

# Dimensional Analysis

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In a typical experiment, we look for how a dependent parameter varies as we change a variety of independent parameters. By independent parameters, we mean those that we can vary conveniently, and the dependent parameter is the one whose behavior we seek to establish. Dimensional analysis permits us to organize the process by which we vary the independent parameters. In fact, it helps us identify the true dependent and independent parameters in a situation; in the process, the number of parameters that we must consider is minimized. But dimensional analysis is not foolproof – we must be careful in considering all possible parameters that can affect the dependent parameter. If we omit a crucial parameter in making the list of independent parameters, dimensional analysis cannot help us find it. We always need common sense and physical intuition in selecting the lists of parameters in a problem.

## Drag on a Sphere

Here, by considering a simple example, I'll show you how to use dimensional analysis. We aim to develop an organized way of examining how the drag on a sphere settling through a fluid varies with the relevant parameters. The first step is to identify the parameters on which the drag  $F_D$  is likely to depend. These are the diameter  $d_p$  of the sphere, the density  $\rho$  and the viscosity  $\mu$  of the fluid, and the settling velocity of the sphere  $V$ .

### Step 1

Make a list of parameters and identify their dimensions using the fundamental dimensions of Mass ( $M$ ) Length ( $L$ ) and Time ( $T$ ). From now on, we'll use the term “variables” to designate these parameters, but recognize that they are parameters in the usual sense of that term.

Variable	Symbol	Dimension
Sphere Diameter	$d_p$	$L$
Fluid Density	$\rho$	$\frac{M}{L^3}$
Settling Velocity	$V$	$\frac{L}{T}$
Fluid Viscosity	$\mu$	$\frac{M}{LT}$
Drag	$F_D$	$\frac{ML}{T^2}$

## Step 2

Identify the repeat variables. There should be as many repeat variables as there are dimensions. These are variables that will, in principle, appear in every dimensionless group that we form. The requirement is that it should not be possible to form a dimensionless group in the form of a product of powers of these variables. The simplest way to meet this requirement is to select the repeat variables so that each has a unique dimension in it. Also, it is helpful to choose repeat variables that are as simple as possible in their dimensions. With this in mind, we choose the first three, namely, the diameter, density, and velocity as our repeat variables, because there are three dimensions in this problem. You can see that density has Mass ( $M$ ) appearing uniquely in it, and velocity has time ( $T$ ) uniquely appearing in it, and the diameter has neither. Therefore, it is not possible to use these three variables to form a dimensionless group.

## Step 3

The number of dimensionless groups is always equal to the number of variables minus the number of repeat variables. Therefore, we can expect to form two dimensionless groups in this problem. The group involving the drag will be the dependent dimensionless group and that involving the viscosity will be the independent dimensionless group. Each group is obtained by forming a product of each repeat variable to an unknown power and then multiplying by one of the remaining variables. If we call these two dimensionless groups  $\Pi_1$  and  $\Pi_2$ , then we might define  $\Pi_1$  as follows.

$$\Pi_1 = \mu d_p^a \rho^b V^c$$

To find the values of  $a, b$ , and  $c$  that would make this group dimensionless, first we write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 = \frac{M}{LT} \times L^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{L}{T}\right)^c = M^{1+b} L^{-1+a-3b+c} T^{-1-c}$$

Now, match the powers of  $M, L$ , and  $T$  on both sides. This yields three algebraic equations for the exponents  $a, b$ , and  $c$  as follows.

$$1 + b = 0$$

$$-1 + a - 3b + c = 0$$

$$-1 - c = 0$$

The solution of these equations is straightforward, and the result is  $a = b = c = -1$ . Therefore,

$$\Pi_1 = \frac{\mu}{d_p \rho V}$$

You can verify that the group in the right side of the above result is indeed dimensionless. Now, there is nothing in dimensional analysis that prevents us from using powers of this group as a dimensionless group in which these four variables appear. It is conventional to invert this group and call it the Reynolds number.

So, let us invent a new dimensionless group to replace  $\Pi_1$ .

Reynolds Number $\text{Re} = \frac{d_p V \rho}{\mu}$
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The Reynolds number is the most important organizing principle in fluid mechanics. It represents the ratio of a characteristic inertial force in a flow to a characteristic viscous force. The transition from laminar to turbulent flow in a confined geometry such as a pipe, or in a boundary layer, is governed by the magnitude of the Reynolds number associated with that flow. In the present example of flow over a sphere, it is known that inertia effects play a negligible role in influencing the drag when the Reynolds number is much smaller than unity.

