

Energy Transport

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Transport of thermal energy in fluids occurs by three mechanisms.

Conduction or molecular transport

Convection or bulk transport

Radiation

Of these, the proper treatment of radiation is beyond the scope of our course. Therefore, we shall only consider conduction and convection.

Conservation of Energy

The energy of a flowing fluid consists of internal and kinetic energy. The rate of increase of the energy content of the fluid present within a control volume at a given instant is equal to the sum of the net flux of energy into the control volume and the work done on the fluid within the control volume by body forces and surface forces acting on it. Bird et al. go through a careful derivation of the mathematical form of the equation of conservation of energy in Chapter 11. The initial balance is written for the total energy and then the part involving the kinetic energy (known as the mechanical energy balance, obtained by taking a dot product of Cauchy's equation with the velocity) is subtracted. After using some thermodynamic relationships, the final form of the equation of conservation of thermal energy is obtained. Various versions are given in Table 11.4-1 of the book. The most common version that we shall use assumes that the density ρ and thermal conductivity k are constant, and is given below.

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) = k \nabla^2 T + \mu \Phi_v + S$$

In the above equation, T is the temperature, C_p is the specific heat at constant pressure, t is time, \mathbf{v} is the (vector) velocity, $\mu \Phi_v$ stands for the rate of irreversible conversion of mechanical energy into internal energy per unit volume by viscous dissipation, and S represents the rate of generation of energy per unit volume by sources such as electrical heating. When the thermal energy equation is obtained for a multicomponent system, the rate of generation (or consumption) of energy per unit volume due to chemical reactions appears naturally in the energy equation when proper accounting is made of the enthalpies of the various species. Detailed expressions for the dissipation function Φ_v in terms of derivatives of the velocity components in common coordinate systems can be found in Table B.7 of Bird et al. In most situations, we can set the viscous dissipation term to zero with negligible error, the exceptions occurring when highly viscous fluids are subjected to

large velocity gradients; an example where viscous heating is important is polymer processing.

There are two fluxes of thermal energy that appear in the energy equation (the prefix “thermal” will be omitted, but implied from here on). One is molecular flux of energy and the other is convective flux of energy.

Molecular transport or Conduction

Molecular transport rates are adequately described for moderate temperature gradients by a linear relationship between the heat flux and the temperature gradient. The phenomenological relationship

$$\mathbf{q} = -k \nabla T$$

is known as Fourier’s law. Here, \mathbf{q} is the (vector) heat flux, and T is the temperature field at a given point. The negative sign tells us that heat flows in the direction opposite to that of the temperature gradient, namely from hot regions to cold regions. The constant of proportionality in Fourier’s law, k is known as the thermal conductivity and is a material property of the fluid. The thermal conductivity depends on temperature and pressure in general, and Bird et al. provide some information regarding this subject in Chapter 9. Table B.2 provides results for the components of \mathbf{q} in common coordinate systems.

Convective or Bulk Transport

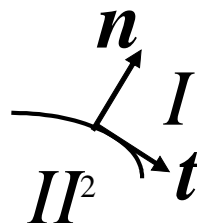
Thermal energy also is transported by the physical movement of an element of fluid from one place to another. This is known as convective transport. In our simplified picture, the (vector) convective flux of thermal energy at a given point can be written as

$$\text{Flux} = \rho C_p \mathbf{v} (T - T_{ref})$$

where T_{ref} is a reference temperature that serves as a datum. A more precise accounting of the convective flux of total energy, is given in Bird et al. in Section 9.7.

Boundary Conditions

At the interface between a solid and a fluid, or that between a fluid and another fluid, it is reasonable to expect thermodynamic equilibrium to prevail between the two phases adjoining the interface, except when the heat flux across the interface is extremely large. A small portion of the interface between phases I and II is shown schematically in the sketch.



