CoOrdinated Math-Physics Assessment for Student Success (COMPASS) assessments on continuing math courses and attitude toward math

Dr. Guangming Yao, Clarkson University

Guangming Yao (Associate Professor and Executive Officer, Clarkson University): Professor Yao received her BSc and MS in Mathematics and Applied Mathematics from Harbin Normal University and PhD in Computational Mathematics from the University of Southern Mississippi. She joined the Department of Mathematics and Computer Science at Clarkson University in 2012. Her research focuses on computational and applied PDEs and radial basis functions and their applications. She has served as a reviewer for Journal of Applied Mathematics and Computing, Journal of Credit Risk, Communications in Nonlinear Science and Numerical Simulation, Computers and Mathematics with Applications, Applied Mathematical Modelling, Applied Mathematics and Computation and many others. Professor Yao organized regional mathematics conference, the Mathematics Conference and Competition of Northern New York (MCCNNY), for undergraduate and graduate students since 2014 at Clarkson (once every two years). She served as faculty advisor for McNair Program, honors program at Clarkson, and advised many students through direct studies. She also published over a dozen peer reviewed journal papers with undergraduate and graduate students.

Dr. Kelly Black, University of Georgia

Kelly Black is a faculty member in the Department of Mathematics at the University of Georgia. His primary interests are in student learning in the introductory mathematics curriculum as well as mathematical modeling in ecological systems.

Dr. Michael W. Ramsdell, Clarkson University

Michael Ramsdell is an Associate Professor of Physics and Director of First Year Physics at Clarkson University. He has over ten years of experience in the design, implementation, and assessment of laboratory curriculum within introductory physics courses. He has also developed, refined and taught a Pre-Freshman Physics course designed to assist student s with the transition to post-secondary education. He is a Co-Director of the NYS STEP Program, IMPETUS which provides economically disadvantaged students the opportunity to pursue their interest in math and science though educational summer camps, workshops, school-year tutoring and mentoring programs. He has helped provide numerous students and teachers with the opportunity to integrate STEM disciplines using real-world problem solving strategies through teacher/coach training institutes.

Dr. Matthew K Voigt, San Diego State University

Matthew Voigt (He, Him, His) is a mathematics education researcher with focus on calculus and equity in mathematics. He has a PhD in Mathematics and Science Education through the joint doctoral program at San Diego State University and the University of California San Diego.

Kalani Kithuliya Rubasinghe Kattadige, Clarkson University, New York

Kalani Rubasinghe is a third year Ph.D. student in Mathematics at Clarkson University, New York and she obtained her Master degree in Mathematics from Clarkson. Her research focuses on numerical solutions for PDEs, using Radial Basis Functions (RBFs).

Dr. Wen Li, Department of Mathematics, University of California, Los Angeles

Wen Li is an Assistant Adjunct Professor of Mathematics at University of California, Los Angele. She is teaching mathematics upper-division courses. She also has several years of experience in teaching lower-division mathematics courses. She received her PhD in Applied Mathematics from Clarkson University.

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Abstract

The COMPASS: CoOrdinated Math-Physics Assessment for Student Success program aims to improve students' understanding of mathematical concepts using physical applications. COMPASS is a first-year calculus course that combines mathematics concepts with physical applications in an effort to improve student understanding of mathematics using their outstanding physics intuitions. The implementation of the COMPASS program is described briefly in this paper. We design and experiment this novel calculus/physics instructional program to understanding whether it is possibly to benefit students in STEM disciplines.

In this paper, we compare students' reports on instructional strategies and beliefs about learning mathematics for COMPASS, COMPASS-eligible and non-COMPASS students during academic year 2017–2018. We track students' performance in their continuing mathematics courses for the groups from academic year 2015-2016, for which most of the students completed calc 3 and Elementary Differential Equations during academic year 2016-2017. The data we collected shows that students who went through COMPASS program reported positive instructional experiences, increase in interest during their year-long training in COMPASS and they performed well in their continuing mathematics courses, regardless of their initial weaknesses in math prior to attending college.

The study is likely to interest a broad group of engineering education researchers and/or practitioners to disseminate knowledge on engineering teaching and learning since mathematics is a common problem throughout engineering education. In addition, even though the improvement of COMPASS students in continuing math courses are not significant, their slightly better performance shows a great improvement in comparison with their low math test scores as high-risk category group students. The COMPASS program may improve instruction through the development of innovative materials and sound instructional designs.

Keywords: CoOrdinated Math-Physics Assessment for Students Success (COMPASS), Differential Equations, Calculus, Progress Through Calculus

Introduction

In the United States (US) and elsewhere introductory mathematics courses, specifically calculus, often serves as a bottleneck, preventing large numbers of STEM-intending students from advancing in their majors [1]. The need for increasing the number of STEM graduates [2, 3, 4, 5, 6, 7] has resulted in various programs to attract and retain students in STEM fields. Bressoud, Mesa and Rasmussen [8] identified several characteristics of successful calculus programs throughout the US which includes incorporating deep and engaging mathematical content as well as the use of variations to traditional calculus courses (e.g., calculus for physics, stretched out calculus). In further research [9] in a national study of precalculus through calc 2 programs has shown that course variations can help support student success outcomes, and level the playing field for students with less mathematical preparation. Additionally, there is emerging evidence that course variations that holistically target students with less preparation can support students in developing productive dispositions to mathematics [10]. Yet these previous studies which focus on national trends fail to capture the impact and implementation considerations of specific course variations. To address this national need Clarkson University has focused on increasing retention rates, and the first step has been to study the needs and outcomes of calculus programs [11] to support student success. We especially focus on the CoOrdinated Math-Physics Assessment for Student Success program (COMPASS). This program aims to improve students' understanding of mathematical concepts through the use of deep and engaging content which utilizes physical applications. As such, COMPASS is a first-year calculus course, which is a variation to the traditional calculus sequence at Clarkson, that combines mathematics concepts with physical applications in an effort to improve student understanding.