Next, let us find the dimensionless group that includes the dependent variable  $F_D$ .

$$\Pi_2 = F_D d_p^a \rho^b V^c$$

Again, we write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 = \frac{ML}{T^2} \times L^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{L}{T}\right)^c = M^{1+b} L^{1+a-3b+c} T^{-2-c}$$

As before, matching the powers of  $M$ ,  $L$ , and  $T$  on both sides yields three algebraic equations for the exponents  $a$ ,  $b$ , and  $c$ .

$$1+b=0$$

$$1+a-3b+c=0$$

$$-2-c=0$$

The solution is  $a = -2$ ,  $b = -1$ ,  $c = -2$ . Therefore,

$$\Pi_2 = \frac{F_D}{d_p^2 V^2 \rho}$$

In practice, a dimensionless drag known as the Drag Coefficient,  $C_D$ , is defined as follows.

$$C_D = \frac{8F_D}{\pi\rho d_p^2 V^2}$$

The reason for the multiplicative factor  $(8/\pi)$  is that the drag coefficient is defined as the drag divided by the product of the projected area of the sphere and the velocity head. Thus,

$$C_D = \frac{F_D}{\frac{\pi}{4}d_p^2 \times \frac{1}{2}\rho V^2} = \frac{8F_D}{\pi\rho d_p^2 V^2}$$

Therefore, dimensional analysis tells us that the drag coefficient is a universal function of Reynolds number, regardless of the choice of fluid, sphere diameter or the settling velocity.

$$\boxed{C_D = \phi(\text{Re})}$$

The nature of the function  $\phi$  has to be established by experiments or theory as appropriate. The universality means that when we plot the drag coefficient against the Reynolds number, we obtain a single curve, regardless of the choice of sphere diameter, fluid, and the velocity at which the sphere moves through the fluid. This is a remarkable result, because it permits us to use any fluid(s) of our choice to carry out the experiments to find the universal relationship between these two dimensionless variables, and once determined, use it to infer the drag on any sphere settling through any fluid at any velocity. You can see how powerful a tool this is.

Figure 12.4 in the textbook by Welty et al. shows the universal curve that results for the drag on a sphere as a function of the Reynolds number.

## **Pressure drop for steady flow of an incompressible Newtonian fluid through a pipe**

Now, let us consider a second example. Suppose we wish to perform experiments on how the pressure drop for achieving the steady flow of an incompressible Newtonian fluid through a pipe varies depends on various parameters. Dimensional analysis helps us determine the relevant dimensionless groups so that we can obtain a maximum amount of information with the smallest number of experiments. As in the previous example, we proceed through the various steps in an organized manner.

### **Step 1**

Make a list of “variables” and identify their dimensions using the fundamental dimensions of Mass ( $M$ ) Length ( $L$ ) and Time ( $T$ ).

Variable	Symbol	Dimension
Diameter of the Pipe	$D$	$L$
Length of the Pipe	$L_{pipe}$	$L$
Fluid Density	$\rho$	$\frac{M}{L^3}$
Average Velocity	$V$	$\frac{L}{T}$
Fluid Viscosity	$\mu$	$\frac{M}{LT}$
Pressure drop	$\Delta P$	$\frac{M}{LT^2}$

## Step 2

Identify the repeat variables. Recall that there should be as many repeat variables as there are dimensions and that these are variables that will, in principle, appear in every dimensionless group that we form. Therefore, we require that it should not be possible to form a dimensionless group in the form of a product of powers of these variables. Let us choose the diameter, density, and velocity as our repeat variables, because there are three dimensions in this problem. If you choose the length of the pipe instead of the diameter, it would be fine, but the dimensionless groups you will obtain will be a bit unconventional. Note that density has the dimension of mass unique to it, and the velocity has the dimension of time unique to it, so that one cannot form a dimensionless group using the density, velocity, and the diameter of the pipe.

## Step 3

The number of dimensionless groups is always equal to the number of variables minus the number of repeat variables. We have six variables and three repeat variables. Therefore, we can expect to form three dimensionless groups in this problem. The group involving the pressure drop will be the dependent dimensionless group, and we shall obtain it last.

