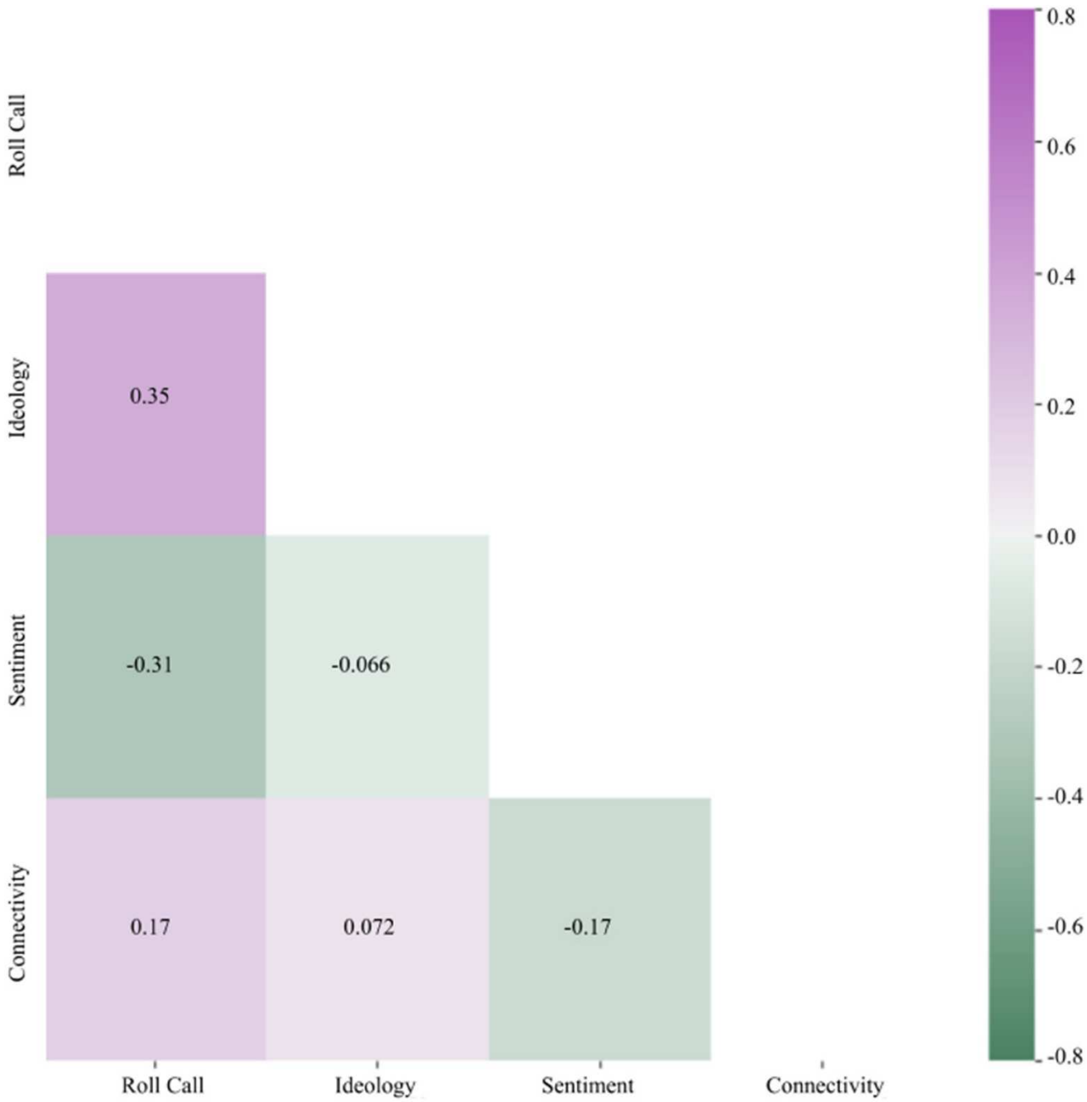

Figure 17. Pearson correlation matrix for scores belonging to Republican users.

Moreover, analysis of a single party across all four scores may yield a new perspective. Thus the Pearson correlation matrix was recalculated using only Republican scores, Figure 17, and only Democratic scores, Figure 18. In Figure 17, the correlation between Connectivity and other scores decreased across the board when focusing on Republicans' data. The correlation between Republican Connectivity and Ideology Scores decreased by 130% (from 0.17 to 0.072). The similar trajectory occurs to Republican Connectivity and Roll Call. This implies that Republicans show a different polarization behavior when following politicians on Twitter versus when they politically interact (e.g., co-sponsoring bills and voting in the Congress).