It is crucial for college educators to find out how to help students to succeed in their beginning years of college, especially those who show potential and engagement within the STEM discipline. We already know that co-teaching physics and calculus, to students who are not fully prepared for calculus, enhances the learning and success in both subjects. Over the past 15 years, we have shown that the best predictor for success in physics is not the students' physics readiness, but their calculus preparation.

There exists a plethora of research reports and papers that focus on students' performance in relation with their background in calculus and physics [12]. Many of the papers even specifically focus on how to improve physics education [13, 14] and calculus-based physics education [15, 16, 17]. Back to 1992, Yeatts *et. al.* published their understanding of challenges at the interface between calculus and physics [18]. For instance, physics instructors may underestimate students computational skills while overestimating their understanding of calculus concepts, while at the same time calc 1nstructors may be less attune to the needs of their students to succeed in physics. We also understand that students initial knowledge of physics could help their understanding in abstract calculus concepts [19], although not much researches have been done on physics based calculus courses [2]. Thus it is important to find out whether the proposed program, which serves a physics based calculus course variation, is working effectively on improving students' retention rates and dispositions towards mathematics. This study serves as a case study analysis on how such a program can be implemented to impact students' outcomes and provide recommendations and insights for the scalability of such program.

More detailed explanation on the COMPASS program can be found in [20]. In this paper, we will give a brief introduction on the COMPASS program structure in Section 2. Detailed explanation on the data we collected through past years of running COMPASS will be given in

Section 3. In Section 4 and Section 5, we will analyzes those data to find the relation between students' pathways in their first year of college and their feedback on their progress and overall performance in their continuing math courses. In Section 6, we will provide some strengths and weaknesses of our approach. A short conclusion will be given in Section 7.

Background and Rational

Prior to the start of classes all STEM students at Clarkson are required to take two preparatory exams. The first exam, the Force Concept Inventory (FCI) [21], is used by the physics department as part of their effort to compare student gains before and after the students take part in the introductory physics course. The second exam is a locally developed mathematics test that measures students' basic computational competency. The skills measured include basic algebra, basic trigonometry, and manipulation of equations with logarithmic and exponential terms.

The expectation was that the scores on the two placement exams would be highly correlated, but surprisingly this was not the case [11]. Instead, four groups were identified in terms of their performance on the two exams: students who did well and those who did not do well on the physics exam; likewise, the students who did and those who did not do well on the math exam were identified. Students with low scores in both exams have been asked to take part in a variety of activities designed to bolster basic skills such as time management and study skills. We analyzed historical data using the preparatory exams after semester I for calc 1 and physics I for our historical cohort of students (2006–2010). The physics score plotted versus the math score reveals a lens through which risk levels may be assigned. A student's math categorization is one of M+ or M-. An M+ indicates that the student earned a score greater than the designated cut-off on the math assessment, while an M- indicates that the student earned a score lower than the designated cut-off. The P+ and P- designation is determined in the same manner based on a student's result on the physics exam. Data on the numbers of individual students indicating their math and physics scores are categorized as (M+, P+), (M-, P-), (M+,P-) and (M-, P+), where (M+, P+) indicates a student within a low risk group while (M-, P-) is a student within the high risk group, and the remaining (M+, P-), and (M-, P+) are categorized as medium risk groups. The categorization points students to one of three risk categories which leads them to different pathways designed to leverage these students' relative strengths in math or physical intuition to develop the other. The target group for COMPASS are students who scored below the cutoff in math and above the cutoff in physics (M-, P+) group.

The students who performed poorly on the math exam but achieved higher scores on the physics exam, (M-, P+), have shown to also have lower retention rates than the other groups [11]. To address this group of students Clarkson University launched COMPASS program starting the fall semester during academic year 2015. The program offers cross-disciplinary STEM students a specific designed first year calculus courses, especially for those who are in (M-, P+) group. The goal is to improve students performance in early STEM courses and increase retention rate. The courses feature a calculus sequence that has been redesigned to coincide with the physics courses, and the mathematical concepts are introduced in the concept of physics problems with the goal of examining more abstract mathematical ideas once the concrete notions are formulated. The program identifies students' needs prior to enrollment using the physics and math scores. The students' high physics intuition may reflect their higher intuitive understanding of the real-world physical problems, while needing support in understanding of abstract mathematical modeling and manipulations skills. COMPASS focuses on taking advantages of students' physics knowledge to enhance their math ability. The cross-disciplinary discussions in calculus courses help students reflect their physics knowledge and in addition improve their ability to understand abstract concepts in math [22].

Previous reports on students' performance on calc 1, calc 2, Physics 1 and Physics 2 can be found in [20]. In this report we examine the students who took part in the program with those who were qualified to take part but chose to take part in the regular sequence in comparison with the larger groups of students (COMPASS eligible and all other students). We investigate possible effects of the COMPASS program on students' performance on calculus class and continuing mathematics courses and on their attitude toward studying mathematics through instructional practice surveys [2].