As before, each group is obtained by forming a product of each repeat variable to an unknown power and then multiplying by one of the remaining variables. Let us define the first dimensionless group  $\Pi_1$  as follows.

$$\Pi_1 = L_{pipe} D^a \rho^b V^c$$

To find the values of  $a$ ,  $b$ , and  $c$  that would make this group dimensionless, first we write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 = L \times L^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{L}{T}\right)^c = M^b L^{1+a-3b+c} T^{-c}$$

Now, match the powers of  $M, L$ , and  $T$  on both sides. This yields three algebraic equations for the exponents  $a, b$ , and  $c$  as follows.

$$b = 0$$

$$1 + a - 3b + c = 0$$

$$-c = 0$$

The solution of these equations is straightforward, and the result is  $a = -1$ ,  $b = c = 0$ . Therefore,

$$\boxed{\Pi_1 = \frac{L_{pipe}}{D}}$$

Now, let us proceed to find the second dimensionless group  $\Pi_2$  by including the viscosity as the fourth variable.

$$\Pi_2 = \mu D^a \rho^b V^c$$

To find the values of  $a, b$ , and  $c$  that would make this group dimensionless, first we write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 = \frac{M}{LT} \times L^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{L}{T}\right)^c = M^{1+b} L^{-1+a-3b+c} T^{-1-c}$$

Match the powers of  $M, L$ , and  $T$  on both sides. This yields three algebraic equations for the exponents  $a, b$ , and  $c$  as follows.

$$1 + b = 0$$

$$-1 + a - 3b + c = 0$$

$$-1 - c = 0$$

The solution of these equations is straightforward, and the result is  $a = b = c = -1$ . Therefore,

$$\Pi_2 = \frac{\mu}{D \rho V}$$

As in the first example, it is conventional to invert this group. So, we shall replace  $\Pi_2$  with its inverse, namely the Reynolds number.

Reynolds Number $Re = \frac{D\rho V}{\mu}$
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For a straight circular pipe, when the Reynolds number is less than approximately 2,300, the flow is laminar. The flow undergoes transition to turbulence in the range of Reynolds numbers from 2,300 to 4,000, and can be considered turbulent for Reynolds number equal to or greater than approximately 4,000.

Next, let us find the dimensionless group that includes the dependent variable  $\Delta P$ .

$$\Pi_3 = \Delta P D^a \rho^b V^c$$

Writing out the equation in terms of the dimensions of each side, we get

$$M^0 L^0 T^0 = \frac{M}{LT^2} \times L^a \times \left(\frac{M}{L^3}\right)^b \times \left(\frac{L}{T}\right)^c = M^{1+b} L^{-1+a-3b+c} T^{-2-c}$$

Match the powers of  $M$ ,  $L$ , and  $T$  on both sides to obtain the following three algebraic equations for the exponents  $a$ ,  $b$ , and  $c$ .

$$1 + b = 0$$

$$-1 + a - 3b + c = 0$$

$$-2 - c = 0$$

The solution is  $a = 0$ ,  $b = -1$ ,  $c = -2$ . Therefore,

$$\Pi_3 = \frac{\Delta P}{\rho V^2}$$

Dimensional analysis tells us that  $\Delta P / (\rho V^2)$  is some function of the two dimensionless groups  $(L_{pipe} / D)$  and the Reynolds number. It cannot help us determine the actual manner in which this dependence occurs. For determining that, we need to either use analysis or experiments. First, let us explore the dependence on the group  $(L_{pipe} / D)$ . Provided the pipe is sufficiently long for entrance effects to be unimportant, we can see that if we double the length of the pipe, all else being the same, we should expect the pressure drop needed to also double. Therefore, the connection between  $\Delta P / (\rho V^2)$  and  $(L_{pipe} / D)$  must be one of simple proportionality. This means that the ratio of  $\Delta P / (\rho V^2)$  and  $(L_{pipe} / D)$  should be fixed if the Reynolds number is fixed, and can only vary if the Reynolds number is changed. So, we can define a new

dimensionless group, namely,  $\frac{\Delta P}{L_{pipe}} \frac{D}{\rho V^2}$ , which will be a function only of the Reynolds number.

