Convective Mass Transfer

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We already have encountered the mass transfer coefficient, defined in a manner analogous to the heat transfer coefficient. It is a parameter that is used to describe the ratio between the actual mass (or molar) flux of a species into or out of a flowing fluid and the driving force that causes that flux. For example, if a liquid flows over a solid surface that is dissolving in the liquid, one might write

$$\boxed{N_A = k_c \left(c_{A,s} - c_{A,\infty} \right) = k_c \, \Delta c_A}$$

where $c_{A,s}$ is the concentration of the solute A in the liquid in contact with the solid surface, which is assumed to be the equilibrium concentration or solubility, and $c_{A,\infty}$ is the concentration of A in the liquid far from the solid surface. Here, k_c is defined as the mass transfer coefficient in this situation, based on a concentration driving force. It is possible to define a mass transfer coefficient in the same situation using a mole fraction driving force.

$$\boxed{N_A = k_x \left(x_{A,s} - x_{A,\infty} \right) = k_x \, \Delta x_A}$$

Given the geometry, the fluid and flow conditions, and the prevailing thermodynamic conditions, the molar flux must be the same, regardless of the type of driving force used. Thus, in this example, the two mass transfer coefficients are related to each other through

$$k_x \, \Delta x_A = k_c \, \Delta c_A$$

We define the mole fraction $x_A = c_A / c$, where c is the total molar concentration of the mixture. Thus, $\Delta x_A = \Delta c_A / c$. Substituting in the above result yields the connection between the two mass transfer coefficients.

$$\boxed{k_x = c k_c}$$

Mass transfer coefficients depend on the relevant physical properties of the fluid, the geometry used along with relevant dimensions, and the average velocity of the fluid if we are considering flow in an enclosed conduit, or the approach velocity if the flow is over an object. Dimensional analysis can be used to express this dependence in dimensionless form. The dimensionless version of the mass transfer coefficient is the **Sherwood number** Sh.

$$Sh = \frac{k_c D}{D_{AB}}$$

where D is a characteristic length scale in the problem, such as the diameter of a tube through which fluid flows, or the diameter of a sphere or cylinder over which fluid flows. In terms of the mass transfer coefficient k_x, we define the Sherwood number as $Sh = k_x D / (c D_{AB})$.

It can be shown in a relatively straightforward manner that in typical mass transfer problems, the Sherwood Number depends on two important dimensionless groups. One is the Reynolds number Re, and the other is the Schmidt number Sc.

$$\boxed{Re = \frac{DV\rho}{\mu}} \qquad \boxed{Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}}$$

The symbol V represents the average velocity of flow in a tube, and the approach velocity if the flow is over a flat plate or an object such as a sphere, ρ is the density of the fluid, μ is the dynamic viscosity, and ν is the kinematic viscosity. We can write

$$Sh = \phi(Re, Sc, \cdots)$$

where the function ϕ needs to be determined from experimental data or by analysis or a combination of both, and the ellipses $(\cdots)$ represent additional dimensionless parameters such as the aspect ratio L/D where L is the length of the tube. We already are familiar with the Reynolds number, but the Schmidt number is a new dimensionless group that we need to discuss at this stage.

Schmidt number

The Schmidt number plays a role in mass transfer that is analogous to that played by the Prandtl number in heat transfer. From its definition, we can infer a physical significance.

$$Sc = \frac{\nu}{D_{AB}} = \frac{\text{Ability of a fluid to transport momentum by molecular means}}{\text{Ability of that fluid to transport species by molecular means}}$$

In gases, molecular transport of momentum and species occur by similar means, namely, by the random movement of molecules moving from one place to another. While some momentum is transmitted through molecular interactions when two molecules come close to each other, the major contribution is from the movement of molecules themselves, which is the only mechanism for species transport by molecular means. Therefore, Schmidt numbers in gases are typically of the order unity. In contrast, in a liquid, molecules are packed closely together, and diffusion is slow, as we know from the order of magnitude of diffusivities in liquids when compared with the order of magnitude of diffusivities in gases. On the other hand, momentum is efficiently transmitted in liquids through molecular interactions with each other. Therefore, Schmidt numbers in liquids are typically three orders of magnitude larger than those in gases.