Data Collection

Once the placement exam results are available students are advised to enroll in their first-year classes. In general, Clarkson offers calculus courses in a large lecture format. Each section of the calculus class would include around 100 students. Students also meet in smaller recitation sections once a week and meet with graduate students who go over example problems and answer questions.

Students who do well in the physics portion of the tests but have not demonstrated strong competency in basic mathematical skills are asked to take part in the COMPASS program. We randomly selected around 30 students that qualified to be enrolled in COMPASS and send the list to student administrative service which is the division who enrolls first year students at Clarkson. The COMPASS program allows Clarkson to offer a small section of calculus class, and the class covers the same topics as the other calculus sections. The order is changed, however, and the topics are first introduced in the context of current topics being covered in their physics class. For example, when the physics class focuses on kinematics, calculus focuses on rate of change and derivative/anti-derivative; when physics focuses on work/energy/momentum/impulse, calculus focuses on anti-derivative/integrals/continuity/limits; and when physics focuses on rotational motion/oscillations, calculus focuses on trigonometric functions. The students will continue in the program for their calc 2 in the following semester with the same group of students and with the same professor unless they fail the calc 1 class at the end of previous semester or if they have scheduling conflicts with other courses.

The students take part in a sequence of two calculus courses: calc 1 and calc 2, and our interest extends to the calc 3 (MA231) and Elementary Differential Equations (diffeq, MA232). The students' groups are given below:

- COMPASS: students who are in COMPASS program (we will use "COMPASS" in following discussions)
- Eligible: students who are not in COMPASS program but who are part of the (M-,P+) group ("eligible" will be used)
- Others: all other students who are not in COMPASS program including COMPASS eligible students ("others" will be used)

In the first study of COMPASS program [20], the students were divided into three groups as described below. We will continue the same group characterization. The progress of the students in the various programs is tracked. At the end of each semester we track whether or not the student is still enrolled in a STEM program of study. Additionally, we ask students to respond to a series of questions to gauge how their attitudes toward mathematics change over time. Finally, we also record student performance on a number of summative assessments including their final exam scores and mid-term test scores. The rationale and details of the specific records examined here are given in the section below.

Methods

The main thrust of our efforts is to understand the impact of COMPASS on improving retention, and we examine students' completion rates for their subsequent academic program, in which most of the students are taking two subsequent mathematics courses during fall semester of 2016 and spring semester of 2017: calc 3 Calculus III and diffeq Elementary Differential Equations. However, those courses are not part of the COMPASS program. We examine student performance by comparing the three groups. The following questions are going to be addressed in this study

1. Is there a significant difference in calc 1 (calc 1) and calc 2 (calc 2) final exam scores between COMPASS students and other students?
2. Is there a significant difference in calc 3 (calc 3) and diffeq (Elementary Differential Equations) final course grades between COMPASS students and other students?
3. What are the attitudes of students toward math and reported instructional experiences in both the COMPASS class and other traditional class for groups in 2017-2018?

Even though all professors follow the same syllabus and grade distribution system, the final course grades can still be influential on the same students in different sections of the same courses. Thus, we focus on students performance on final exam grade in calc 1, calc 2, PH131 and PH132 instead of course grades. However, calc 3 and diffeq are taught from professors who are not related to COMPASS program, and all students (whether they were COMPASS students or not), are distributed in different sections of the courses in two different semesters (fall semester of 2016 and spring semester of 2017). Thus, we evaluate students courses grades for calc 3 and diffeq.

The data consists of comparisons to scores on the students final exams for calculus and physics courses as well as the students' scores on individual questions. Additionally, we collected the students' final course grades in the courses of interest (calc 3 and diffeq). The final exams were graded using teams of graduate students and course instructors. Each question was graded by the same person, and the scoring was checked to insure it was uniform for the whole class. The scores for all students were recorded immediately after the completion of the grading. The students were given the tests in the same session under identical conditions.

In addition to students' academic performance their attitudes toward the subject of mathematics is also of interest. The students' attitudes toward math at their first semester fall 2017 and their second semester, spring 2018, are compared, by analysis on the student post-secondary instructional practices survey mathematics (SPIPS-M) results. This set of data was found from the students' responses to the survey administered as part of a larger NSF-funded study, Progress through Calculus (PtC). The SPIPS-M survey [23] was developed based on previously reliable and validated student survey instrument scales used to asses STEM instructional practices [24]. The survey seeks to broadly understand how students describe instructional practices in precalculus and calculus courses, their attitudes towards mathematics, as well as items related to inclusive and equitable learning experiences. Students were asked to complete the SPIPS-M survey questions administered via Qualtrics three quarters of the way through the term, and were asked to respond using Likert scale responses. It is assumed that the students were consistent and honest in their responses, and erroneous data was removed from subsequent analysis.

Results

We first examine the descriptive statistics for the collected information on students' experience in the COMPASS program in comparison with other groups. The numbers of students in each groups are listed in Table 1. The significance level for testing all hypotheses was set at 0.05.

Table 1: Number of students within each of three groups in academic cohort 2015–2016 and cohort 2017–2018 at Clarkson University, where cohort 2015–2016 data was used to understand students' continuing math courses performance, whereas cohort 2017–2018 data was used for analyzing students' attitudes toward math at beginning and at the end of the compass program.