The assumption of thermodynamic equilibrium at the interface leads to the boundary condition

$$T_I = T_{II}$$

at the interface. In addition, because the interface is assumed to have no mass, any heat flux crossing the interface from one phase must necessarily be transmitted to the other phase. Therefore,

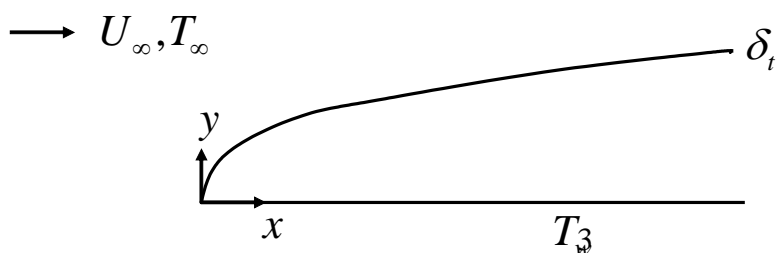
$$\mathbf{n} \cdot \mathbf{q}_I = \mathbf{n} \cdot \mathbf{q}_{II}$$

at the interface. The most general way to formulate a problem is to write the governing energy equation for each phase and the above pair of conditions at each phase interface, along with any other applicable initial and boundary conditions. In practice, it is far more convenient to study heat transport in controlled conditions wherein the temperature at a solid boundary in contact with a fluid is prescribed, or the heat flux from the solid to the fluid is prescribed. Experimentally, we can achieve a condition of prescribed uniform wall temperature in a pipe by choosing a highly conducting wall material and surrounding it with either a phase change system (such as condensing steam) or a segmented electrical heating system with a controller that maintains the temperature of the wall at a constant value. A uniform heat flux can be achieved most conveniently by using electrical heating. These two boundary conditions represent the two extremes of a family of boundary conditions that are common in heat transport problems.

The heat transfer coefficient

The heat transfer coefficient h and its dimensionless counterpart, the Nusselt Number Nu , are in common use in engineering work. Here, I discuss how the heat transfer coefficient is defined in typical situations.

Consider a fluid at a temperature T_∞ flowing at a uniform velocity U_∞ that encounters a rigid wall, which is maintained at a uniform temperature T_w . Let us assume that $T_w > T_\infty$ for the sake of definiteness; however, the results given below are equally valid when $T_w < T_\infty$.



Just as a momentum boundary layer forms at the wall and grows in thickness with distance x along the plate, a thermal boundary layer forms at the wall; the temperature of the fluid changes from T_w to T_∞ in the thermal boundary layer. The thickness of the thermal boundary layer δ_t also grows with distance x .

At the rigid wall, the normal velocity is zero and the heat flux from the wall to the fluid consists only of the conduction flux q_y . From Fourier's law, this can be written as

$$q_w(x) = -k \frac{\partial T}{\partial y}(x, 0)$$

Note that $q_w > 0$ when $T_w > T_\infty$, and $q_w < 0$ when $T_w < T_\infty$. Even though q_w is purely a conduction flux, it is modified by flow. As the velocity of the fluid increases, the ability of the fluid to carry away heat supplied by the plate increases, and the temperature gradient at the wall becomes sharper, consistent with a larger heat flux. You can see that a sharper temperature gradient at the wall is consistent with a thinner thermal boundary layer.

If we can solve the energy equation for the temperature distribution in the flowing fluid, the heat flux $q_w(x)$ can be evaluated as a function of x . A heat transfer coefficient $h(x)$ is defined for this system as follows.

$$q_w(x) = h(x)(T_w - T_\infty)$$

Therefore, the heat transfer coefficient is seen to be directly related to the temperature gradient at the wall.

$$h(x) = \frac{-k \frac{\partial T}{\partial y}(x, 0)}{(T_w - T_\infty)}$$

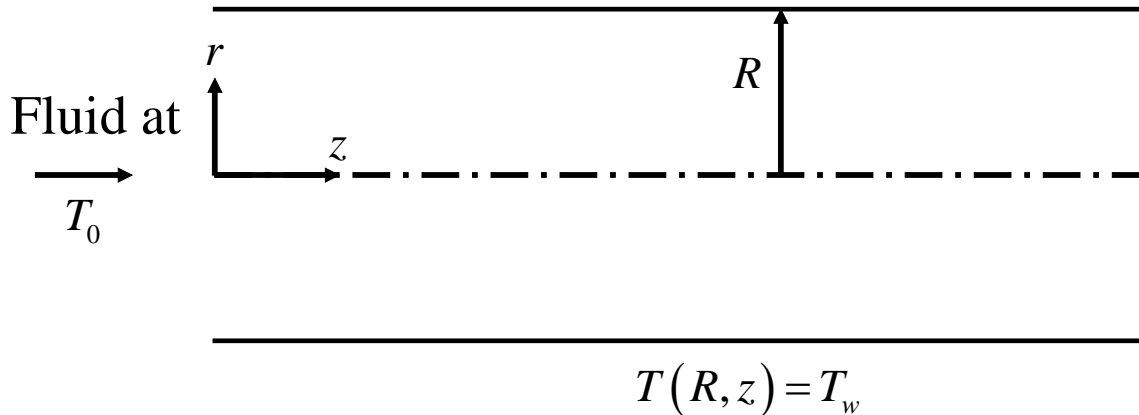
This definition of $h(x)$ holds regardless of the sign of $(T_w - T_\infty)$.

