Pipe Flow Calculations

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We begin with some results that we shall use when making friction loss calculations for steady, fully developed, incompressible, Newtonian flow through a straight circular pipe.

Volumetric flow rate $\boxed{Q = \dfrac{\pi}{4} D^2 V}$ where D is the pipe diameter, and V is the average velocity.

Reynolds Number: $\boxed{\text{Re} = \dfrac{DV\rho}{\mu} = \dfrac{DV}{\nu} = \dfrac{4Q}{\pi D \nu} = \dfrac{4\dot{m}}{\pi D \mu}}$ where ρ is the density of the fluid, μ is its dynamic viscosity, and $\nu = \mu / \rho$ is the kinematic viscosity.

The pressure drop ΔP is related to the loss in the Engineering Bernoulli Equation, or equivalently, the frictional head loss h_f, through $\boxed{\Delta P = \rho \times \text{loss} = \gamma h_f}$

Here, the specific weight $\gamma = \rho g$, where g is the magnitude of the acceleration due to gravity.

Power

The power required to overcome friction is related to the pressure drop through

$\boxed{\text{Power} = \Delta P\, Q}$ or we can relate it to the head loss due to pipe friction via $\boxed{\text{Power} = \gamma h_f Q}$

Head Loss/Pressure Drop

The head loss h_f is related to the Fanning friction factor f through

$$\boxed{h_f = 2 f \left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right)}$$

or alternatively we can write the pressure drop as $\boxed{\Delta P = 2 f \left(\dfrac{L}{D}\right)\left(\rho V^2\right)}$

Friction Factor

In laminar flow, $\boxed{f = \dfrac{16}{\text{Re}}}$.

In turbulent flow we can use either the Colebrook or the Zigrang-Sylvester Equation, depending on the problem. Both give equivalent results well within experimental uncertainty. In these equations, ε is the average roughness of the interior surface of the pipe. A table of roughness values recommended for commercial pipes given in a textbook on Fluid Mechanics by F.M. White is provided at the end of these notes.

Colebrook Equation

$$\frac{1}{\sqrt{f}} = -4.0\,\log_{10}\left[\frac{\varepsilon / D}{3.7} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right]$$

Zigrang-Sylvester Equation

$$\frac{1}{\sqrt{f}} = -4.0\,\log_{10}\left[\frac{\varepsilon / D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\varepsilon / D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right]$$

Non-Circular Conduits

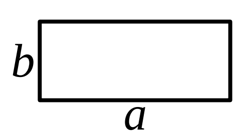
Not all flow conduits are circular pipes. An example of a non-circular cross-section in heat exchanger applications is an annulus, which is the region between two circular pipes. Another is a rectangular duct, used in HVAC (Heating, Ventilation, and Air-Conditioning) applications. Less common are ducts of triangular or elliptical cross-sections, but they are used on occasion. In all these cases, when the flow is turbulent, we use the same friction factor correlations that are used for circular pipes, substituting an equivalent diameter for the pipe diameter. The equivalent diameter D_e, which is set equal to four times the "Hydraulic Radius," R_h is defined as follows.

$$D_e = 4R_h = 4\times\frac{\text{Cross-Sectional Area}}{\text{Wetted Perimeter}}$$

In this definition, the term "wetted perimeter" is used to designate the perimeter of the cross-section that is in contact with the flowing fluid. This applies to a liquid that occupies part of a conduit, as in sewer lines carrying waste-water, or a creek or river. If a gas flows through a conduit, the entire perimeter is "wetted."

Using the above definition, we arrive at the following results for the equivalent diameter for two common cross-sections. We assume that the entire perimeter is "wetted."