Semester	COMPASS	eligible	others
calc 1 F2015	25	146	492
calc 2 S2016	18	128	420
calc 3 F2016/S2017	13	82	276
diffeq F2016/S2017	14	99	328
calc 1 F2017	25	76	322
calc 2 S2018	19	65	251

Survey analysis

As part of a national project examining undergraduate math classes, Progress through Calculus (PtC), students are asked to complete the survey. From 2015 through 2019, the MAA is conducting a study of the precalculus through calc 2 sequence in U.S. colleges and universities, sponsored by NSF.

The survey takes about 20-30 minutes to finish. The analysis in this section provides descriptive statistics of the survey data for the three groups of students (COMPASS, COMPASS eligible, and all other students) who completed the survey for Fall 2017 and Spring 2018. There were $n = 15$ COMPASS responses, $n = 64$ COMPASS eligible responses and $n = 303$ all other students responses. Given the variation in sample sizes, descriptive averages are reported for some of the survey questions. Those questions from the survey were selected, based on whether it is related to students' experience in the course offered, and their self-evaluation on their ability to do mathematics. The preferred scores in the tables below and the lowest standard deviations are highlighted.

Q21 Indicate the degree to which the following statements describe your experience in regular course meetings (5 Very descriptive, 4 Mostly descriptive, 3 Somewhat descriptive, 2 Minimally descriptive, 1 Does not occur)

Table 2 and Table 3 shows the students experiences regarding Q21 during academic year 2016-2017. Due to the nature of the questions, very descriptive statement will receive 5 points, mostly descriptive statement will receive 4 points and so on. Since the statements are worded positively, the bigger the averages are, the more descriptive or frequent those particular instructional practices occurred. As we can see from both tables, most students in group 1 (COMPASS students) had more frequent beneficial or ambitious instructional experiences than other students in the traditional classes. For example, students in group 1 (COMPASS students) were more likely to describes the class activities connecting to their life and future work, which is one of the aims of the COMPASS program to link the mathematical content to physical applications. Additionally, COMPASS students are more likely to describe discussing their mathematical difficulties with other students a feature which is supported in the COMPASS program by placing students with similar preparation levels.

Q29 I believe that my math ability can be improved through dedication and hard work. (1 Strongly Agree $\cdots$ 6 Strongly Disagree)

We now analyze the average and standard deviation on Q29. The lower the average is, the more confident the students are in terms of believing dedication and hard work can improve their math ability, which is an indication of a growth mindset[25]. The COMPASS students have an average 1.5, and standard deviation 0.6, the COMPASS eligible students and all others have average 1.8, and their standard deviations are 0.8 and 1.1, respectively. This shows that the COMPASS students believe they have the ability to improve math skills with a smaller standard deviation. Again, we believe that students received more attention and got reminded frequently on the logistics of the classes more often in a smaller classroom setting.

Q30 Please indicate your level of agreement for the following statements from the beginning of the course and now. (1 Strongly Agree $\cdots$ 6 Strongly Disagree)

Table 4 shows the percentage of students whose attitudes (e.g., interest, enjoyment, confidence and ability) towards mathematics increased, decreased or remained the same as a result of the course. The 27% COMPASS students reported increases in their interest

Table 2: Students survey results for Q21 Indicate the degree to which the following statements describe your experience in regular course meetings (5 Very descriptive, 4 Mostly descriptive, 3 Somewhat descriptive, 2 Minimally descriptive, 1 Does not occur) –part 1.

	Average			Std. Dev		
	compass	eligible	others	compass	eligible	others
I listen as the instructor guides me through major topics	4.3	3.9	4.2	0.7	1.1	0.9
The class activities connect course content to my life and future work	3.3	3.0	3.0	1.0	1.2	1.3
I receive immediate feedback on my work during class (e.g., student response systems such as clickers or voting systems; short quizzes)	2.9	2.1	2.1	1.3	1.0	1.2
I am asked to respond to questions during class time	3.7	2.6	2.6	1.0	1.3	1.2
In my class a variety of means (models, drawings, graphs, symbols, simulations, tables, etc.) are used to represent course topics and/or solve problems	3.5	3.7	3.7	1.0	1.0	1.1
I talk with other students about course topics during class	3.3	2.6	2.8	1.3	1.3	1.3
I constructively criticize other students' ideas during class	2.5	2.0	1.8	1.5	1.2	1.1
I discuss the difficulties I have with math with other students during class	3.5	2.8	2.9	1.4	1.4	1.4
I work on problems individually during class time	3.6	3.1	3.1	1.5	1.2	1.2
I work with other students in small groups during class	2.2	2.0	2.0	1.0	1.2	1.2
I receive feedback from my instructor on homework, exams, quizzes, etc.	3.9	3.3	3.1	1.1	1.2	1.2
I share my ideas (or my group's ideas) during whole class discussions	3.1	2.2	2.1	1.2	1.0	1.2
I have enough time during class to reflect about the processes I use to solve problems	3.4	3.0	3.0	1.2	1.2	1.2

in mathematics as a result of the course, compared to the 12% of COMPASS eligible and 14% of other students. This is a promising finding, since the format and aim of the COMPASS program is foster student success by engaging them in mathematics through physical applications.