The Nusselt number Nu is a dimensionless version of the heat transfer coefficient. It is defined in the above problem as follows.

$$Nu = \frac{hL}{k}$$

Here L is a characteristic length scale, which can be taken as the axial distance x or the length of the plate, depending on our needs.

Now, consider heat transfer to a fluid flowing through a pipe. In this case, let the fluid enter the pipe at some temperature T_0 , encountering a step change in wall temperature to T_w . A sketch of the system is given below.



At any given axial position z , the heat flux from the wall to the fluid is given by

$$q_w = k \frac{\partial T}{\partial r}(R, z)$$

Do you see why a positive sign is used in the right side? It is because the heat flux from the wall to the fluid is in the negative r - direction.

For defining the heat transfer coefficient, we need a driving force at any location z . While it is possible to use $(T_w - T_0)$, the more common choice is $(T_w - T_b)$ where T_b is known as the bulk or cup-mixing average temperature. The bulk average temperature is experimentally determined by collecting the fluid coming out of the system at a given axial location and mixing it completely, and then measuring its temperature. The following mathematical definition directly follows from this physical definition.

$$T_b = \frac{\int_0^R 2\pi r V(r) T(r, z) dr}{\int_0^R 2\pi r V(r) dr}$$

Here, $V(r)$ represents the velocity field.

The heat transfer coefficient in this system is defined as follows.

$$q_w = h(z)(T_w - T_b)$$

so that we can evaluate $h(z)$ using

$$h(z) = \frac{k \frac{\partial T}{\partial r}(R, z)}{(T_w - T_b)}$$

if we know the detailed temperature distribution in the fluid.

We see from these two examples that the heat transfer coefficient will depend on position, the system parameters, and on time in unsteady state problems. This concept of a heat transfer coefficient is extended to many practical heat transfer situations. Typically, in any given system, heat transfer rates and suitably defined driving forces are both measured. The ratio of the flux of thermal energy to the driving force expressed as a temperature difference is reported as the heat transfer coefficient.

$$h = \frac{\text{heat flux}}{\text{driving force}}$$

We can think of the heat transfer coefficient as the conductance (using an electrical analogy) of the system. Typically, spatial averages are easier to measure and report; it is not common in engineering design to use local values.

In problems amenable to analysis from first principles, temperature distributions and heat fluxes can be calculated directly from the solution. There is really no need to define a heat transfer coefficient. But, to make it convenient to report and use the results, even in such problems, suitably defined heat transfer coefficients are calculated from the theoretical results and reported.

Important dimensionless groups in heat transfer

We already have defined a dimensionless group commonly used in engineering practice, namely, the Nusselt number Nu . Considering it to be an average value for a given heat transfer setting such as heat transfer to fluid flowing through a circular pipe of diameter D , we can use dimensional analysis to identify the dimensionless groups on which Nu will depend. Thus, we obtain

$$Nu = Nu\left(\text{Re}, \text{Pr}, \frac{L}{D}\right)$$

where $Nu = \frac{hD}{k}$, the Reynolds number $Re = \frac{UD}{\nu}$, and the Prandtl number $Pr = \frac{\nu}{\alpha}$. The velocity U appearing in the Reynolds number is a characteristic velocity in the system such as the average velocity of flow, and ν and α are the kinematic viscosity and the thermal diffusivity of the fluid, respectively. The thermal diffusivity $\alpha = k / (\rho C_p)$. The symbol L stands for the length of the pipe. For non-circular cross-sections, the Nusselt number will also depend on additional aspect ratio parameters.