$$Sc_{liquid} \sim O(10^3)$$

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When mass transfer occurs from a solid surface to a fluid flowing past it, a concentration boundary layer is formed along the solid surface, just like the momentum boundary layer. The sketch given below illustrates the case for a liquid with a large Schmidt number.
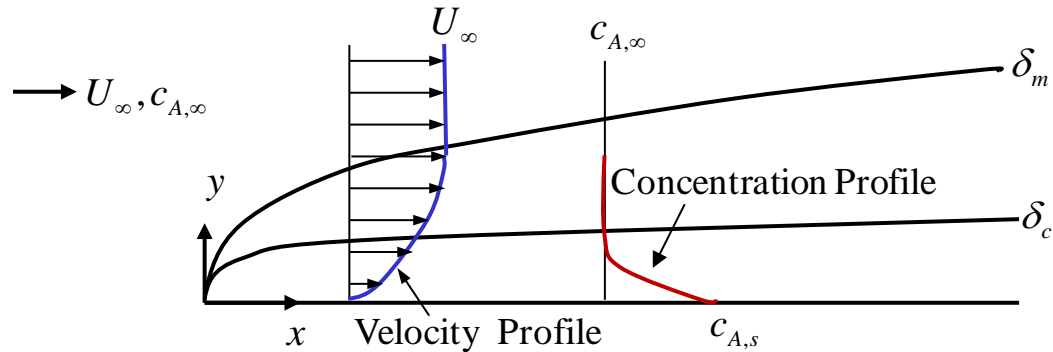
Just as the velocity changes in magnitude from zero at the solid surface to U_∞ at the edge of the momentum boundary layer, the concentration of the dissolving species A changes from $c_{A,s}$ at the solid surface, which is the equilibrium solubility at the prevailing temperature, to $c_{A,\infty}$, the concentration of the solute in the incoming stream, at the edge of the concentration boundary layer. Note that when the Schmidt number is large, momentum is transported by molecular means across a liquid much more effectively than species. This is why the concentration boundary layer is relatively thin, when compared with the momentum boundary layer. In a gas, the two boundary layers would be of comparable thickness.

A detailed analysis of this mass transfer problem for a laminar boundary layer leads to the following correlation for the average Sherwood number over the length of a plate, L.

$$Sh_L = \frac{k_{c,average}\, L}{D_{AB}} = 0.664\, \text{Re}_L^{1/2}\, Sc^{1/3}$$

Here, $\text{Re}_L = LV/\nu$ is the Reynolds number based on the length of the plate as the length scale. If you examine the above correlation for the average Sherwood number, you'll see that it is identical to the correlation provided for heat transfer in laminar boundary layer flow over a flat plate. This analogy, of course, holds only when the motion arising from diffusion can be neglected, that is, when the solution is dilute. This is a restriction that we must impose on the above result.

Analogies among mass, momentum, and energy transfer

The example problem of mass transfer in laminar boundary layer flow over a flat plate points to the possibility that a heat transfer experiment can be used to predict mass transfer performance, when the mass transfer problem involves a dilute solution. Earlier, we learned about the $j-$ factor introduced by Colburn (1) in the context of heat transfer.

$$j_H = \frac{f}{2} = \frac{h}{\rho C_p V} \Pr^{2/3}$$

The Colburn analogy permits the prediction of heat transfer performance from friction factor results. A similar analogy between heat and mass transfer was proposed by Chilton and Colburn (2). This analogy has been used widely by chemical engineers in mass transfer, even though predictions obtained using it are not always very good. Chilton and Colburn defined a j – factor for mass transfer j_D as

$$j_D = \frac{k_c}{V} Sc^{2/3}$$

and suggested that $j_D = j_H$, so that

$$\frac{h}{\rho C_p V} \Pr^{2/3} = \frac{k_c}{V} Sc^{2/3}$$

For an external flow, the velocity $V = U_\infty$, the approach velocity. For flow inside a conduit, V is the average velocity across the cross-section of the conduit.