Rectangular Duct



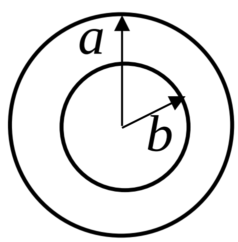
For the duct shown in the sketch, the cross-sectional area is ab, while the perimeter is $2(a+b)$ so that the equivalent diameter is written as follows.

$$D_e = 4\times\frac{ab}{2(a+b)} = \boxed{\frac{2}{\left(\frac{1}{a}+\frac{1}{b}\right)}}$$

If the flow is laminar, a result similar to that for circular tubes is available for the friction factor, which can be written as $\boxed{f = C/\mathrm{Re}}$, where C is a constant that depends on the aspect ratio a/b, and the Reynolds number is defined using the equivalent diameter. A few values of the constant C for selected values of the aspect ratio are given in the Table below (Source: F.M. White, Fluid Mechanics, 7th Edition). For other aspect ratios, you can use interpolation.

a/b	C	a/b	C
1.0	14.23	6.0	19.70
1.33	14.47	8.0	20.59
2.0	15.55	10.0	21.17
2.5	16.37	20.0	22.48
4.0	18.23	∞	24.00

Annulus



The cross-sectional area of the annulus shown is $\pi\left(a^2-b^2\right)$, while the wetted perimeter is $2\pi\left(a+b\right)$. Therefore, the equivalent diameter is obtained as

$$D_e = 4\frac{\pi\left(a^2-b^2\right)}{2\pi\left(a+b\right)} = \boxed{2\left(a-b\right)}$$

Again, for laminar flow, we find that $\boxed{f = C/\mathrm{Re}}$, where C is a constant that depends on the aspect ratio a/b, and the Reynolds number is defined using the equivalent diameter. As with the rectangular cross-section, a few values constant C for selected values of the aspect ratio are given in the Table that follows (Source: F.M. White, Fluid Mechanics, 7th Edition). For other aspect ratios, you can use interpolation.

a/b	C	a/b	C
1.0	24.00	10,0	22.34
1.25	23.98	20.0	21.57
1.67	23.90	100	20.03
2.5	23.68	1000	18.67
5.0	23.09	∞	16.00

Minor Losses

Minor losses is a term used to describe losses that occur in fittings, expansions, contractions, and the like. Fittings commonly used in the industry include bends, tees, elbows, unions, and of course, valves used to control flow. Even though these losses are called minor, they can be substantial compared to those for flow through short straight pipe segments. Losses are commonly reported in velocity heads. A velocity head is $V^2/(2g)$. Therefore, we can write minor losses as $h_m = K_L \dfrac{V^2}{2g}$, where K_L is called the loss coefficient.

Typical values of K_L for some common fittings are given below. Usually, the values depend upon the nominal pipe diameter, the Reynolds number, and the manner in which the valve is installed (screwed or flanged). Manufacturers' data should be used wherever possible.
Globe Valve (fully open): 5.5 - 14
Gate Valve (fully open): 0.03 - 0.80
Swing Check Valve (fully open): 2.0 - 5.1

Standard 45^o Elbow: 0.2 - 0.4
Long radius 45^o Elbow: 0.14 - 0.21

Standard 90^o Elbow: 0.21 - 2.0
Long radius 90^o Elbow: 0.07 - 1.0

Tee: 0.1 - 2.4

When solving homework problems, use the values given in Table 13.1 in the textbook by Welty et al.

Sudden Expansion and Sudden Contraction

A sudden expansion in a pipe is one of the few cases where the losses can be obtained from the basic balances. The expression for K_L is given by

$$K_L = \left[1 - \frac{d^2}{D^2}\right]^2$$

Here, d and D represent the diameters of the smaller and larger pipes, respectively. For a sudden contraction, we can use the same result if $d/D \geq 0.76$. For smaller values of d/D we can use the empirical relation $K_L = 0.42\left[1 - d^2/D^2\right]$.

In both cases, we should multiply K_L by the velocity head in the pipe segment of diameter d. The losses would be smaller if the expansion or contraction is gradual.

When a pipe empties into a reservoir, all the kinetic energy in the fluid coming in is dissipated, so that you can treat this as a sudden expansion with the ratio $d/D = 0$, yielding $K_L = 1$.

Typical Pipe Flow Problems

In typical pipe flow problems, we know the nature of the fluid that will flow through the pipe, and the temperature. Therefore, we can find the relevant physical properties immediately. They are the density ρ and the dynamic viscosity μ. Knowing these properties, we also can calculate the kinematic viscosity $\nu = \mu/\rho$.