The physical significance of the Nusselt number is simply that it represents a dimensionless heat flux at the wall or a ratio of the actual heat transfer rate to that prevailing in a hypothetical system in which the same driving force applied across the characteristic distance drives a conduction flux. We can see this by recasting it as follows.

$$Nu = \frac{h\Delta T}{k \frac{\Delta T}{D}}$$

The Prandtl number

To appreciate the physical significance of the Prandtl number, we note that it is the ratio of the intrinsic transport coefficients ν and α for molecular transport of momentum and energy, respectively. Consider a simple one-dimensional transport situation in which fluid flows in the x -direction with a velocity $v_x(y)$. In this case, Newton's law of viscosity for the momentum flux τ_{yx} (note that we are interpreting this symbol as the negative of the shear stress) can be written as

$$\tau_{yx} = -\mu \frac{\partial v_x}{\partial y}$$

and in a similar one-dimensional conduction problem, Fourier's law for the heat flux q_y can be written as

$$q_y = -k \frac{\partial T}{\partial y}$$

If we assume that the density and specific heat at constant pressure are constant, we can rewrite the above results in the following form.

$$\tau_{yx} = -\nu \frac{\partial(\rho v_x)}{\partial y}$$

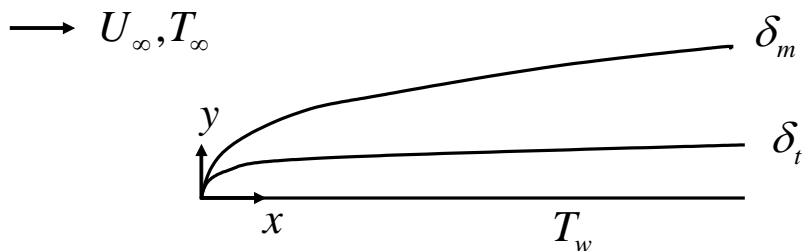
$$q_y = -\alpha \frac{\partial(\rho C_p \{T - T_{ref}\})}{\partial y}$$

The product ρv_x represents the amount of x – momentum in unit volume of fluid, and can be regarded as the concentration of x – momentum. Therefore, we see that the flux of x – momentum is proportional to the gradient of the concentration of that momentum, and occurs in the direction opposite to that gradient. The coefficient of proportionality is the kinematic viscosity ν . In a like manner, the flux of thermal energy is proportional to the gradient of the concentration of thermal energy, with a coefficient of proportionality equal to the thermal diffusivity α . So, molecular transport leads to momentum and energy flowing “downhill” in the direction opposite to that of the concentration gradient of each, with coefficients of proportionality that represent the ability of the fluid to transport momentum or energy by molecular means. This leads to the following physical interpretation of the Prandtl number.

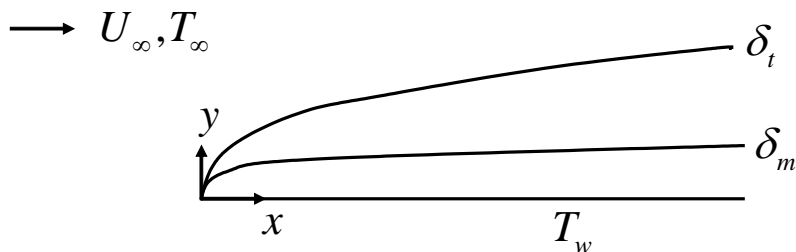
$$\text{Pr} = \frac{\text{Ability of a fluid to transport momentum by molecular means}}{\text{Ability of that fluid to transport energy by molecular means}}$$

Thus, in flow over an object, the relative thicknesses of the momentum and thermal boundary layers reflect the magnitude of the Prandtl number, as the examples given below show.

Large Prandtl number, $\text{Pr} \gg 1$ ($\nu \gg \alpha$)



Small Prandtl number, $\text{Pr} \ll 1$ ($\nu \ll \alpha$)



For gases, the Prandtl number is typically of $O(1)$, while for common liquids Prandtl numbers are found to vary from 10 to 1000. The Prandtl numbers of very viscous liquids such as polymer melts can be even larger than 1000, reaching values of 10^5 or greater. Because liquid metals are great conductors of thermal energy, Prandtl numbers for liquid metals are typically of $O(10^{-2})$.