The Dittus-Boelter correlation for turbulent heat transfer in a circular tube can be extended to mass transfer in a dilute system by setting $j_D = j_H$. This led Linton and Sherwood to correlate data for turbulent mass transfer in a tube of diameter D using

$$Sh = \frac{k_c D}{D_{AB}} = 0.023 \, \mathrm{Re}^{0.8} \, Sc^{1/3}$$

Compare this with the Dittus-Boelter correlation that we encountered earlier

$$Nu = \frac{hD}{k} = 0.023 \, \mathrm{Re}^{0.8} \, \Pr^{n}$$

where the exponent n is adjusted depending on whether the fluid is being heated or cooled in the tube, to accommodate cross-sectional variations in the fluid viscosity arising from temperature variations. You will see that they are essentially the same correlation that is being applied to heat and mass transfer situations. You can learn more about the analogies among momentum, heat, and mass transfer from Section 28.6 of the textbook by Welty et al. (3).

As we noted, the analogy between heat and mass transfer is good only when mass transfer occurs in a dilute system in which the role of convection caused by diffusion is negligible. It is possible, however, to correct the mass transfer coefficient obtained in the dilute case so that it is approximately applicable to the case when the convection caused by diffusion is significant. To learn more about this subject, you can read Section 22.8 in Bird et al. (4).

Lewis-Whitman Two-Film model

A two-resistance model was originally proposed by Whitman (5) in 1923 and introduced as the two-film model for interphase mass transfer by Lewis and Whitman (6) in 1924. Lewis and Whitman postulated nearly motionless "films" on the two sides of the interface. To quote from their article: "It is now becoming generally recognized that wherever a liquid and a gas come into contact there exists on the gas side of the interface a layer of gas in which motion by convection is slight compared to that in the main body of the gas, and that similarly on the liquid side of the interface there is a surface layer of liquid which is practically free from mixing by convection. This phenomenon is frequently expressed by assuming the existence of stationary films of gas and liquid respectively on the two sides of the interface." As we know from having studied fluid mechanics earlier in this course, this is not true. The gas as well as the liquid at the interface are both in motion, and cannot be considered to be stationary. On the other hand, both the gas and liquid at the interface move at the same velocity. Furthermore, the regions of sharp concentration changes adjoining each interface (called films by Lewis and Whitman) are usually thin, especially on the liquid side, so that one might assume as a simple first approximation that the gas and liquid are both moving at the same velocity as that prevailing at the interface throughout these regions, and mass transfer occurs by molecular diffusion across these regions. This is the concept proposed by Lewis and Whitman in their two-film model, as is made clear by subsequent statements in reference (6). Quoting again from the article: "In the main body of either liquid or gas, except under special conditions which will not be considered here, mixing by convection is so rapid that the concentration of solute in the fluid is essentially uniform at all points… On the other hand, the surface films are practically free from convection currents and consequently any transfer of solute through these films must be effected by the relatively slow process of diffusion. These films, therefore, offer the controlling resistances to transfer of a material from one phase to another." Thus, a fictitious model that uses the ideas presented in their article has proved useful in interpreting data on mass transfer rates in complex geometries such as packed columns. Interestingly, the article by Lewis and Whitman deals with interpretation of data from absorbers in which gas is bubbled through a liquid, and experiments in which gas is passed over a liquid body that is either quiescent or stirred. Nowadays, such systems would be modeled using more sophisticated descriptions. As an aside, Lewis and Whitman (6) use the term "diffusion coefficient per unit area" to describe what we would now call a "mass transfer coefficient."

Now, we shall return to packed columns. A packed column consists of solid particles of a variety of shapes, termed "the packing," that are distributed inside a cylindrical column. Figure 31.4 in the textbook by Welty et al. (3) provides sketches of some common types of packings used. Typically, a packed column is vertical, and liquid flows down the column covering the surface of the packing particles and forming a liquid film on this surface that flows downward. During this downward transit, the liquid meets the gas mixture, which flows upward through the interstitial spaces among the particles of packing. In gas absorption operations, the gas introduced at the bottom of the column is rich in a component that we want to remove from the gas mixture by dissolution into the liquid; this dissolved gas is taken away with the liquid that exits the column at the bottom. The liquid enters the column with none of this dissolved gaseous solute, or only a small concentration of it, so that it can absorb the solute during its passage through the column. Because the rate of mass transfer between the two phases depends on the area of contact, the

liquid is spread around the surfaces of the packing particles to increase the area of contact as much as possible. Packed columns also are used in distillation, ion exchange, and several other important mass transfer operations. You'll learn more about packed columns in a subsequent course CH370: Transfer Process Design. Here, we shall focus on the two-film model that is used to interpret mass transfer data from such equipment. At the outset, it is worth noting that the description of mass transfer rates we shall use does not require the concept of "stationary films" or even the film concept. Thus, the two-film picture of Lewis and Whitman is presented here mainly for historical reasons.