It is conventional to introduce a multiplicative factor of 1/2 in this group, and to drop the subscript on the length of the pipe, simply designating it as  $L$  to define the so-called Fanning Friction Factor  $f$ , as

$$f = \frac{1}{2} \frac{\Delta P}{L} \frac{D}{\rho V^2}$$

Thus, dimensional analysis, combined with some common sense thinking about how the pressure drop varies with the length of a tube of given diameter leads us to the following inference.

$$f = \phi(\text{Re})$$

The actual dependence of the friction factor on the Reynolds number must be established by experiment or theory or a combination thereof. As it happens, there is one more dimensionless group that influences the friction factor. If the interior wall of the pipe is not absolutely smooth, it is found that the friction factor is not the same as in a smooth pipe if the Reynolds number is sufficiently large to cause turbulent flow. After some investigation, scientists established that if we define the average height of the roughness of the interior surface of the pipe as  $\varepsilon$ , then the relative roughness  $\varepsilon / D$  affects the values of the friction factor in turbulent flow. If the flow is laminar, then the friction factor is independent of  $\varepsilon / D$ .

Figure 13.1 in the textbook by Welty et al. shows the curves that result when the Fanning friction factor is plotted against the Reynolds number. Note that logarithmic axes are used, because both groups vary over a wide range in practice. The importance of dimensional analysis is to show that regardless of the nature of the fluid, the material of the pipe, and the actual value of the velocity, all that matter are the values of two dimensionless groups, namely, the relative roughness and the Reynolds number. These two values uniquely establish the value of the friction factor, from which one can calculate the pressure drop needed. Incidentally, Welty et al. suggest that turbulent flow is achieved by the time the Reynolds number exceeds a value of 3,000, but it is generally accepted that one requires a Reynolds number of 4,000 to be able to use friction factor correlations developed for turbulent flow. It is best not to operate in the transition regime  $2,300 \leq \text{Re} \leq 4,000$ , because of the uncertainty in the predicted friction factor values.

Chemical Engineers use the Fanning friction factor we have defined here. There is another friction factor that is widely used by Mechanical and Civil Engineers. It is called the Darcy friction factor, and it is four times as large as the Fanning friction factor. Whenever you use the friction factor from a correlation or a graph to calculate the pressure drop, you need to check and make sure that you are using the correct relationship with the pressure drop for that friction factor.



## Steady Heat Transfer to a Fluid Flowing Through a Circular Pipe

Now, let us consider an example in heat transfer. We wish to perform experiments on steady heat transfer to a fluid that is in steady flow through a circular pipe. Our objective is to find the average heat transfer coefficient as a function of the various parameters in the system. The new twist is the appearance of temperature differences in variables such as thermal conductivity and the specific heat, which are properties relevant in determining heat transfer rates. This requires that we introduce a fourth dimension  $\theta$  to stand for temperature or temperature differences, besides the original three dimensions of Mass ( $M$ ) Length ( $L$ ) and Time ( $T$ ). Except for this change, we proceed exactly as we did in the earlier examples.

We can expect the average heat transfer coefficient to depend on the dimensions of the tube, the density, specific heat, and thermal conductivity of the fluid, and the average velocity of the fluid. While not immediately obvious, it also will depend on the viscosity of the fluid. This is because the viscosity plays an important role in the fluid mechanics, and the flow in turn plays a crucial role in determining the rate of heat transfer to the fluid.

### Step 1

First, let us make a list of all the variables, along with the symbols and dimensions. We use  $M, L, T$ , and  $\theta$  for Mass, Length, Time, and Temperature Differences, respectively.

Variable	Symbol	Dimension
Diameter of the pipe	$D$	$L$
Length of the pipe	$L_{pipe}$	$L$
Density	$\rho$	$\frac{M}{L^3}$
Velocity	$V$	$\frac{L}{T}$
Viscosity	$\mu$	$\frac{M}{LT}$
Thermal conductivity	$k$	$\frac{ML}{T^3\theta}$
Specific heat	$C_p$	$\frac{L^2}{T^2\theta}$
Average Heat transfer coefficient	$h$	$\frac{M}{T^3\theta}$

## Step 2

Identify the repeat variables. There should be as many repeat variables as there are dimensions. Because we have four fundamental dimensions in this situation, we must choose four repeat variables.

