# Flow through Packed and Fluidized Beds

## R. Shankar Subramanian Department of Chemical and Biomolecular Engineering Clarkson University

Chemical engineering operations commonly involve the use of packed and fluidized beds. These are devices in which a large surface area for contact between a liquid and a gas (absorption, distillation) or a solid and a gas or liquid (adsorption, catalysis) is obtained for achieving rapid mass and heat transfer, and particularly in the case of fluidized beds, catalytic chemical reactions. You will find a good deal of information about flow through packed and fluidized beds in the book by McCabe, Smith, and Harriott (1) and Perry's Handbook (2). Here, only a brief summary is given. First, let us consider flow through a packed bed.

### **Packed Beds**

A typical packed bed is a cylindrical column that is filled with a suitable packing material. You can learn about different types of packing materials from Perry's Handbook. The liquid is distributed as uniformly as possible at the top of the column and flows downward, wetting the packing material. A gas is admitted at the bottom, and flows upward, contacting the liquid in a countercurrent fashion. An example of a packed bed is an absorber. Here, the gas contains some carrier species that is insoluble in the liquid (such as air) and a soluble species such as carbon dioxide or ammonia. The soluble species is absorbed in the liquid, and the lean gas leaves the column at the top. The liquid rich in the soluble species is taken out at the bottom.

From a fluid mechanical perspective, the most important issue is that of the pressure drop required for the liquid or the gas to flow through the column at a specified flow rate. To calculate this quantity we rely on a friction factor correlation attributed to Ergun. Other fluid mechanical issues involve the proper distribution of the liquid across the cross-section, and developing models of the velocity profile in the liquid film around a piece of packing material so that heat/mass transfer calculations can be made. Design of packing materials to achieve uniform distribution of the fluid across the cross-section throughout the column is an important subject as well. Here, we only focus on the pressure drop issue.

The **Ergun equation** that is commonly employed is given below.

$$f_p = \frac{150}{Re_p} + 1.75$$

Here, the friction factor  $f_p$  for the packed bed, and the Reynolds number  $Re_p$ , are defined as follows.

The various symbols appearing in the above equations are defined as follows.

 $\Delta p$ : Pressure Drop

*L*: Length of the Bed

 $D_p$ : Equivalent spherical diameter of the particle defined by  $D_p = 6 \frac{\text{Volume of the particle}}{\text{Surface area of the particle}}$ 

- $\rho$ : Density of the fluid
- $\mu$ : Dynamic viscosity of the fluid

 $V_s$ : Superficial velocity ( $V_s = \frac{Q}{A}$  where Q is the volumetric flow rate of the fluid and A is the cross-sectional area of the bed)

 $\varepsilon$ : Void fraction of the bed ( $\varepsilon$  is the ratio of the void volume to the total volume of the bed)

Sometimes, we may use the concept of the interstitial velocity  $V_i$ , which is related to the superficial velocity by  $V_i = \frac{V_s}{\varepsilon}$ . The interstitial velocity is the average velocity that prevails in the pores of the column.

Two simpler results, each obtained by ignoring one or the other term in the Ergun equation also are in use. One is the **Kozeny-Carman equation**, used for flow under very viscous conditions.

$$f_p = \frac{150}{Re_p} , \qquad Re_p \le 1$$

The other is the **Burke-Plummer** equation, used when viscous effects are not as important as inertia.

$$f_p = 1.75$$
,  $Re_p \ge 1,000$ 

It is suggested that the student simply use the Ergun equation. There is no need to use these other two approximate results, even though they continue to be reported in textbooks.

Several improvements to the Ergun equation have been suggested since its original appearance. A new combined correlation that includes these changes can be found in Harrison et al. (3).

#### **Fluidized Beds**

A fluidized bed is a packed bed through which fluid flows at such a high velocity that the bed is loosened and the particle-fluid mixture behaves as though it is a fluid. Thus, when a bed of particles is fluidized, the entire bed can be transported like a fluid, if desired. Both gas and liquid flows can be used to fluidize a bed of particles. The most common reason for fluidizing a bed is to obtain vigorous agitation of the solids in contact with the fluid, leading to excellent contact of the solid and the fluid and the solid and the wall. This means that nearly uniform temperatures can be maintained even in highly exothermic reaction situations where the particles are used to catalyze a reaction in the species contained in the fluid. In fact, fluidized beds were used in catalytic cracking in the petroleum industry in the past. The catalyst is suspended in the fluid by fluidizing a bed of catalytic particles so that intimate contact can be achieved between the particles and the fluid. Nowadays, you will find fluidized beds used in catalyst regeneration, solid-gas reactors, combustion of coal, roasting of ores, drying, and gas adsorption operations.



First, we consider the behavior of a bed of particles when the upward superficial fluid velocity is gradually increased from zero past the point of fluidization, and back down to zero. Reference is made to the figure on page 3.

At first, when there is no flow, the pressure drop zero, and the bed has a certain height. As we proceed along the right arrow in the direction of increasing superficial velocity, tracing the path ABCD, at first, the pressure drop gradually increases while the bed height remains fixed. This is a region where the Ergun equation for a packed bed can be used to relate the pressure drop to the velocity. When the point B is reached, the bed starts expanding in height while the pressure drop levels off and no longer increases as the superficial velocity is increased. This is when the upward force exerted by the fluid on the particles is sufficient to balance the net weight of the bed and the particles begin to separate from each other and float in the fluid. As the velocity is increased further, the bed continues to expand in height, but the pressure drop stays constant. It is possible to reach large superficial velocities without having the particles are typically much larger than the largest superficial velocities used.