The sketch schematically shows the process of mass transfer between the gas and the liquid at some point inside the column. We are looking at just a small portion of the surface of a packing particle somewhere in the column. As noted earlier, the liquid flows down the surface of a packing particle, shown on the right and magnified to make it look flat.
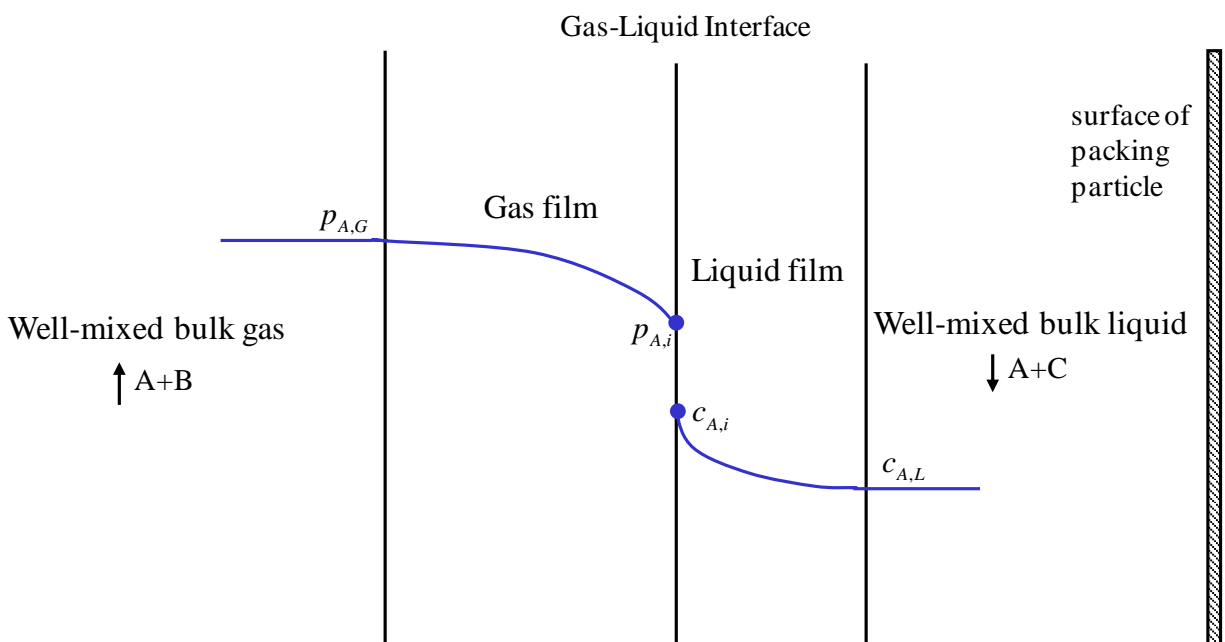
In the two-film model, it is assumed that all the resistance to mass transfer is confined to regions adjoining the interface that are called "films." There is a gas film on the gas-side and a liquid film on the liquid-side in the sketch. The liquid that covers the surface of a packing also is called a film in fluid mechanics, but this should not cause any confusion in the present discussion. As noted earlier, the liquid phase and gas phase "films" adjoining the interface in the model are entirely fictitious, and are invented just to model mass transfer in a simple way. When we say the resistance on each side is confined to the film, what we mean is that outside these films, the fluid is assumed to be completely mixed and at a uniform concentration, labeled the "bulk concentration" at that location. So, at this particular location in the absorption column, the bulk liquid concentration is $c_{A,L}$. Likewise, the partial pressure of the soluble species A in the well-mixed bulk gas phase is $p_{A,G}$. We assume that at the interface between the gas and liquid, thermodynamic equilibrium prevails. What does this mean? Imagine bringing the gas mixture

and the liquid together in a closed container kept at a certain fixed temperature and pressure. Under these conditions, eventually, gas A will saturate the liquid at the prevailing conditions, and there will be a dynamic equilibrium concentration of A in the liquid, which we shall label $c_{A,i}$. This equilibrium concentration corresponds to the partial pressure of A in the gas phase, $p_{A,i}$ at the prevailing temperature. By varying this partial pressure of A at that fixed temperature, we can vary the equilibrium concentration of A that would be obtained in the liquid. These data points are usually plotted in the form of an equilibrium curve, and fitted to a suitable model that describes the compositions in equilibrium.