Repeat variables will, in principle, appear in every dimensionless group that we form. The requirement is that it should not be possible to form a dimensionless group in the form of a product of powers of these variables. The simplest way to meet this requirement is to select the repeat variables so that each has a unique dimension in it. Also, it is helpful to choose repeat variables that are as simple as possible in their dimensions. Here, we shall select the diameter ( $D$ ), viscosity ( $\mu$ ), velocity ( $V$ ), and thermal conductivity ( $k$ ) as our repeat variables. You can see that viscosity has Mass ( $M$ ) appearing uniquely in it, and velocity has time ( $T$ ) uniquely appearing in it, thermal conductivity has temperature ( $\theta$ ) uniquely appearing in it, and the diameter has none of these three dimensions in it. Therefore, it is not possible to use these four variables to form a dimensionless group.

## Step 3

The number of dimensionless groups is always equal to the number of variables minus the number of repeat variables. Therefore, we can expect to form 4 dimensionless groups in this problem. The group involving the average heat transfer coefficient will be the dependent dimensionless group and that involving each of the other three variables (length of the pipe, density, and specific heat) will be the independent dimensionless groups. Each group is obtained by forming a product of each repeat variable to an unknown power and then multiplying by one of the remaining variables.

We begin with the group  $\Pi_1$  containing the heat transfer coefficient.

$$\Pi_1 = h D^a V^b \mu^c k^d$$

To find the values of  $a, b, c$ , and  $d$  that would make this group dimensionless, first we write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 \theta^0 = \frac{M}{T^3 \theta} \times L^a \times \left(\frac{L}{T}\right)^b \times \left(\frac{M}{LT}\right)^c \times \left(\frac{ML}{T^3 \theta}\right)^d = M^{1+c+d} L^{a+b-c+d} T^{-3-b-c-3d} \theta^{-1-d}$$

Now, match the powers of  $M, L, T$ , and  $\theta$  on both sides. This yields four algebraic equations for the exponents  $a, b, c$ , and  $d$  as follows.

$$\begin{aligned}
1 + c + d &= 0 \\
a + b - c + d &= 0 \\
-3 - b - c - 3d &= 0 \\
-1 - d &= 0
\end{aligned}$$

Solving these equations, we find  $a = 1, b = c = 0, d = -1$ . Therefore,

$$\boxed{\Pi_1 = \frac{hD}{k}}$$
 This dimensionless group is known as the Nusselt number, abbreviated as  $Nu$ . If we

had chosen  $L_{pipe}$  as a repeat variable, the group would have been  $hL_{pipe}/k$ . In this problem, the key length scale is the diameter, because the important velocity variations occur across the cross-section of the pipe, and not along the length of the pipe. These variations exert a strong influence on the rate of heat transfer between the pipe wall and the fluid. Therefore, when there is a choice for a repeat variable, it is sensible to choose the crucial scale involved, rather than make the choice arbitrarily. In any case, after all the groups have been identified, it would be possible to multiply the groups to any powers we like to form new groups, so the Nusselt number can always be obtained as the dependent dimensionless group in the end.

Now, we must systematically obtain the remaining (independent) dimensionless groups.

Using the density  $\rho$ , define  $\Pi_2 = \rho D^a V^b \mu^c k^d$  and write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 \theta^0 = \frac{M}{L^3} \times L^a \times \left(\frac{L}{T}\right)^b \times \left(\frac{M}{LT}\right)^c \times \left(\frac{ML}{T^3 \theta}\right)^d = M^{1+c+d} L^{-3+a+b-c+d} T^{-b-c-3d} \theta^{-d}$$

As before, matching the powers of  $M, L, T$ , and  $\theta$  on both sides yields four algebraic equations for the exponents  $a, b, c$ , and  $d$ .

$$\begin{aligned}
1 + c + d &= 0 \\
-3 + a + b - c + d &= 0 \\
-b - c - 3d &= 0 \\
-d &= 0
\end{aligned}$$

The solution is  $a = 1, b = 1, c = -1, d = 0$ . Therefore,  $\boxed{\Pi_2 = \frac{\rho D V}{\mu}}$ , and we recognize this as the

Reynolds number  $Re$ .

Continuing with the specific heat  $C_p$ , define  $\Pi_3 = C_p D^a V^b \mu^c k^d$  and write out the equation in terms of the dimensions of each side.

$$M^0 L^0 T^0 \theta^0 = \frac{L^2}{T^2 \theta} \times L^a \times \left(\frac{L}{T}\right)^b \times \left(\frac{M}{LT}\right)^c \times \left(\frac{ML}{T^3 \theta}\right)^d = M^{c+d} L^{2+a+b-c+d} T^{-2-b-c-3d} \theta^{-1-d}$$

Matching exponents, we obtain the set of equations

$$c + d = 0$$

$$2 + a + b - c + d = 0$$

$$-2 - b - c - 3d = 0$$

$$-1 - d = 0$$

and the solution is  $a = b = 0, c = 1, d = -1$ . Therefore,  $\Pi_3 = \frac{C_p \mu}{k}$ , which is the Prandtl number  $\text{Pr}$ .