The length of the pipe L can be estimated from process equipment layout considerations. The nature of the fluid to be pumped will dictate corrosion constraints on the pipe material. Other considerations are cost and ease of procurement. Based on these, we can select the material of the pipe to be used, and once we do, the roughness ε can be specified. This leaves us with three unspecified parameters, namely the head loss h_f or equivalently, the pressure drop required to pump the fluid Δp, the volumetric flow rate Q (or equivalently the mass flow rate), and the pipe diameter D. Unless we plan to also optimize the cost, two of these must be specified, leaving only a single parameter to be calculated. Thus, pipe flow problems that do not involve cost optimization will fall into three broad categories.

1. Given D and Q, find the head loss h_f

2. Given D and h_f, find the volumetric flow rate Q

3. Given Q and h_f, find the diameter D

Each of these three types of problems is illustrated next with a numerical example.

Example 1

Find the head loss due to the flow of 1,500 gpm of oil ($\nu = 1.15 \times 10^{-4}\ ft^2/s$) through 1,600 feet of 8" diameter cast iron pipe. If the density of the oil $\rho = 1.75\ slug/ft^3$, what is the power to be supplied by a pump to the fluid? Find the BHP of the pump if its efficiency is 0.85.

Solution

We have the following information.

$\rho = 1.75\ slug/ft^3$ $\qquad \nu = 1.15 \times 10^{-4}\ ft^2/s$ $\qquad D = 0.667\ ft$

Therefore, the cross-sectional area is

$$A = \pi D^2/4 = \pi \times (0.667\ ft)^2/4 = \boxed{0.349\ ft^2}$$

$$Q = 1500\,(gpm) \times \frac{1\left(ft^3/s\right)}{448.8(gpm)} = \boxed{3.34\ \frac{ft^3}{s}}$$

Therefore, the average velocity through the pipe is $V = \dfrac{Q}{A} = \dfrac{3.34\left(ft^3/s\right)}{0.349\left(ft^2\right)} = \boxed{9.58\ \dfrac{ft}{s}}$

We can calculate the Reynolds number.

$$\mathrm{Re} = \frac{DV}{\nu} = \frac{0.667\,(ft) \times 9.58\,(ft/s)}{1.15 \times 10^{-4}\left(ft^2/s\right)} = \boxed{5.55 \times 10^4}$$ Therefore, the flow is turbulent.

For cast iron, $\varepsilon = 8.5 \times 10^{-4}\ ft$. Therefore, the relative roughness is

$$\frac{\varepsilon}{D} = \frac{8.5 \times 10^{-4}\,(ft)}{0.667\,(ft)} = \boxed{1.27 \times 10^{-3}}$$

Because we have the values of both the Reynolds number and the relative roughness, it is efficient to use the Zigrang-Sylvester equation for a once-through calculation of the turbulent flow friction factor.

$$\frac{1}{\sqrt{f}} = -4.0 \log_{10}\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}} \log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right]$$

$$= -4.0 \log_{10}\left[\frac{1.27 \times 10^{-3}}{3.7} - \frac{5.02}{5.55 \times 10^4} \log_{10}\left(\frac{1.27 \times 10^{-3}}{3.7} + \frac{13}{5.55 \times 10^4}\right)\right] = 12.8$$

which yields $\boxed{f = 0.00612}$

The head loss is obtained by using

$$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right) = 2\times 0.00612\times\frac{1,600\,(ft)}{0.667\,(ft)}\times\frac{(9.58\ ft/s)^2}{32.2\left(ft/s^2\right)} = \boxed{83.7\ ft}$$

The mass flow rate is $\dot{m} = \rho\,Q = 1.75\left(\frac{slug}{ft^3}\right)\times 3.34\left(\frac{ft^3}{s}\right) = \boxed{5.85\,\frac{slug}{s}}$

The power supplied to the fluid is calculated from

$$\text{Power to Fluid} = \dot{m}\,h_f\,g = 5.85\left(\frac{slug}{s}\right)\times 83.7\,(ft)\times 32.2\left(\frac{ft}{s^2}\right) = \boxed{1.58\times 10^4\ \frac{ft\bullet lb_f}{s}}$$

