Notes/Corrections/Clarifications for lecture given on 1/13/2011 (Thursday)

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1 Arguments in Oz procedures and functions

Question: Can Oz procedures return multiple values?

Answer: Quoting from [2, §2.3.4]:

> A function always has exactly one output. **A procedure can have any number of inputs and outputs, including zero.**

2 Pascal's triangle

Question: Pascal's formula, [4], states that $\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$, where the formula for choosing k items out of a total of n items is: $\binom{n}{k} = \frac{n!}{k! * (n-k)!}$. Now consider, say, the last entry in row $n = 3$ (see Figure 1); this value is $\binom{3}{3}$[1]. By the Pascal's formula, $\binom{3}{3} = \binom{2}{3} + \binom{2}{2}$. However, if we apply the definition of Combinations on $\binom{2}{3}$ we get $\frac{2!}{3! * (-1)!}$ *which is undefined* since the factorial function is defined only for positive integers, [3]. So this means we have an undefined entry in the Pascal's triangle!

Answer:

- First, recall that the entries in row n of a Pascal's triangle are: $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$. These are all non-negative integers, so all values in triangle are well-defined.

[1] The entries in row n of the Pascal's triangle are: $\binom{n}{0}$ to $\binom{n}{n}$.

$n = 0$:					1					
$n = 1$:				1		1				
$n = 2$:			1		2		1			
$n = 3$:		1		3		3		1		
$n = 4$:	1		4		6		4		1	

Figure 1: Pascal's triangle

- The prescribed textbook, [2, §1.4], describes Pascal's triangle as follows:

  > It starts with 1 in the first row. Each element is the sum of the two elements just above it to the left and right. (If there is no element, as on the edges, **then zero is taken**).

  In this way, the book sidesteps this problem by not using Pascal's formula for the entries at the "boundaries" of the triangle. Therefore, the identities $\binom{n}{0} = \binom{n}{n} = 1$ have be used (along with Pascal's formula) to calculate the entries of a Pascal's triangle, starting from the initial row.

- Rosen, [1, §5.3], defines $\binom{n}{k}$ only for non-negative n and $0 \leq r \leq n$. If we follow this convention, the troublesome expression (the factorial of a negative number), would never arise. Following this convention, $\binom{2}{3}$ is undefined and hence cannot be used to determine the value of $\binom{3}{3}$ (and therefore we will have to rely on using the identity $\binom{3}{3} = 1$).

References

[1] Kenneth H. Rosen. *Discrete Mathematics and Its Applications.* McGraw-Hill Higher Education, 6th edition, 2006.

[2] Peter Van Roy and Seif Haridi. *Concepts, Techniques, and Models of Computer Programming.* MIT Press, 2004.

[3] Eric W. Weissstein. Factorial. `http://mathworld.wolfram.com/Factorial.html`. From MathWorld–A Wolfram Web Resource.

[4] Eric W. Weissstein. Pascal's formula. http://mathworld.wolfram.com/PascalsFormula.html. From MathWorld–A Wolfram Web Resource.