The only remaining variable is the length of the pipe  $L_{\text{pipe}}$ . We can go through the formal procedure to find the dimensionless group  $\Pi_4$ , but by inspection it can be seen that it must be

$$\Pi_4 = \frac{L_{\text{pipe}}}{D}.$$

Therefore, dimensional analysis reveals that the Nusselt number must be a function of the Reynolds number, the Prandtl number, and an aspect ratio  $L_{\text{pipe}} / D$ .

$$\text{Nu} = \phi \left( \text{Re}, \text{Pr}, \frac{L_{\text{pipe}}}{D} \right)$$

When we perform experiments by varying the three dimensionless parameters on which the Nusselt number depends, we find that for relatively long pipes, the Nusselt number based on the average heat transfer coefficient depends only weakly on the group  $L_{\text{pipe}} / D$ . This is indicative that the local heat transfer coefficient, which varies with position along the pipe, settles into a constant asymptotic value beyond a certain distance along the pipe. This “entrance length” for heat transfer conditions to reach an asymptotic state depends on whether the flow in the tube is laminar or turbulent. If we assume the flow to be fully-developed for the sake of simplicity, in laminar flow the only mechanism for radial transport of heat is conduction. This leads to long entrance lengths. In contrast, in turbulent flow, eddy transport, which permits rapid transport of heat across the cross-section, leads to a great shortening of the entrance length.

The local heat transfer coefficient is largest near the inlet, and decreases to its asymptotic value at some distance from the inlet. Therefore, in laminar flow heat transfer, it is best to use a short heat exchanger to take advantage of the relatively large heat transfer coefficients that prevail near the inlet. In turbulent flow, the thermal entrance length is usually very short. Therefore, virtually all of the heat transfer in turbulent flow occurs in the asymptotic region wherein the

heat transfer coefficient is constant. Thus, you'll not find the ratio  $L_{pipe} / D$  appearing in a typical turbulent flow heat transfer correlation.

One aspect of heat transfer that we have neglected to consider is that physical properties depend on the temperature. Therefore, they vary along the pipe and across the cross-section. One usually selects average values between the inlet and the outlet and the wall and the fluid for use in calculating the dimensionless groups in the correlations. This usually leads to reasonably good estimate of the heat transfer coefficient. But, there is one property that depends strongly on temperature that exerts an additional influence that must be considered. It is the fluid viscosity. The viscosity varies rapidly with changing temperature, especially in liquids, but the variation is not negligible even in gases. You may recall that the viscosity of a liquid typically decreases as the temperature is increased, while that of a gas increases with increasing temperature.

The variation of viscosity across the cross-section of a pipe introduces a correction to the heat transfer coefficient that must be accommodated. Therefore, heat transfer correlations usually include a ratio of the viscosity of the fluid at the wall temperature to that at the bulk average temperature as another dimensionless group, besides the Reynolds and Prandtl numbers.

## Mass Transfer

Finally, a word about the analogous mass transfer situation. Dimensional analysis of a similar mass transfer problem is very similar. Instead of thermal conductivity and specific heat, we use the mass diffusivity  $D_{AB}$ , and instead of the heat transfer coefficient, we use a mass transfer coefficient. If the mass transfer coefficient  $k_c$  is based on a concentration difference driving force, it is easily established that the dimensions of that mass transfer coefficient are  $(L/T)$ . For mass transfer, the analog of the Nusselt number in heat transfer is the Sherwood Number  $Sh$  defined as

$$Sh = \frac{k_c D}{D_{AB}}$$

Dimensional analysis then reveals that

$$Sh = \phi \left( \text{Re}, Sc, \frac{L_{pipe}}{D} \right)$$

where the Schmidt number  $Sc$  plays a role in mass transfer that is precisely analogous to that of the Prandtl number in heat transfer.

$$Sc = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}}$$

Here  $\nu = \mu / \rho$  is the kinematic viscosity of the fluid.