We know that $1\,\text{HorsePower} = 550\,\frac{ft\bullet lb_f}{s}$. Therefore, $\text{Power to Fluid} = \boxed{28.7\ hp}$

The efficiency of the pump $\eta = 0.85$. Therefore,

$$\text{Brake Horse Power} = \frac{\text{Power to Fluid}}{\eta} = \frac{28.7\,(hp)}{0.85} = \boxed{33.7\ hp}$$

Example 2

Water at $15^\circ\,C$ flows through a $25-cm$ diameter riveted steel pipe of length $450\,m$ and roughness $\varepsilon = 3.2\,mm$. The head loss is known to be $7.30\,m$. Find the volumetric flow rate of water in the pipe.

Solution

For water at $15^\circ\,C$, $\rho = 999\,kg/m^3$ $\mu = 1.16\times 10^{-3}\ Pa\bullet s$ so that the kinematic viscosity can be calculated as $\nu = \mu/\rho = \boxed{1.16\times 10^{-6}\ m^2/s}$

The pipe diameter is given as $D = 0.25\,m$, so that the cross-sectional area is

$A = \pi D^2/4 = \pi\times(0.25\,m)^2/4 = \boxed{4.91\times 10^{-2}\ m^2}$

The length of the pipe is given as $\boxed{L = 450\,m}$

We do not know the velocity of water in the pipe, but we can express the Reynolds number in terms of the unknown velocity.

$$\text{Re} = \frac{DV}{\nu} = \frac{0.25\,(m)\times V}{1.16\times 10^{-6}\,(m^2/s)} = \boxed{2.16\times 10^5\,V}$$ where V must be in m/s.

At this point, we do not know whether the flow is laminar or turbulent. Given the size of the pipe and the head loss, it is reasonable to assume turbulent flow and proceed. In the end, we need to check whether this assumption is correct.

Now, we are given the head loss h_f. Let us write the result for h_f in terms of the friction factor.

$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right)$ Substitute the values of known entities in this equation.

$7.30\,(m) = 2\ f \times \left(\frac{450\,(m)}{0.25\,(m)}\right) \times \left(\frac{V^2}{9.81\left(m/s^2\right)}\right)$ This can be rearranged to yield

$\boxed{fV^2 = 1.99\times10^{-2}\ \frac{m^2}{s^2}}$ where V must be in m/s.

Taking the square root, we find $\boxed{\sqrt{f} = \frac{0.141}{V}}$

We can see that the product $\mathrm{Re}\sqrt{f}$ can be calculated, even though we do not know the velocity V.

$$\mathrm{Re}\sqrt{f} = 2.16\times10^5\ V \times \frac{0.141}{V} = \boxed{3.05\times10^4}$$

Given $\varepsilon = 3.2\ mm$, the relative roughness is

$$\frac{\varepsilon}{D} = \frac{3.2\times10^{-3}\ (m)}{0.25\,(m)} = \boxed{1.28\times10^{-2}}$$

Therefore, the entire right side in the Colebrook Equation for the friction factor is known. We can use the Colebrook Equation to evaluate the friction factor in an once-through calculation.

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{\varepsilon/D}{3.7} + \frac{1.26}{\mathrm{Re}\sqrt{f}}\right] = -4.0\log_{10}\left[\frac{1.28\times10^{-2}}{3.7} + \frac{1.26}{3.05\times10^4}\right] = \boxed{9.82}$$

Therefore, the friction factor is $\boxed{f = 0.0104}$

Using $\sqrt{f} = \frac{0.141}{V}$, we can evaluate the velocity as

$V = \frac{0.141\left(m/s\right)}{\sqrt{f}} = \frac{0.141\left(m/s\right)}{0.102} = \boxed{1.39\ m/s}$ so that the volumetric flow rate is obtained as

$$Q = VA = 1.39\ (m/s)\ \times\ 4.91\times10^{-2}\ \left(m^2\right) = \boxed{6.80\times10^{-2}\ \frac{m^3}{s}}$$

We must check the Reynolds number. $\mathrm{Re} = 2.16\times10^5\ V = \boxed{3.00\times10^5}$. This is well over $4,000$ so that we can conclude that the assumption of turbulent flow is correct.