Beyond correlation with Connectivity, in Figure 17, there is a notable 47% decrease in the correlation between Republican Sentiment and Roll Call Scores. This emphasizes the differences between polarization analysis of politicians' Tweets versus voting behavior analysis. For Democrats in Figure 18, their Twitter following behavior (Connectivity), bill-voting behavior (Roll Call), and bill co-sponsorship behavior (Ideology) are all

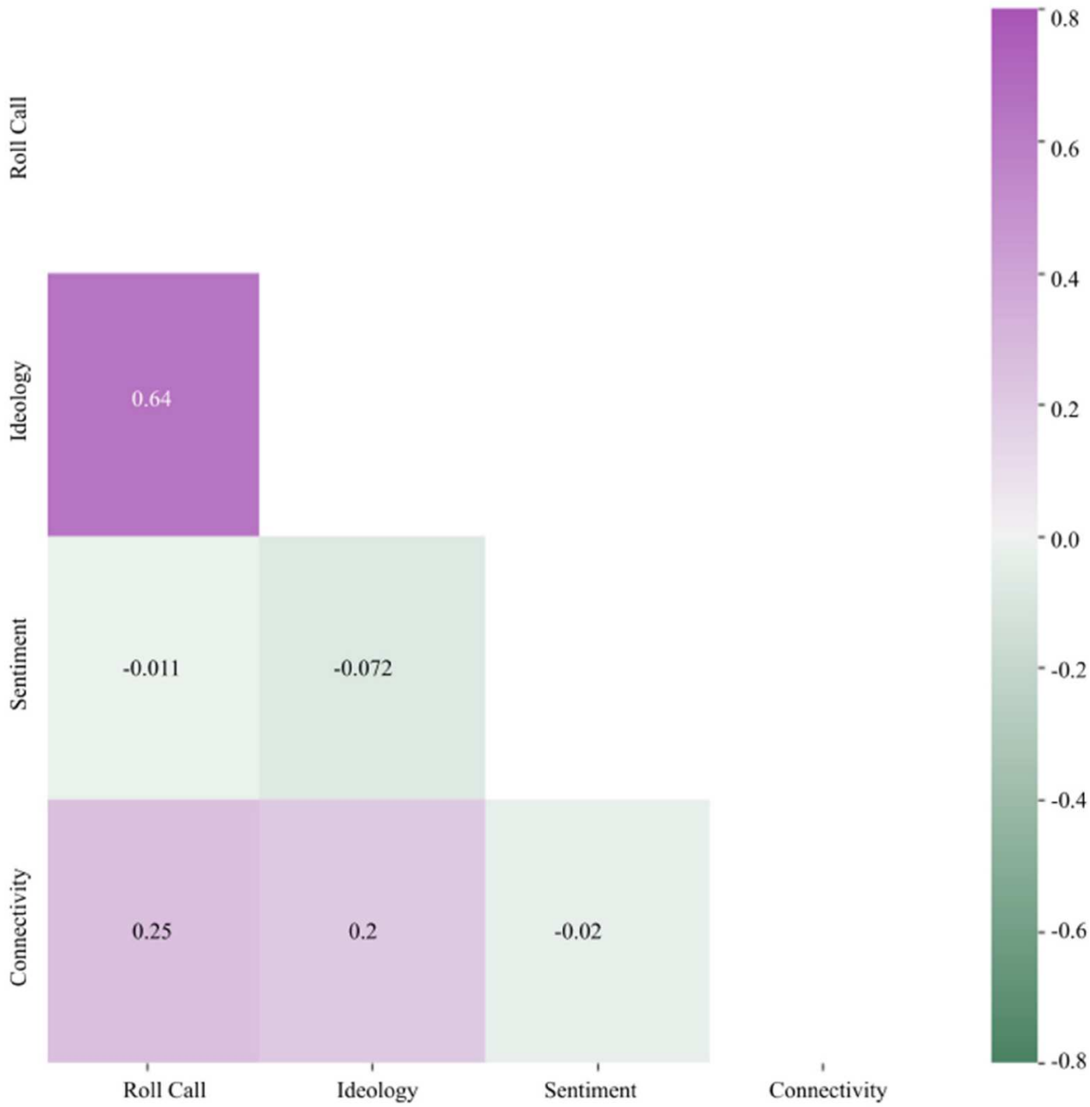

Figure 18. Pearson correlation matrix for scores belonging to Democratic users.

significantly more correlated than Republicans. Beyond Connectivity, almost all other correlation coefficients calculated for Democrats (in Figure 18) moved in the more positive direction. In Figure 16, Figure 17, and Figure 18, Sentiment Scores do not show any positive correlation with other scores.