Concentration of A in the liquid phase
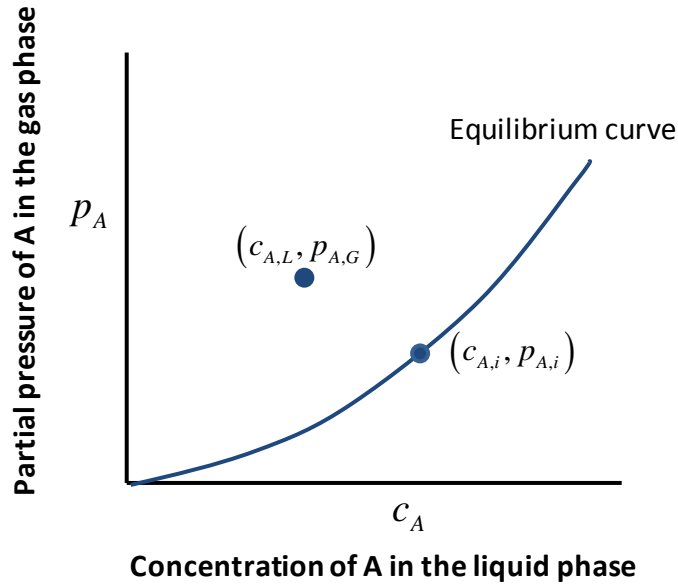
In the textbook by Welty et al. (3), the partial pressure and liquid-phase concentration at points along this curve are related through Henry's law in Figures 29.4 and 29.5, but this is not necessary at this stage. All that matters from our perspective is that points on the equilibrium curve represent the equilibrium concentration of A in the liquid corresponding to the partial pressure of A in the gas. Sometimes, to designate the fact that the two concentrations are in equilibrium, an asterisk is used, as for example, when we use the notation $\left(c_A^*, p_A^*\right)$ to designate a point on the equilibrium curve. Here, we shall assume the interfacial compositions are in equilibrium, and use the subscript i to designate those compositions.

The key idea to grasp is that it is departure from equilibrium that drives mass transfer. Thus, the partial pressure of A in the bulk gas phase must be larger than $p_{A,i}$ for A to diffuse from the bulk to the interface. That is, $\boxed{p_{A,G} > p_{A,i}}$. Likewise, for species A to diffuse from the interface to the bulk liquid phase, there must be a concentration driving force. Therefore, $\boxed{c_{A,i} > c_{A,L}}$. These features are shown in the sketch used here to represent the two-film model. The concentration distribution in the film is not known at this point, and is shown by a suitable curve in each film.

In the equilibrium diagram, I have shown both the equilibrium point $\left(c_{A,i}, p_{A,i}\right)$ and the point representing the compositions in the bulk gas phase and the bulk liquid phase that are in contact at the given location in the absorption column $\left(c_{A,L}, p_{A,G}\right)$. You can see that this latter point must lie to the left of and above the equilibrium curve to satisfy the inequalities we have noted. If the gaseous species A were being removed from the liquid by a gas that contains very little A (a process known as stripping), the driving forces would be in the opposite direction in each phase and this point would then lie to the right of and below the equilibrium curve.

What is the slope of the line joining the two points $\left(c_{A,L}, p_{A,G}\right)$ and $\left(c_{A,i}, p_{A,i}\right)$? It is possible to write this slope in terms of the individual phase mass transfer coefficients at the given location. We can write the molar flux of A in the gas film as

$$N_A = k_G \left(p_{A,G} - p_{A,i}\right)$$

where k_G is the mass transfer coefficient for partial pressure driving forces in the gas phase. We assume steady operation so that this flux N_A remains constant throughout the gas "film." This flux of A has to enter the liquid, because A cannot accumulate at the interface, which has no volume. At steady state, N_A must remain constant throughout the liquid "film." We can write this molar flux of A that crosses over into the liquid phase as