Now, if we trace our path backward, gradually decreasing the superficial velocity, in the direction of the reverse arrows in the figure, we find that the behavior of the bed follows the curves DCE. At first, the pressure drop stays fixed while the bed settles back down, and then begins to decrease when the point C is reached. The bed height no longer decreases while the pressure drop follows the curve CEO. A bed of particles, left alone for a sufficient length of time, becomes consolidated, but it is loosened when it is fluidized. After fluidization, it settles back into a more loosely packed state; this is why the constant bed height on the return loop is larger than the bed height in the initial state. If we now repeat the experiment by increasing the superficial velocity from zero, we'll follow the set of curves ECD in both directions. Because of this reason, we define the velocity at the point C in the figure as the **minimum fluidization** velocity  $V_f$ . We can calculate it by balancing the net weight of the bed against the upward force exerted on the bed, namely the pressure drop across the bed  $\Delta p$  multiplied by the cross-sectional area of the bed A. In doing this balance, we ignore the small frictional force exerted on the wall of the column by the flowing fluid.

Upward force on the bed =  $\Delta p A$ 

If the height of the bed at this point is L and the void fraction is  $\varepsilon$ , we can write

Volume of particles =  $(1 - \varepsilon)AL$ 

If the acceleration due to gravity is g, the net gravitational force on the particles (net weight) is

Net Weight of the particles =  $(1 - \varepsilon) (\rho_p - \rho_f) ALg$ 

Balancing the two yields

$$\Delta p = (1 - \varepsilon) (\rho_p - \rho_f) Lg$$

By using an expression relating  $\Delta p$  to the superficial velocity, which is the fluidization velocity at this point, we can obtain a result for the latter.

Typically, for a bed of small particles ( $D_p \le 0.1 \text{ mm}$ ), the flow conditions at this stage are such that the Reynolds number is relatively small ( $\text{Re} \le 10$ ) so that we can use the Kozeny-Carman Equation, applicable to the viscous flow regime, for establishing the point of onset of fluidization. This yields

$$V_f = \frac{\left(\rho_p - \rho_f\right)gD_p^2}{150\mu}\frac{\varepsilon^3}{1-\varepsilon}$$

When the superficial velocity  $V_s$  is equal to  $V_f$ , we refer to the state of the bed as one of **incipient fluidization**. The void fraction  $\varepsilon$  at this state depends upon the material, shape, and size of the particles. For nearly spherical particles, McCabe, Smith, and Harriott (1) suggest that  $\varepsilon$  lies in the range 0.40–0.45, increasing a bit with particle size.

For large particles  $(D_p \ge 1 \text{ mm})$ , inertial effects are important, and the full Ergun equation must be used to determine  $V_f$ . When in doubt, use the Ergun equation instead of a simplified version of it.

Now, we consider the condition we must impose on the superficial velocity so that particles are not carried out with the fluid at the exit. This would occur if the superficial velocity is equal to the settling velocity of the particles. Restricting attention to small particles so that Stokes law can be used to calculate their settling velocity, we can write

$$V_{\text{settling}} = \frac{\left(\rho_p - \rho_f\right)gD_p^2}{18\mu}$$

If we now use the result for the minimum fluidization velocity for the case of small particles, given above, we see that the ratio

$$\frac{V_{settling}}{V_f} = \frac{25}{3} \frac{1-\varepsilon}{\varepsilon^3}$$

For  $\varepsilon$  lying in the range 0.40–0.45, this yields a ratio ranging from 78–50. McCabe et al. suggest that it is common to operate fluidized beds at velocities as high as  $30V_f$ , and values as large as  $100V_f$  are used on occasion. Recognizing that not all particles are of the same size and

that  $D_p$  is only an average size, we see that fine particles are likely to be carried out with the exiting fluid in such a situation. They can be recovered by filters or cyclone separators and returned, in order to obtain the benefits of operating a bed at such large superficial velocities.

Fluidization can be broadly classified into **particulate fluidization** or **bubbling fluidization**. Particulate fluidization occurs in liquids. As the velocity of the liquid is increased past the minimum fluidization velocity, the bed expands uniformly, and uniform conditions prevail in the liquid solid mixture. In contrast, bubbling fluidization occurs in gas-fluidized beds. Here, when the bed is fluidized, large pockets of gas, free of particles, are seen to rise through the bed. Where there are particles, the bed void fraction is approximately at the value that prevails at the point of incipient fluidization. The bubbles grow until they fill the cross-section, and then successive bubbles move up the column, a condition known as slugging.

The above classification should not be interpreted rigidly. Sometimes, very dense particles in a liquid can show "bubbling" and gases at high pressure when flowing through beds of fine particles, can give rise to particulate fluidization. Usually, this occurs at lower velocities, and at higher velocities, the bed shows bubbling.

### References

1. W.E. McCabe, J.C. Smith, and P.Harriott 2001. Unit Operations of Chemical Engineering, McGraw Hill, New York.

2. Perry's Chemical Engineers Handbook, 7<sup>th</sup> Edition. 1997 (Ed: R.H. Perry, D.W. Green, and J.O. Maloney), McGraw-Hill, New York.

3. L.D. Harrison, K.M. Brunner, and W.C. Hecker, A combined packed-bed friction factor equation: extension to higher Reynolds number with wall effects, AIChE J., 59, No.3, 703-706 (2013).