Now, we ranked the politicians in our sample based on their polarization level using all four scores to see if the scores agreed on the specific ranking of any individual politician. We analyzed the scores to identify the top 10% most polarized politicians across each score. Given the lack of correlation between Sentiment and other scores, the Sentiment Scores are excluded. The results show that there are four politicians, all Republican, who are in the top 10% across the three scores (Connectivity, Roll Call, and Ideology). Their scores are listed below in Table 5.

In comparison to expectations, Representative Brian Babin ranking highly is expected given his most recent behavior in the 2020 election (Otten et al., 2022).

Table 5. Politicians ranked as top 10% most polarized by Connectivity, Roll Call, and Ideology.

Name	Party	Normalized Connectivity	Normalized Roll Call	Normalized Ideology
Representative Brian Babin	R	1.00	0.722	0.853
Representative Greg Steube	R	0.632	0.685	0.899
Senator Mike Braun	R	0.630	0.834	0.947
Representative Ralph Norman	R	0.625	0.884	1.00

Table 6. A summary for connectivity, sentiment, ideology, and roll call scores.

Aspects	Connectivity	Sentiment	Ideology	Roll Call
The input metadata (per politician)	Twitter connection adjacency matrix & party affiliation	Tweet content	Bill co-sponsorship adjacency matrix	Bill-voting adjacency matrix
Sensitivity to Data Variation	Stable (Twitter following data is relatively stable comparing to content)	Sensitive (dependent on the size of logged data/content)	Semi-stable (stable for longer-term politicians)	
Replication	Doable: access to users' connections data	Difficult: dependent on dynamic content & lack of access to content	Doable, but large dataset required	
Range of non-normalized score	[1, $+\infty$) e.g., the higher score means more polarized	[0, 1] e.g., the higher score means more polarized	[0, 1] e.g., 0: 'most politically left'; 1: 'most politically right'	[−1, 1] e.g., -1: more economically liberal; 1: more economically conservative
Limitation	• Subject to potential variation depending on dynamic trends • Requires knowledge of node belongness to sub-graph (party affiliation) • Less effective for sparse network	• Subject to potential variation depending on dynamic trends • Bias in data collection and algorithm training • Focus on words out of context • Massive size input data	• Sensitive without large dataset • Applicable to only politicians and no other polarized network	
Advantage	• Extendable to other polarized networks (beyond political), e.g., Network of fans of rival sports teams, pro/anti vaccination, pro/anti celebrity trials, etc. • Scalability to larger network due to using small memory storage for input data	• Extendable to other polarized networks (beyond political)	• Based on real-world behavioral data • Good at separating politicians into political sub-groups	

Finally, we summarized different aspects of these four scores in Table 6. As for the limitations (Table 6, the fifth row), it is noteworthy that social media content is subject to more frequent and potentially significant variation (which is relevant to Sentiment) in comparison to the number of following/followers (which is relevant to Connectivity).

5.4.1. The impact of availability of data on social media

The availability and access to data on social media platforms have consistently posed obstacles to research in the field of polarized networks, driven by factors such as platform policies, user privacy concerns, and the high costs associated with obtaining users' data. Two situations might occur regarding the data availability on social media platforms:

(1) When more data is available (including metadata and content), then multi-dimensional investigation of network is more beneficial (Xiao et al., 2020). This means, ideally, multiple methods can be used simultaneously (Connectivity Score along with content-based analysis, with different type of input data) to enhance our comprehension of different dimensions of social media polarization and overcome some limitations of using each method in isolation.
(2) When dealing with limited data availability on platforms, the applicability of methods relying on minimal input data becomes crucial. The Connectivity Algorithm stands out by requiring relatively less input data compared to Sentiment and other content-based methods in the literature. Its input parameters include only the following/follower relationship between users in the network. Notably, on many platforms, the number of followers/following is public data, even for private accounts, whereas accessing content can be impossible for private accounts on most platforms. This characteristic makes the application of the Connectivity Algorithm more practical and adaptable in scenarios with restricted content access. The ease of collecting relationship (following/follower) data is dependent on density of the network considered.