$$N_A = k_L \left(c_{A,i} - c_{A,L}\right)$$

using a liquid-phase driving force. We have adopted the convention that this liquid-phase mass transfer coefficient should be designated k_L to maintain consistency with common notation in absorption problems, even though the designation k_c would be equally acceptable. Equating the results for N_A in the two phases yields
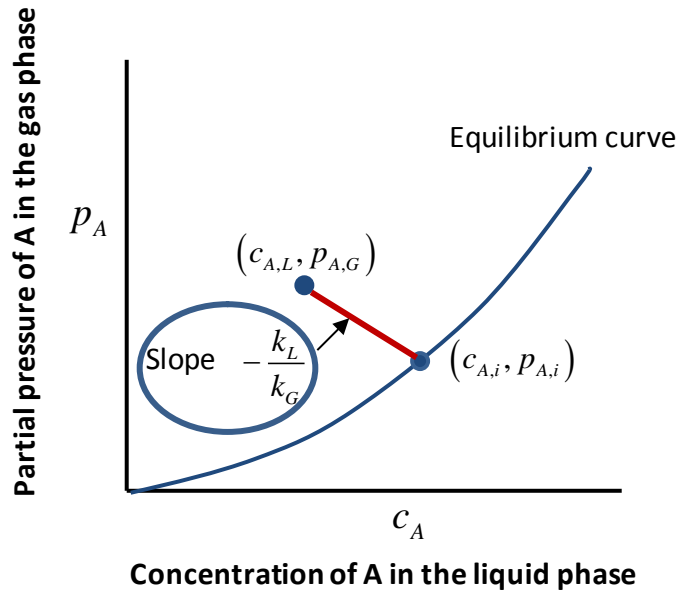
$$N_A = k_G \left(p_{A,G} - p_{A,i}\right) = k_L \left(c_{A,i} - c_{A,L}\right)$$

so that we find the slope of the straight line joining the two points $\left(c_{A,L}, p_{A,G}\right)$ and $\left(c_{A,i}, p_{A,i}\right)$ to be given by

$$\boxed{\frac{p_{A,i} - p_{A,G}}{c_{A,i} - c_{A,L}} = -\frac{k_L}{k_G}}$$

as displayed in the sketch on the following page.
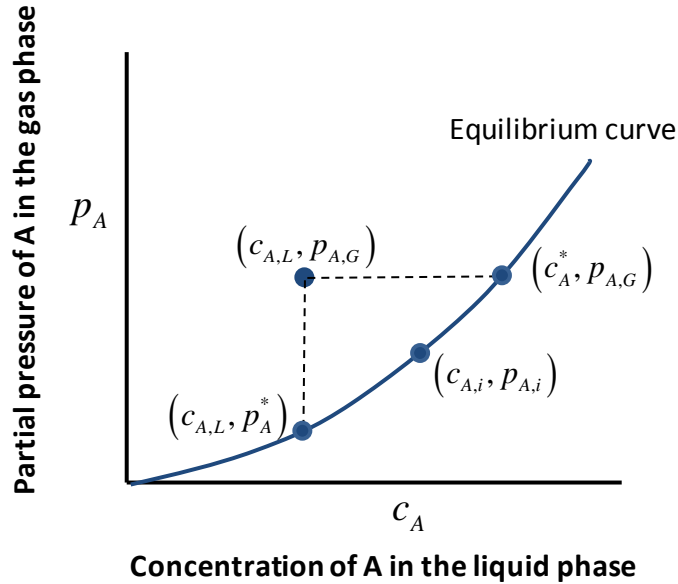
Concentration of A in the liquid phase

Overall mass transfer coefficients

No doubt you remember the concept of the overall heat transfer coefficient. When heat transfer occurs through resistances in series, we employ the overall heat transfer coefficient in the rate equation, along with the overall driving force. The reason for using the overall driving force is usually that it can be measured more easily than the driving forces in individual phases. Likewise, in mass transfer, we cannot expect to know the interfacial compositions at various locations in a column, but the bulk concentrations can be measured. Furthermore, it is the change in these bulk concentrations from one end to the other that is of interest in determining how much of a species has been transferred from one phase to another in a column. Therefore, it would be good to be able to write the molar flux N_A in terms of the difference between the compositions of the two bulk phases. The immediate problem you'll notice is that the compositions are measured in entirely different units in the two phases. This problem will not be resolved even if we use molar concentration units (or mole fractions) in both phases, because in phase I, it would be moles of A per mole of phase I, and likewise in phase II, it would be moles of A per mole of phase II. It would be like mixing apples and oranges to use differences in these concentrations. Thus, we must be creative in coming up with an overall driving force for mass transfer.