Example 3

Determine the size of smooth 14-gage BWG copper tubing needed to convey $10\ gpm$ of a process liquid of kinematic viscosity $\nu = 2.40 \times 10^{-5}\ ft^2/s$ over a distance of $133\ ft$ at ground level using a storage tank at an elevation of $20\ ft$. You can assume minor losses from fittings in the line to account for $5\ ft$ of head.

In this problem, we are asked to calculate the diameter D of the tube. We are given $\boxed{L = 150\ ft}$ and $Q = 10\ gpm \times \dfrac{1\left(ft^3/s\right)}{448.8\left(gpm\right)} = \boxed{2.23\times 10^{-2}\ \dfrac{ft^3}{s}}$. Given that the storage tank is located at an elevation of $20\ ft$ above ground, we can infer that the available head loss for friction in the flow through the tube is $h_f = \left(20-5\right)\ ft = \boxed{15\ ft}$.

The diameter appears in both the Reynolds number and the result for the head loss in terms of the friction factor. Let us begin with the head loss and write it in terms of the volumetric flow rate, which is known.

$$h_f = 2f\left(\frac{L}{D}\right)\left(\frac{V^2}{g}\right) = 2f\left(\frac{L}{D}\right)\left(\frac{\left(4Q/\pi D^2\right)^2}{g}\right) = f \times \frac{32\ LQ^2}{\pi^2 g D^5}$$

Substituting known entities in this equation, we obtain

$$15\ ft = f \times \frac{32\times 133\left(ft\right)\times\left(2.23\times 10^{-2}\ ft^3/s\right)^2}{\pi^2 \times 32.2\left(ft/s^2\right)\times D^5} = 6.65\times 10^{-3}\ \frac{f}{D^5} \quad \text{so that} \quad \boxed{f = 2.26\times 10^3\ D^5}$$

where D must be in feet.

The Reynolds number can be written as

$$\mathrm{Re} = \frac{4Q}{\pi \nu D} = \frac{4\times 2.23\times 10^{-2}\left(ft^3/s\right)}{\pi \times 2.40\times 10^{-5}\left(ft^2/s\right)\times D} = \boxed{\frac{1.18\times 10^3}{D}}$$ where D must be in feet.

We can make further progress if we assume the type of flow, so that we can use a correlation for the friction factor. It is reasonable in process situations with this flow rate to assume turbulent flow. So, we shall proceed with that assumption, to be verified later when we can calculate the Reynolds number.

It does not matter which correlation we use, because we must solve an implicit equation for the diameter in either case. So, let us use the Colebrook equation because it is simpler. For a smooth tube, the roughness , $\varepsilon = 0$, so that we can set the relative roughness $\varepsilon / D = 0$ in the Colebrook equation to obtain

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{1.26}{\mathrm{Re}\sqrt{f}}\right]$$

In this equation, substitute for both the friction factor and the Reynolds number in terms of the diameter, to obtain

$$\frac{1}{47.5D^{5/2}} = -4.0\log_{10}\left[\frac{1.26}{\left(1.18\times10^{3}/D\right)\times\left(47.5\,D^{5/2}\right)}\right] = -4.0\log_{10}\left[\frac{2.25\times10^{-5}}{D^{3/2}}\right]$$

or

$$D^{-5/2} = -190\log_{10}\left[2.25\times10^{-5}\,D^{-3/2}\right]$$

Solving this equation, we obtain $D = 7.91\times10^{-2}\ ft = \boxed{0.949"}$

A table of standard tubing dimensions for specified nominal diameters and Birmingham Wire Gage (BWG) values can be found in many places. The textbook by Welty et al. provides it as Appendix N. From the table, we find that for 14-gage tubing with an outside diameter of 1", the inside diameter is 0.834". The next higher outside diameter available is $1\frac{1}{4}-\text{inch}$, and for this OD, 14-gage tubing comes with an inside diameter of 1.084". Therefore, we must select one of these two tubes. If we want to be sure to obtain the desired flow rate, we must choose the value that is larger than 0.949" . You may wonder why. Here is an approximate answer.