Although, the COMPASS group shows increased interests in math, there is little difference in their reported levels of enjoyment, confidence and interest compared to COMPASS

Table 3: Students survey results for Q21 Indicate the degree to which the following statements describe your experience in regular course meetings (5 Very descriptive, 4 Mostly descriptive, 3 Somewhat descriptive, 2 Minimally descriptive, 1 Does not occur) –part 2.

	Average			Std. Dev		
	compass	eligible	others	compass	eligible	others
Multiple approaches to solving a problem are discussed in class	3.6	3.6	3.5	1.1	1.1	1.2
A wide range of students respond to the instructor's questions in class	3.4	3.0	2.6	1.1	1.0	1.1
The instructor knows my name	4.3	2.8	2.8	1.4	1.4	1.4
Class is structured to encourage peer-to-peer support among students (e.g., ask peer before you ask instructor, having group roles, developing a group solution to share)	3.2	2.6	2.6	1.3	1.3	1.3
There is a sense of community among the students in my class	4.0	2.8	2.8	1.5	1.1	1.2
The instructor adjusts teaching based upon what the class understands and does not understand	3.9	3.4	3.1	1.2	1.3	1.3
The instructor explains concepts in this class in a variety of ways	3.6	3.5	3.4	1.2	1.3	1.2
A wide range of students participate in class	3.6	2.8	2.5	1.1	1.1	1.2
My instructor uses strategies to encourage participation from a wide range of students	3.6	3.1	3.0	1.4	1.2	1.3
Helpful-I listen as the instructor guides me through major topics	2.4	2.6	2.6	0.8	0.6	0.6
Helpful-The class activities connect course content to my life and future work	2.3	2.3	2.2	0.7	0.6	0.7
Helpful-I receive immediate feedback on my work during class (e.g., student response systems such as clickers or voting systems; short quizzes)	2.6	2.2	2.3	0.5	0.7	0.7

eligible students. Although there was minimal change reported in COMPASS students ability to do mathematics, as noted earlier COMPASS students reported overall higher levels of growth mindest believing that hard work and dedication could improve their math ability.

Q77 Do you think your previous math courses adequately prepared you? (1 Yes, 2 No)

The survey results from Q77 shows that 14% of the COMPASS students think that their previous math courses adequately prepared them for calculus at Clarkson, but 11% and

Table 4: Students survey results for Q30 Please indicate your level of agreement for the following statements from the beginning of the course and now. (1 Strongly Agree $\cdots$ 6 Strongly Disagree).

		Total	Inc.	Same	Dec.	Inc.	Same	Dec.
interest	compass	15	4	9	2	27%	60%	13%
	eligible	60	7	37	16	12%	62%	27%
	others	280	40	165	75	14%	59%	27%
enjoyment	compass	15	0	10	5	0%	67%	33%
	eligible	60	4	37	19	7%	62%	32%
	others	279	45	158	76	16%	57%	27%
confidence	compass	15	2	9	4	13%	60%	27%
	eligible	57	9	29	19	16%	51%	33%
	others	278	64	120	94	23%	43%	34%
ability	compass	15	1	13	1	7	87%	34%
	eligible	55	7	39	9	13%	71%	16%
	others	279	48	173	58	17%	62%	21%

19% for the COMPASS eligible students and all others, respectively. It is not surprising that the COMPASS and COMPASS eligible students report lower levels of preparation, since these students are specifically targeted based on their mathematical preparation.

The survey results from Q77 shows that 14% of the COMPASS students think that their previous math courses adequately prepared them for calculus at Clarkson, but 11% and 19% for the COMPASS eligible students and all others, respectively. It is not surprising that the COMPASS and COMPASS eligible students report lower levels of preparation, since these students are specifically targeted based on their mathematical preparation.

Analysis on final exam grades

Table 5: Comparison between three groups of students on their calc 3/diffeq final course GPA during F2016 and S2017, where the maximum GPA is 4.0 and NA indicating the numbers of students who did not take those two courses during those two semesters.

Group	Min.	1st Qu.	Median	Mean	3rd Qu.	Max	NA's
COMPASS	1.667	2.334	2.667	2.847	3.334	4.000	1
Eligible	0.000	2.000	2.334	2.382	3.251	4.000	23
Others	0.000	2.000	2.667	2.632	3.667	4.000	48

Table 1 shows the numbers of students in all three groups during Fall 2015, S2016 and F2016 together with S2017 for four courses: calc 1, calc 2, calc 3 and diffeq. We combined students who enrolled calc 3 during F2016 and S2017 as a single group. The same was done for diffeq. The sample size was reduced from 18 students in COMPASS group for calc 2 to 13 or 14 in F2016/S2017 for both calc 3 and diffeq. There are no requirements for computer science students to enroll in any of those two courses which accounts for the difference.

We analyzed the final exam grades instead of course grades to evaluate student performance in calc 1 and calc 2. The maximum final exam grade is 100, while the minimum is 0. The data shows that the mean grades on final exam on calculus classes in COMPASS group was 77.63

and 75.82, compared to 76.52 and 69.60 in the COMPASS Eligible group and 73.42 and 69.26 in the all others group. The variances on final exam on calc 1 and calc 2 in COMPASS group was 10.89 and 9.95, compared to 17.52 and 16.44 in the COMPASS eligible group and 16.80 and 17.54 in the all others group.