5.4.2. Computational efficiency of algorithms

This section briefly compares the efficiency of our algorithm to sentiment analysis. Unlike content-based methods (including sentiment analysis), which demand large amounts of memory storage while logging the entire corpus (including the content and related metadata), our algorithm does not require the whole corpus as input data (Wankhade et al., 2022). In terms of computational complexity, our algorithm's runtime depends on the size of the network, whereas sentiment analysis's runtime is contingent on both the complexity of the training model used and the size of the corpus. The computational differences can be elaborated as follows.

The shortest path calculation in our Connectivity Algorithm utilizes a priority queue data structure (refer to set Q, Lines 9 and 18, Connectivity Algorithm) to keep track of visited nodes and find the distance from the center. In this calculation, all nodes (users) and edges (connections between nodes) are eventually visited. Therefore, the time complexity of this shortest path calculation is $O(|E_i| + |N_i| \log |N_i|)$ for graph $G_i(N_i, E_i)$ demonstrating that the computational time of our algorithm increases with the size of the network (number of users) and the number of connections (edges).

For sentiment analysis, achieving more accuracy necessitates increasing the training data size and complicating the training model, leading to an exponential rise in the computational complexity of the sentiment analysis model. High-end graphics processing units (GPUs) may be required to train a model with a huge corpus. Even using less computationally costly learning algorithms such as SVM (Support Vector Machine) and NB (Naïve Bayes) for sentiment analysis has its disadvantages, such as long training times for large datasets and reliance on assumptions like attributes are mutually independent (Wankhade et al., 2022).

6. Conclusion and future work

This study proposes a novel methodology for quantifying polarization of users on social media networks, focusing on network structure rather than content analysis. We model polarized networks as a directed graph where nodes and directed edges denoted users and following/follower connections between them. It is assumed that the two sub-graphs (referring to contrary polarized groups) are already identified. We proposed a practical algorithm that can be applied to each sub-graph and calculate a Connectivity Score for each node solely based on its following/follower connections. This score can represent the polarization level of the node, providing a base for comparison against other nodes.

We applied our algorithm individually to Democratic and Republican sub-graphs of the 116th Congressmembers Twitter network, and the results were compared to three other existing methods used to quantify polarization: Sentiment Score, Ideology Score, and Roll Call Score. In the case of Independents, Congressmembers were grouped with the party they associate and caucus with historically. This is to ensure accurate political ideology grouping and reduce potential bias. Our analysis demonstrated that polarization behavior on social media can be defined by examining users' connections. Additionally, we confirmed that users' real political behavior has some reflection on how they make connections on social media (observing a meaningful correlation between Connectivity and Ideology or Roll Call Scores). Moreover, we observed that Democrats' Twitter following behavior (Connectivity), bill-voting behavior (Roll Call), and bill co-sponsorship behavior (Ideology) are all significantly more correlated than Republicans.

We contribute to the literature through the following advantages: (1) the main required input data for our algorithm is users' connections which are often public, quick to obtain on most platforms, and does not require much storage which can make it scalable for larger network. (2) Our method is based on objective data between users, unlike content-based methods which need to deal with language capacity, interpretation, and biases related to training. (3) We analyze the network for polarization at the user level, rather than the community or network level. (4) Our approach may require less input data compared to existing content- based approaches.

Finally, we recommend utilizing multiple methods (*CS* along with content-based analysis for nuanced opinions, community detection when community division is not known a priori (Blondel et al., 2008), or Guerra's network and community level metrics (Guerra et al., 2021)) which can enhance our comprehension of different dimensions of social media polarization and overcome some limitations of using each method in isolation. One example could be defining polarization at all layers: (1) at the network level, (2) at the community level, and (3) at the user level using *CS*. Using our method could pave the way for structural solutions to mitigate social media polarization, i.e., modifying the 'connection suggestion' of social media platforms which will be a future direction for this study.

Notes

1. X is still commonly referred to by its prior name Twitter, and will be throughout this paper to maintain continuity (Ivanova, 2023).

2. Operator '=' denotes equivalence between the left- and right-hand side of the operator, while '←' is used to denote that the value on the right side of the operator is assigned to the item on the left. Furthermore, the initial value of all parameters and variables is set as 0, except s_n, which is initially set as $+\infty$ for all $n \in Ni$ (Line 2).
3. Since 1935, meetings of Congress, dubbed 'nth Congress', last for two years, starting and ending at noon on January 3rd of subsequent odd-numbered years (unless law designates a starting or ending date other than January 3rd). For instance, the 116th Congress ran between 3 January 2019 and 3 January 2021 (US Senate, 2023).

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No potential conflict of interest was reported by the author(s).

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Data availability statement

The authors confirm that our data is available within the article and its supplementary materials.

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