In turbulent flow, the friction factor $f \propto V^{-a}$, where $0 \le a < 1$. In laminar flow, $f \propto V^{-1}$. In both cases, we can write $fV^2 \propto V^b$ where $b > 0$. Therefore, the head loss from pipe flow friction $h_f = \frac{1}{D} fV^2\left(\frac{2L}{g}\right) \propto \frac{1}{D} fV^2 \propto \boxed{\frac{1}{D}V^b}$

For a fixed volumetric flow rate, as the diameter is increased, V^b decreases and $1/D$ also decreases. Therefore, the head loss decreases for a given volumetric flow rate as the diameter is increased. This means that with a fixed head loss available, we can comfortably achieve the desired flow rate using a suitable valve. On the other hand, if we choose a diameter that is smaller than the calculated value, we would need a larger head available for driving the flow than is available.

Now, let us use the actual inside diameter of the selected tube, $D = 1.084" = 9.03\times10^{-2}\ ft$ to evaluate the Reynolds number of the flow.

$$\mathrm{Re} = \frac{4Q}{\pi \nu D} = \frac{4\times 2.23\times 10^{-2}\ \left(ft^3/s\right)}{\pi\times 2.40\times 10^{-5}\ \left(ft^2/s\right)\times 9.03\times 10^{-2}\ \left(ft\right)} = \boxed{1.31\times 10^4}$$

Therefore, the flow is turbulent as assumed.

The actual friction factor can be calculated from the Zigrang-Sylvester equation.

$$\frac{1}{\sqrt{f}} = -4.0\log_{10}\left[\frac{\varepsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{13}{\mathrm{Re}}\right)\right]$$

$$= -4.0\log_{10}\left[0 - \frac{5.02}{1.31\times 10^4}\log_{10}\left(0 + \frac{13}{1.31\times 10^4}\right)\right] = 11.8$$

yielding $\boxed{f = 0.00724}$

The actual head loss for the desired volumetric flow rate will be

$$h_f = f\times\frac{32\,LQ^2}{\pi^2 g D^5} = 0.00724\times\frac{32\times 133\left(ft\right)\times\left(2.23\times 10^{-2}\ ft^3/s\right)^2}{\pi^2\times 32.2\left(ft/s^2\right)\times\left(9.03\times 10^{-2}\ ft\right)^5} = \boxed{8.03\ ft}$$

which is less than available head of $15\ ft$.

Therefore, we must specify 14-gage, $1\frac{1}{4}-$inch tubing for this application.

Roughness values for Commercial Pipes

These roughness values are given in Table 6.1 from a textbook by White (1). Because of the variation in roughness in these materials depending on the source, the roughness values reported here have uncertainties ranging from $\pm 20\,\%$ for new wrought Iron to $\pm 70\,\%$ for riveted steel. A typical uncertainty in the roughness values can be assumed to be in the range $\pm 30-50\,\%$.

Material	**Condition**	**ft**	**mm**
Steel	Sheet metal, new	1.6×10^{-4}	5×10^{-2}
	Stainless, new	7×10^{-6}	2×10^{-3}
	Commercial, new	1.5×10^{-4}	4.6×10^{-2}
	Riveted	1×10^{-2}	3.0
	Rusted	7×10^{-3}	2.0
Iron	Cast, new	8.5×10^{-4}	2.6×10^{-1}
	Wrought, new	1.5×10^{-4}	4.6×10^{-2}
	Galvanized, new	5×10^{-4}	1.5×10^{-1}
	Asphalted, cast	4×10^{-4}	1.2×10^{-1}
Brass	Drawn, new	7×10^{-6}	2×10^{-3}
Plastic	Drawn tubing	5×10^{-6}	1.5×10^{-3}
Glass		Smooth	Smooth
Concrete	Smoothed	1.3×10^{-4}	4×10^{-2}
	Rough	7×10^{-3}	2.0
Rubber	Smoothed	3.3×10^{-5}	1×10^{-2}
Wood	Stave	1.6×10^{-3}	5×10^{-1}

Reference

1. F.M. White, Fluid Mechanics, 7th Edition, McGraw-Hill, New York, 2011.