Table 5 shows data obtained during F2016 and S2017 for calc 3 and diffeq. Note that the data for the subsequent mathematics courses after COMPASS program are from the course grades in terms of GPA. The maximum GPA for a course is 4.0, while the minimum is 0. The data shows that the mean grades on course average in COMPASS group was 2.847, compared to 2.382 in the COMPASS eligible group and 2.632 in the all others group. This shows a slight better performance in COMPASS students on their continuing math courses. Notice that COMPASS students have relatively higher mean and smaller standard deviation, compared to both COMPASS eligible group and all others group. However, this is not sufficient to claim there is any significant differences in students' performance on their continuing math courses.

Hypothesis test on students' course grades

The descriptive statistics of the collected information are given in the previous section. We now provide the results of the hypothesis tests when comparing the results across the three groups. The hypothesis test examined is the comparisons for the students' final grades in the course. The statistical comparisons are given below. We examine the results of the statistical tests in order.

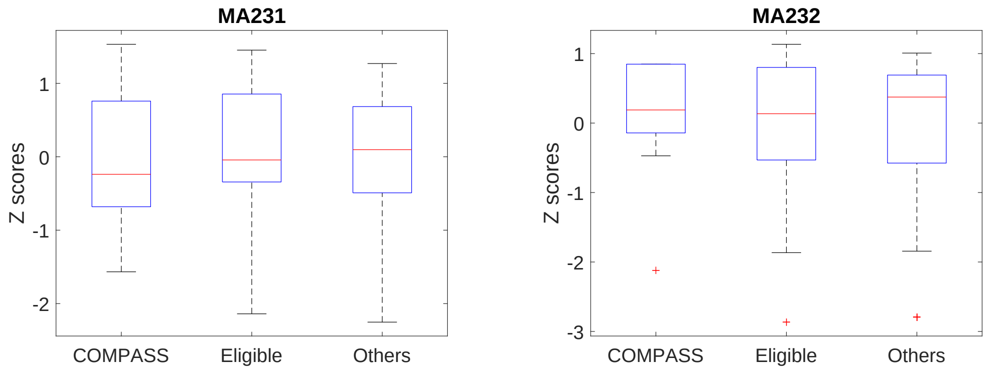
1. H_1 There is no statistically significant difference on calc 3 final course grades between three groups in Fall 2016 plus Spring 2017
2. H_2 There is no statistically significant difference on diffeq final exam scores between three groups in Fall 2016 plus Spring 2017.

Unlike the previous section we examine the two semesters separately. For each semester we examine the performance of the three groups of students, COMPASS, eligible, and others. As noted above the numbers of students within each group for each semester is given in Table 1. The final exam scores for each of the three groups are compared. A comparison of the means is made using an ANOVA test. The variance within each group is found to be similar, but the normality assumption for the residual errors is not normally distributed. Because of this discrepancy the results of a comparison of the means using a non-parametric Kruskel-Wallis rank sum test is also given.

One difficulty in comparing students in the follow up courses is that students take them in a different order. For example, students who successfully finished calc 2 may enroll in calc 3 or they may enroll in diffeq during F2016 and S2017. Each of these courses offer three sections. Thus, students' performance on those courses may differ by semester, or by instructor.

Figure 1 shows the box plots of z-scores of students' performance on calc 3 and diffeq by groups after standardization. Note that the higher the box is, the better students' grades are. Although COMPASS students have higher maximum and minimum in comparison with the other two groups for both calc 3 and diffeq, COMPASS students perform better in diffeq than others.

Figure 1: A comparison of the students' performance on calc 3 (left) and on diffeq (right) during F2016 and S2017.



However, it is hard to tell the differences between groups through the box plot. We also analyzed students' raw GPA scores on both courses (figure not shown), which seems to be consistent with the standardized z-score analysis.

To compare the difference in means between the three groups one way is an analysis of variance (which is calculated in ANOVA). The test was run to compare the effect of placement between the three groups on the students' final grade in the course. The first is the group of students who are not eligible for the COMPASS program, the second is the group of students who are eligible for the program but did not take part, and the third is the group of students who are eligible and took part in the program.

The analysis of variance was run through ANOVA for both the first semester (calc 3) and the second semester (diffeq). An analysis of variance showed that the effect of placement for calc 3 on the students' final grade was not significant, F(2,285)=1.851, p=0.159. An analysis of variance showed that the effect of placement for diffeq on the students' final grade was not significant, F(2,337)=0.607, p=0.546. The full table is given in Table 6.

Two tests were conducted of the underlying assumptions for the ANOVA. First, the assumption of the homogeneity of the variances was tested using a Fligner-Killeen test. The original data is clearly not normally distributed so the Fligner-Killeen test was chosen as a more robust option to test whether or not the residuals are normally distributed. With respect to the Fall (calc 3) data, the test resulted in a chi-squared value of 3.11 with 2 degrees of freedom which resulted in a p-value of 0.21. With respect to the Spring (MA 232) data, the test resulted in chi-squared value of 0.82 with 2 degrees of freedom which resulted in a p-value of 0.66. In both cases we cannot reject the null hypothesis that the variances are equal.

The second test is the assumption that the residuals between the models are normally distributed in the ANOVA test. In this case, a quick examination of the QQ-normal plot of the residuals, Figure 2, demonstrates that the residuals are not normal. This is problematic due to the low number of students in the COMPASS group, and the assumptions of the ANOVA test are not consistent with the data.