Let us examine the equilibrium diagram again, but this time with a couple of additional points marked on it.

Concentration of A in the liquid phase

You'll notice that the additional points are $\left(c_{A,L}, p_A^*\right)$ and $\left(c_A^*, p_{A,G}\right)$. The partial pressure p_A^* is the equilibrium partial pressure corresponding to the bulk liquid phase concentration $c_{A,L}$. Likewise, the concentration c_A^* is the equilibrium concentration in the liquid that would correspond to the partial pressure of A in the bulk gas phase, $p_{A,G}$.

Let us first consider p_A^*. This is a unique partial pressure that can be used as an equivalent gas phase partial pressure corresponding to the bulk liquid phase concentration $c_{A,L}$. Thus, an overall driving force for mass transfer can be written purely in gas phase partial pressures of A as $\left(p_{A,G} - p_A^*\right)$. We can now express the molar flux in terms of this overall driving force for mass transfer and an overall mass transfer coefficient.

$$N_A = K_G\left(p_{A,G} - p_A^*\right)$$

where K_G is termed the "overall mass transfer coefficient based on gas phase driving forces." In an equivalent manner, we also can write

$$N_A = K_L\left(c_A^* - c_{A,L}\right)$$

where K_L is termed the "overall mass transfer coefficient based on liquid phase driving forces." It is fine to use either K_L or K_G to describe the rate of mass transfer. Sometimes, the choice is made based on which resistance dominates. We shall discuss this issue shortly. First, we must find a way to relate these overall mass transfer coefficients to the individual phase mass transfer coefficients we used earlier. For this, we need to have a suitable mathematical connection

between the equilibrium partial pressure and the concentration in the liquid phase. For this purpose, we assume the solutions are dilute, and use Henry's law.

$$p_{A,G} = m\, c_A^*$$

where m is the Henry's law constant. Henry's law assumes that the equilibrium curve is a straight line.

Now, let us work out a connection among K_G, k_G, and k_L. First, we rewrite $N_A = k_G\left(p_{A,G} - p_{A,i}\right)$ as $p_{A,G} - p_{A,i} = \dfrac{N_A}{k_G}$. Likewise, rewrite $N_A = k_L\left(c_{A,i} - c_{A,L}\right)$ as $c_{A,i} - c_{A,L} = \dfrac{N_A}{k_L}$. Now, substitute the equilibrium relationships in this result, replacing $c_{A,i}$ with $p_{A,i}/m$, and $c_{A,L}$ with p_A^*/m to obtain $p_{A,i} - p_A^* = \dfrac{m\, N_A}{k_L}$. Add the two partial pressure differences to obtain

$$p_{A,G} - p_{A,i} + p_{A,i} - p_A^* = p_{A,G} - p_A^* = \frac{N_A}{k_G} + \frac{m\, N_A}{k_L} = N_A\left(\frac{1}{k_G} + \frac{m}{k_L}\right)$$

But we already know that the overall driving force in terms of gas phase partial pressures of A can be written in terms of the molar flux and the overall mass transfer coefficient K_G as

$$p_{A,G} - p_A^* = \frac{N_A}{K_G}$$

Thus, we obtain a connection among the individual mass transfer coefficients and the overall mass transfer coefficient as

$$\boxed{\frac{1}{K_G} = \frac{1}{k_G} + \frac{m}{k_L}}$$

Each term here can be interpreted as a resistance to mass transfer. The left side is the overall resistance; the first term on the right side represents the gas phase resistance, and the second term the liquid phase resistance.

In a completely analogous development, we can obtain the following result for the overall mass transfer coefficient based on liquid phase driving forces.

$$\boxed{\frac{1}{K_L} = \frac{1}{m k_G} + \frac{1}{k_L}}$$

The same interpretation applies to this result. The left side represents the overall resistance to mass transfer, while the first term in the right side stands for the gas phase resistance, and the second term, for the liquid phase resistance.