A non-parametric Kruskal-Wallis rank sum test was employed to examine the difference in

Figure 2: The Normal QQ plot of the residuals for the final exam grades of the Fall MA 231 course as well as the Spring MA 232 course.

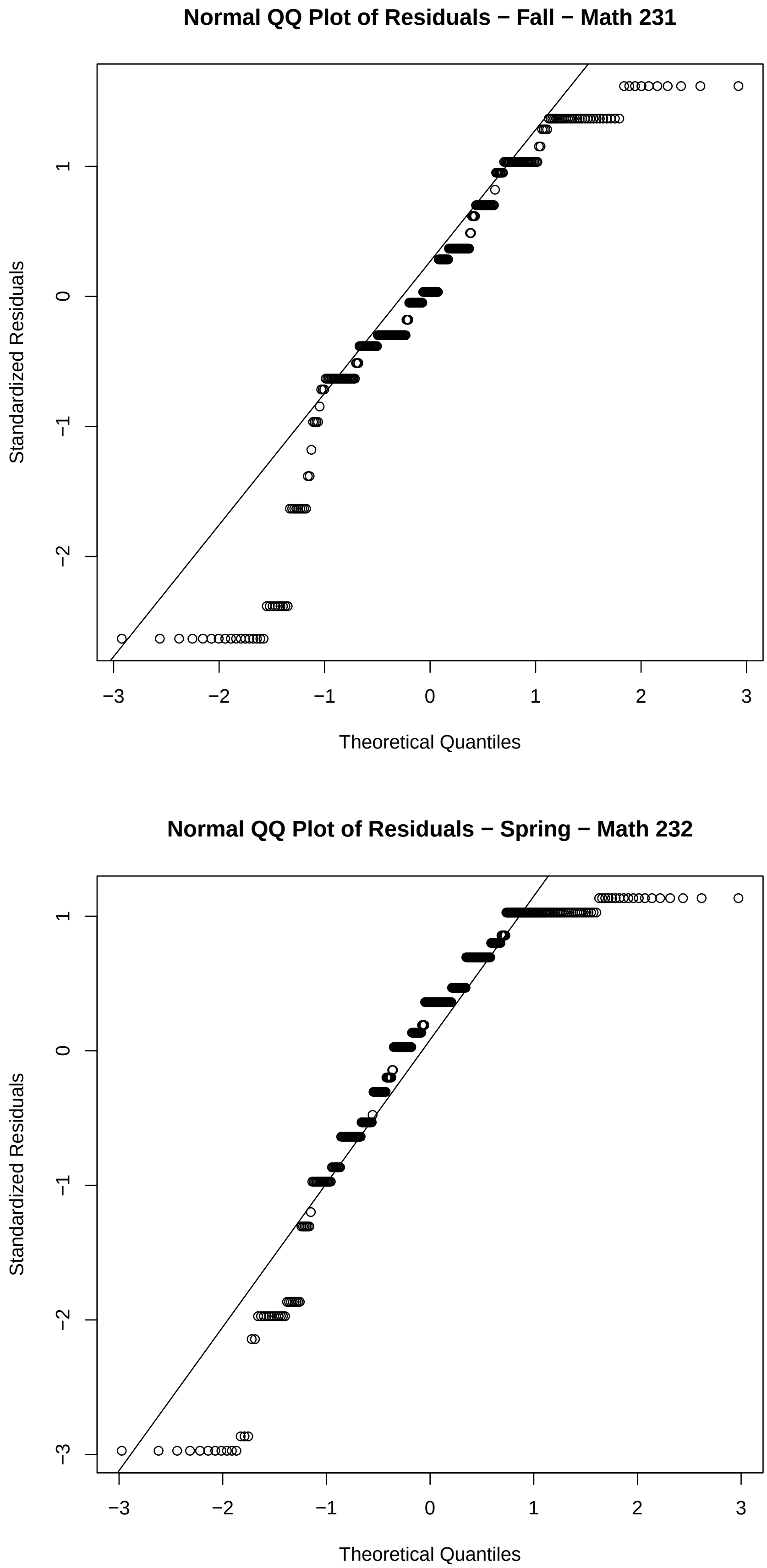


Table 6: ANOVA table for the effect of placement for first semester (calc 3) and second semester (diffeq) students on their final score.

calc 3	Df	Sum of Squares	Mean Square	F	Pr(>F)
Between Groups	2	4.6	2.316	1.851	0.159
Within Groups	285	356.6	1.251		
diffeq	Df	Sum of Squares	Mean Square	F	Pr(¿F)
Between Groups	2	1.3	0.6698	0.607	0.546
Within Groups	337	372.1	1.1041		

the means between the three groups as a secondary evaluation of the results. For the Fall (calc 3) groups, the result is a chi-squared value of 4.8 with 2 degrees of freedom and a p-value of 0.09. For the Spring (diffeq) groups, the result is a chi-squared value of 2.70 with 2 degrees of freedom and a p-value of 0.26. As expected, these results do not allow us to reject the null hypotheses.

The statistical tests comparing the means does not adequately express the complete story. The aims of the study are to bring the students classified to be in the (M-,P+) group closer to the same level as their (M+,P+) peers. It is more important to compare the performance of the COMPASS students to the students in the Other group and compare the performance of the Eligible students to the students in the Other group. To do this we examine the confidence intervals of the three groups of students.

With respect to the three groups, the confidence intervals for the means of the students' final grades in both courses for each of the three groups are analyzed, with the confidence level of 95%. The 95% confidence interval for the mean in calc 3 (Fall) for the COMPASS students is (2.39, 3.30). The 95% confidence interval for the mean in calc 3 for the Other groups is (2.47, 2.79). Finally the 95% confidence interval for the mean in calc 3 for the Eligible group is (2.13, 2.63). The 95% confidence interval for the COMPASS group extends beyond to the right of the 95% confidence interval for the Other group, and the COMPASS students are performing at a similar level as the Other group. At the same time, the high end of the 95% confidence interval for the Eligible group falls to the left of the middle of the 95% confidence interval of the Other group. Given the small number of students in this initial survey the results are encouraging.

The 95% confidence interval for the mean in diffeq (Spring) for the COMPASS students is (2.56, 3.73). The 95% confidence interval for the mean in diffeq for the Other groups is (2.83, 3.11). Finally the 95% confidence interval for the mean in diffeq for the Eligible group is (2.67, 3.07). The 95% confidence interval for the COMPASS group again extends beyond to the right of the 95% confidence interval for the Other group, and again the COMPASS students are performing at a similar level as the Other students. During the Spring semester the students that were still in the program and in the Eligible group were performing at similar levels. The program's impact for the students remaining in the program for the second semester was less impactful.

Validation and Limitation

We analyzed students instructional practice survey results, students' final exam grades and students performance on their continuing mathematics courses. We tried to find answers to the three questions described in Section 3 through those data we collected. The analyses suggest that the COMPASS students do slightly better than others, although we have a small sample size, and the differences are not significant statistically. Thus, the question on whether the COMPASS program impacts student retention and performance remains unclear; however, COMPASS students' positive descriptions of their instructional practices and attitudes towards mathematics supports the continued implementation of the program. The COMPASS program is replicable if readers check our previous publication on PRIMUS [20]. The difficulties and constraints are that we do not have the ability to make this to a larger scale unless some flipped classroom ideas or active learning techniques are used [26, 27, 28]. We believe it is important to recall students' attention to physical intuitions when dealing with abstract calculus concepts [29].

Conclusion

It is crucial for college educators to help students to succeed in their beginning years of college, especially as it relates to courses that have traditionally served to exclude students from experiencing success. We already know that co-teaching physics courses and calculus courses, to students who are not fully prepared for calculus, enhances the learning and success in both subjects. Over the past 15 years, we have shown that the best predictor for success in physics is not the students' physics readiness, but their pre-calculus assessment. We have developed a number of innovative ways to help students succeed. The CoOrdinated Math-Physics Assessment for Student Success program (COMPASS) was designed at Clarkson to help students who are slightly under-performing in math but strong in physics intuition. The students are identified through two pre-college surveys: the Force Concept Inventory (FCI) and the math diagnostic exam. The COMPASS program randomly selected around 25 students each academic year and placed them in the program so students are going through calc 1/II and physics I/II within the program during that academic year. In the program we teach a highly coordinated calculus and physics sequence in a way that reinforces the calculus concepts.

Through our study, we conclude that

1. COMPASS students scored better than the other students on their final exam grades on calc 1 and calc 2 courses. However, hypothesis tests show that there is no statistically significant difference.
2. Similar to the previous conclusion, COMPASS students' (from academic year 2015-2016) performance on continuing mathematics courses including calc 3 and elementary differential equations courses (during academic year 2016-2017) are higher on average in comparison with other students. This is consistent with their final exam grades on calc 1 and calc 2 courses. However, hypothesis tests show that there is no statistically significant difference.

3. From the student instructional practices math survey for groups from academic year 2017-2018, we found that students in COMPASS program had better classroom experience and became more confident and their attitude toward learning mathematics has been increased.

Although these specific analysis on students performance are not replicable it indicates that students enrolled in a physics based calculus courses benefit from being in the small classroom, making close connection with their peers and seeing the application of mathematical concepts to their life and future career. Over the last decade, the first year experience for STEM majors has been restructured to be more individual-based on the students' preparation. This has resulted in the development of special courses and sequences to encourage success. The first year has been transformed from the older tradition of gatekeeper to creating gateways for all our students. Academic factors no longer dominate STEM retention issues; we have approximately 90% retention of first-year STEM majors. The fundamentals of this pedagogical paradigm shift are greater attention to the needs of each individual student, more opportunities for active learning such as research and projects, and better programmatic plans that are matched to students' abilities and preparation. Our COMPASS program represents an innovation of the interventions for better calculus preparation, which improved passing scores from 30% to 70% of the most at-risk students. We wish to institutionalize this approach in a new way of teaching that would increase its benefits for all students.

The study is likely to interest a broad group of engineering education researchers and/or practitioners to disseminate knowledge on engineering teaching and learning since mathematics is a common problem throughout engineering education. In addition, even though the improvement of COMPASS students in continuing math courses are not significant, their slightly better performance shows a great improvement in comparison with their low math test scores as high-risk category group students. It is also possible that there are impacts in students' continuing physics or other physical science course sequences. However, Clarkson University as a major engineering school where the most of our students do not take continuing physics courses unless they are majoring or minoring in physics. We are very much interested in developing freshman calculus courses by students' academic background and pre-college math knowledge. The COMPASS program may improve instruction through the development of innovative materials and sound instructional designs.

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