The following results can be obtained immediately from the above.

$$\frac{\text{resistance to mass transfer in the gas phase}}{\text{total resistance to mass transfer}} = \frac{1/k_G}{1/K_G}$$

$$\frac{\text{resistance to mass transfer in the liquid phase}}{\text{total resistance to mass transfer}} = \frac{1/k_L}{1/K_L}$$

When one or the other resistance dominates, it is called the controlling resistance. As a general rule, for highly soluble gases, the mass transfer process is likely to be gas-phase controlled, whereas for sparingly soluble gases, it might be liquid-phase controlled. These are general statements, with exceptions that can occur, depending on the relative magnitudes of the individual phase mass transfer coefficients and the Henry's law constant. If the gas phase is pure A, then there is no resistance to diffusion in the gas phase, and the entire resistance lies in the liquid phase, that is, $\boxed{1/k_G = 0}$.

Film model

You may have noticed that the properties of the "films" in the gas and liquid phases were never used in the above development, nor was the postulate made by Lewis and Whitman that these films are stationary. In fact, as you can see from the above development, the film model is not necessary for developing rate equations based on overall mass transfer coefficients, nor for relating those overall mass transfer coefficients to the individual phase mass transfer coefficients through the use of an equilibrium relationship and some algebra. But it is usually introduced in this context. Why then do we need the film model?

The film model becomes useful when we need to extrapolate mass transfer information obtained from experiments on dilute systems to mass transfer in concentrated systems wherein the convection caused by diffusion is important, or to multicomponent mass transfer situations in which Fick's law cannot be used, or to mass transfer with chemical reaction. In all these cases, a simple film model in each phase is used to relate the rate of mass transfer to the concentration driving force in that phase and the film thickness. This permits one to relate the fictitious film thickness in a phase directly to the mass transfer coefficient in that phase. Thereafter, experiments that permit the correlation of the Sherwood number, which is the dimensionless version of the mass transfer coefficient in a phase, with groups such as the Reynolds and Schmidt numbers, allow one to infer the way the fictitious film thickness in that phase, made dimensionless using a length scale, is to be correlated with these groups. The film model is then used with the same film thickness to predict mass transfer rates in concentrated systems, multicomponent systems, or chemically reacting systems. This is where the model has predictive capability. You can learn more about this topic from Bird et al. (4).

Penetration model

A key prediction of the film model is that mass transfer rates depend on the first power of the diffusivity. While this is consistent with experimental observations in certain mass transfer situations, other mass transfer systems exhibit a dependence of the rate on the square root of the diffusivity. Higbie (7) appears to have been the first to present a logical picture of mass transfer for short contact times between a gas and a liquid that leads to this square root dependence. Higbie was concerned with situations such as when a gas bubble rises through a liquid, as in bubble absorbers, and those in which a liquid is sprayed as fine drops into a gas, as in spray absorbers. Also, Higbie envisioned that in a packed absorber, the liquid and gas come into contact for short periods when the liquid flows over a single piece of packing. He assumed that subsequently each phase becomes well-mixed before passing on to the next piece of packing. In all these cases, the key idea is that of short contact times between the two phases. Because liquid-phase diffusivities are small, the distance to which the absorbed gas penetrates into the liquid film is small when the contact time is small as well. Within this concentration boundary layer, the fluid velocity can be assumed to be uniform so that the entire region of liquid into which the dissolved gaseous species penetrates moves like a solid body into which unsteady diffusion occurs. Using such ideas, Higbie was able to show that the mass transfer rate in this situation is actually proportional to the square root of the molecular diffusivity of the dissolving species in the liquid. Higbie also performed experiments on long gas bubbles that were forced downward through a vertical glass tube filled with the liquid to verify these predictions. Higbie's model has come to be known as the "penetration model" of mass transfer. In certain mass transfer experiments, it provides a better description of the results than the film model. You can learn more about the film model and the penetration model from Section 28.7 in the textbook by Welty et al. (3).

Mass transfer correlations

Chapter 30 of the textbook by Welty et al. (3) provides a variety of correlations for the Sherwood number for mass transfer to plates, spheres and cylinders, mass transfer in flow through pipes and in wetted-wall columns, mass transfer in packed and fluidized beds, and mass transfer in stirred tanks. You are encouraged to look through this chapter to learn about these correlations that you can use when working on the design of mass transfer equipment. Another good resource for mass transfer correlations is a book by Sherwood, Pigford, and Wilke (8).

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